BASIC MATHEMATICS

FOR

CADASTRAL MAPPING

Chapter 5

2015 Cadastral Mapping Manual

Introduction

Data which a mapper must use to solve problems comes from a myriad of sources both new and old. The general need for the precise location of land did not exist until the 1960’s when the values of land began to increase at a rapid rate. At the present time most land values are 10 - 50 times higher than they were in the early 1960's. Careful licensing of land surveyors in addition to the high technology equipment, ie: G.P.S. equipment, electronic distance meters, computers, etc ... developed for their use, has greatly increased the precision of the land survey. Many legal descriptions which appear on older documents and some newer documents were determined by “self surveys”, (ie. rough measurements of bearings and distances by untrained persons with improper equipment), “table top surveys', (descriptions made up by information from GLO maps, deeds or tax descriptions).

How do mappers handle a set of data? How do they reconcile data from an old inaccurate description with data from a more precise description? How do they calculate acreage of irregular parcels? This section on mathematics will deal with these and other problems commonly encountered by the cadastral mapper.

The old saying, “There are more ways than one to skin a cat,” was never more applicable than it is in solving mathematics problems. Problems and their solutions presented in this section are based on the most concise approach. If the reader wishes to solve the problems with a different approach, that is entirely his/her option as long as sound principles are followed.

Many cadastral mappers have found calculators, to be most valuable in the day to day work routine. If these problems can be solved more efficiently through the use of a calculator, by all means use one. Some Recorder's office mapping departments have graphics computers with good coordinate geometry software which have the ability to solve problems by direct interaction with the drawing on the computer screen. Although a computer is not allowed for the exam, a non programmable calculator may be used on the exam to assist with the mathematical calculations.

Treatment of Data - Significant Figures:

When the length of a line in a legal description is described as being 243.32 feet long, we can assume that the individual doing the measuring had equipment capable of measuring within ±0.01 feet of the absolute length unless otherwise specified. The absolute value of the above measurement lies at some point between 243.31 feet and 243.33 feet. The length of a line given as 100.00 feet is also accurate to within ±0.01 feet even though the numeric place holders are zeros. This number is significant to the nearest hundredth (0.01) of a foot. A measurement reported as 1178.7 feet is only accurate to within ±0.1 feet, significant to the nearest tenth of a foot. 423 is significant to the nearest one foot (±1.0 feet). When adding a group of numbers, the result is only as accurate as the one number of the group having the least significant figures to the right of the decimal.

	Add	372.2
Example:		21.03
		14.637
	Result	407.8~~67~~

The last two digits of this result are insignificant. The result should be rounded off to the nearest tenth (0.1) ie. The result should be rounded to 407.9 because one of the original numbers, 372.2 is the number of the group being added with least significant figures to the right of the decimal.

When multiplying a measured distance by a factor, the resulting answer should only be expressed to as many places to the right of the decimal as the original distance. For example, when multiplying 271.14 feet by 0.29993 the result is 81.32 feet (not 81.32398 ... feet).

In order to preserve accuracy without imposing artificial accuracy, the following rules should be followed when converting between any units of measure. Carry the resulting answer to as many places of accuracy as the smallest of the original units.

This same principle can be applied when dealing with bearings. For example, the bearing N *89°27′* W implies that the angle measured is only accurate to within 1' (one minute) of the reported bearing. However, if the bearing is reported as N 89°42'00" E, then you can assume that this bearing is accurate to within 1" (one second). The mapper should use what information is reported and consider that in their calculations.

Definitions & Formulas

Coordinates: Coordinates are a set of numbers which describe the distance north (or south) and east (or west) from a monument or reference point. Generally north and east distances are considered positive and south and west are considered negative. For the purpose of this text we will label coordinates as ordered pairs as follows:

1. The coordinate of a point north 200 feet and east 50 feet from a monument or reference point will be labeled (E 50, N 200) or (50, 200).

2. For a point south 143 feet and east 270 feet it will be labeled (E 270, S 143) or (270, -143).

*NOTE: South and west coordinates are always considered negative because of their locations on their respective number lines.

In counties set up with state plane coordinates, the coordinates can be expressed as grid coordinates. The convention used for expressing state plane coordinates is a line with the easting on the top of the line and the northing on the bottom as follows:

$$\frac{1{,}945{,}372.43 \quad (E)}{764{,}006.72 \quad (N)}$$

This allows the coordinates to be placed in a smaller space than they would if expressed as ordered pairs.

<u>Course</u>: A course is a unit of measure used in mapping which gives a direction (bearing) and length (distance) of a line.

An example of a course is S 29°18'30” E 21.18 feet. The bearing is S 29°18'30” E and the distance is 21.18 feet. A course may also describe a radius curve, a spiral curve, or an irregular line defined by a canal, stream, river, ridge or gully etc.

Converting degrees, minutes and seconds to decimal degrees.

To perform some calculations in this and the advances Math sections it may be necessary to convert angles from degrees, minutes and seconds to decimal degrees. At a later stage in some problems you will then need to convert angles back from decimal degrees to degrees, minutes and seconds. If you have a calculator which will perform these conversions directly - use it. If not, convert degrees, minutes and seconds to decimal degrees as follows:

Degrees, Minutes and Seconds Conversion		Conversion Example 27°37'14"		Example Converted to Decimal Equivalent	
Degrees	=	27°	=	27°	Deg
$\frac{\text{Minutes}}{60}$	=	$\frac{37}{60}$	=	0.61667	Deg
$\frac{\text{Seconds}}{3600}$	=	$\frac{14}{3600}$	=	0.00389	Deg
Add Above Amounts			Total	27.62056	Deg
27.62056 Degrees is the decimal equivalent of 27°37'14"					

To convert decimal degrees back to degrees, minutes and seconds use the following steps:

1. For the angle expressed in decimal degrees, 48.25667°, take that portion of the number to the left of the decimal - 48. that will be the degrees (48°).

2. Take that portion of the number in the example which lies to the right of the decimal and multiply it by 60.

   0.25667 x 60 = 15.40002

   The portion of the number to the left of the decimal is the minutes (15').

3. Take the portion of the product (in step 2) to the right of the decimal and multiply it by 60.

   0.40002 x 60 = 24.0001

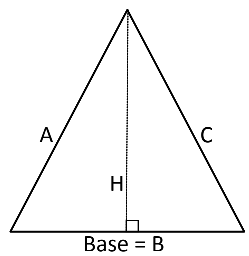
   Round off, the portion of the product to the left of the decimal is the seconds (24”).

The example decimal degrees, 48.25667° converts to 48°15'24”

<u>Traverse</u>: To travel from one point to another. In mapping it means to move along the course of a legal description to define or trace a property boundary.

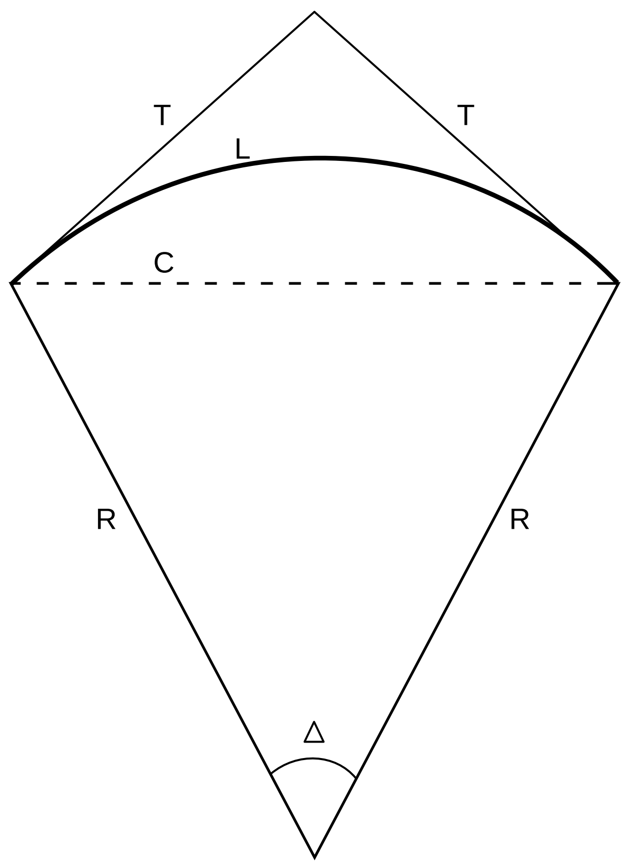
<u>Inverse Traverse</u>: The calculation of bearing and distance between two points of known coordinates. (See the trigonometry portion of the advanced mathematics.)

<u>Triangle</u>: Any three (3) sided figure.



Area = ½ Base x Height

Perimeter = A + B + C



Simple Curve Definitions:

R = Radius:

△ = (Delta) Central angle

L = Length along curve or Arc Length

T = Tangent length

C = Chord or Long Chord

Simple curves, when degree of curve is given.

$$R = \frac{50}{\sin\left(\frac{\textit{degree curve}}{2}\right)} \qquad\qquad \textit{Degree Curve} = \frac{5729.578}{R}$$

(Railroad Practice) (Highway Practice)

These both give the same results

A one degree (1°) curve in highway or railroad practice is one with a radius of 5729.578 feet. A curve with this radius and a 1° central angle yields 100.00 ft. of arc length.

Highway and railroad construction engineers in years past used this convention (degrees of curvature) as an easy reference to curves with radius lengths which were commonly used in highway construction.

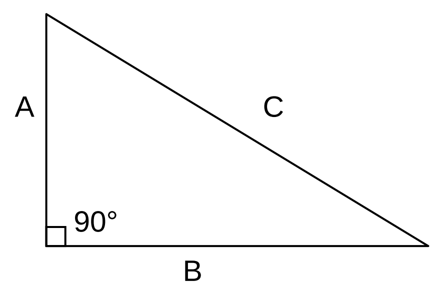
Geometry: is a branch of mathematics concerned with questions of shape, size, relative position of figures, and the properties of space. The word Geometry comes from a combination of the Greek words geo, "earth" and metron, "measure"

Geometry elements of a right triangle are as follows:

A = Altitude (or height)

B = Base

C = the long side



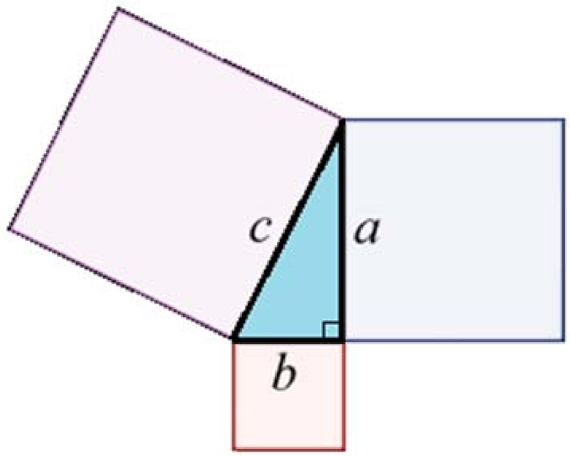
Geometric relations involving a right triangle are as follows:

The Pythagorean Theorem says that $A^2 + B^2 = C^2$

If the length of A and B are known then C can be calculated as follows: $C = \sqrt{A^2 + B^2}$

If the length of B and C are known then A can be calculated as follows: $A = \sqrt{C^2 - B^2}$

If the length of A and C are known then B can be calculated as follows: $B = \sqrt{C^2 - A^2}$

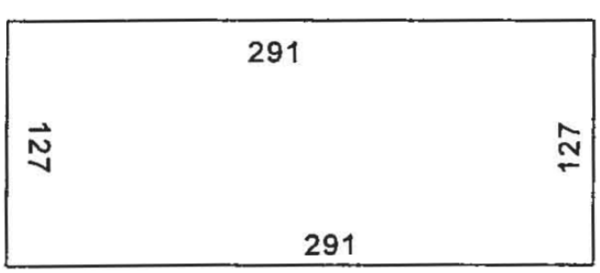


Sample Problems Involving Basic Mathematic Functions

1. Find acreage of parcels of ground with the following legal descriptions. (Included are Solutions)

   A. Beginning at a point North 1163 feet and East 2034 feet from the Southwest corner of section 16, Township 10 South, Range 10 East, Salt Lake Base & Meridian. Running thence North 127 feet; thence East 291 feet; thence South 127 feet; thence West 291 feet to the point of beginning.

Solution: Parcel described is a rectangle which would appear as follows:



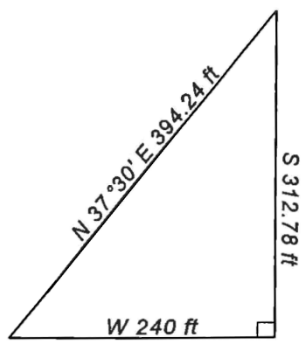
Area of a rectangle is: $A = L \times W$

$A = (291) \times (127)$ ft
$A = 36957$ sq ft

Area in acres: $ac. = \frac{sq\ ft}{43560}$

$$\frac{36957}{43560} = 0.85ac$$

B. Com N 17.5 ft and W 23 ft from the E ¼ Cor Sec 16, T 10 S, R 10 E, SLM; W 240 ft; N 37°30' E 394.24 ft; S 312.78 ft to beg.



Solution: $A = \frac{1}{2}BH$

$A = \frac{1}{2}(240\ ft)(312.78\ ft)$
$A = 37533.6$ sq ft

Area in acres: $ac. = \frac{sq\ ft}{43560}$

$\frac{37533.6}{43560} = 0.86$ ac

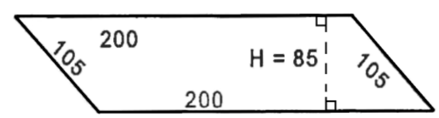
2. Find acreage for the following figures.

A. Solution: Area of a parallelogram = Base x Height

$A = (200\ ft)(85\ ft)$

$A = 17{,}000$ sq ft

Area in acres: $\frac{17000}{43560} = 0.39$ac.



B. Solution: Area of a trapezoid = $\frac{(A+B)}{2}\ h$

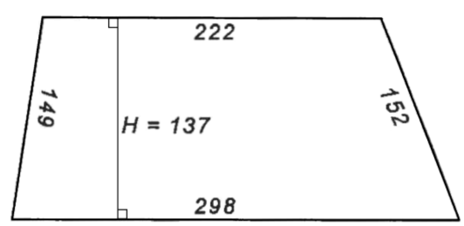
$A = (\frac{222+298}{2})137$

$A = (\frac{520}{2})137$

$A = (260ft)\ (137ft)$

$A = 35{,}620$ sq ft

Area in acres: $\frac{35620}{43560} = 0.82$ac.



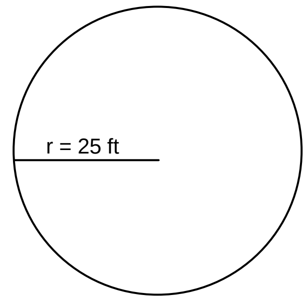
C. Solution: Area of a circle = πr^2

$A=(3.1416)(25ft)^2$

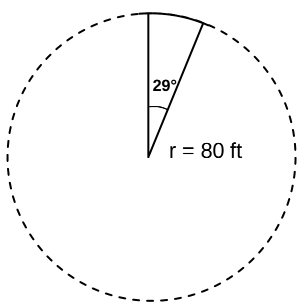
$A=(3.1416)(625\ sq\ ft)$

$A=1963.50\ sq\ ft$

Area in acres: $\frac{1963.50}{43560} = 0.05ac.$



D. Finding the area of part or a sector of a circle. A full circle has central angle of 360° and a sector is a fraction of a full circle. Hence, the area of the sector is that fraction times the area. If the central angle is expressed in degrees, minutes and seconds, convert the angle to decimal degrees.



Area of a sector = $(\frac{\Delta}{360})(\pi r^2)$

$A= (\frac{29}{360})\ (3.1416)(80ft)^2$

$A= (0.08)(3.1416)(6400\ sq\ ft)$

$A= (0.08)(20106.24\ sq\ ft)$

$A=1608.50\ sq\ ft$

Area in acres: $\frac{1608.50}{43560} = 0.04ac.$

Rotation of Bearings

Occasionally a legal description will traverse along a section line but call out a different basis of bearing than the one customarily used for that section. For the purpose of obtaining coordinates which are compatible with those of other parcels of land in that area, there are two methods to rotate the description.

Method 1 — Mathematically calculate the rotated bearing for each course of the description to match with section bearings of all parcels for that section. Then draw the parcel using the mathematically rotated bearings.

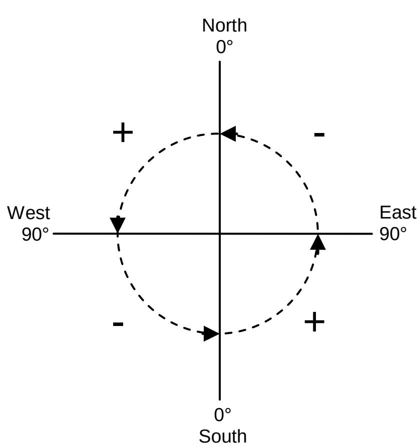
Method 2 — Draw the parcel using the original description and then physically rotate that description and its basis of bearing to match the one customarily used for that section. We will discuss two versions of this method when we talk about mapping descriptions and then with GIS.

Do not label that parcel on the plats with the rotated bearings nor reflect the rotation in the tax description. The tax description should always reflect the legal description on the document. This will help prevent the tax payer from being confused over the difference between the deed description and tax description.

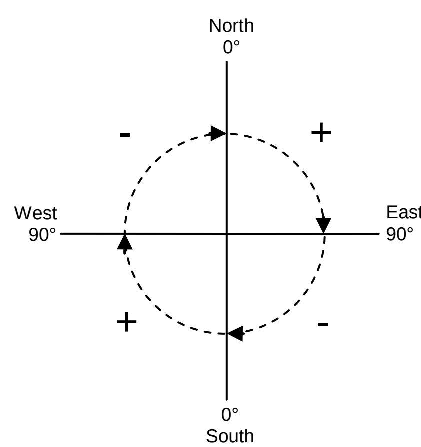
If drawing up a description with a drafting machine, requiring rotation from the normal bearing, that rotation can be accomplished by re-adjusting the head of the drafting machine to coincide with the description, and then draw the description as it reads.

For those using computers, there is normally a provision for rotating the courses of a legal description. (Refer to Chapter 7 on GIS)

When rotating a description, you rotate each bearing in the description the same direction. The same rotation will be positive or negative depending on the direction of rotation and quadrant it is in. The quadrant is the same as the direction of the bearing. Sometimes you will need to add the rotation angle amount or subtract the rotation angle amount depending on the quadrant. See the diagram below for your rotation and quadrant to determine if you need to add or subtract the rotation angle.



Rotating Counter Clockwise



Rotating Clockwise

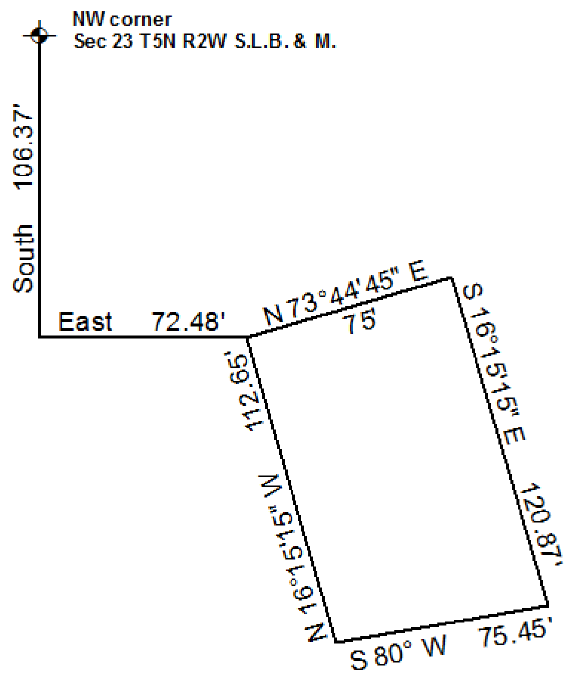
For more advanced details on how to rotate a bearing in a legal description mathematically see the advanced mathematics section in the appendix.

Method 1 Example: Basic mathematical rotation of a legal description

For this example we will be using the same legal description we will use in the mapping section but this one we will be rotating it 10° counter clockwise. Each bearing in the description will have to be individually rotated.

<u>Original Legal Description</u>

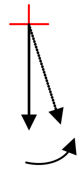
Part of the NW quarter of Section 23, Township 5 North, Range 2 West, Salt Lake Base and Meridian. Beginning at a point South 106.37 feet and East 72.48 feet from the Northwest corner of said section. Running thence North 73°44'45" East 75 feet; thence South 16°15'15" East 120.87 feet; thence South 80° West 75.45 feet; thence North 16°15'15" West 112.65 feet' to the point of beginning.



Now we need to separate out each bearing from the example legal description and mathematically calculate the rotation of each one of them for 10° counter clockwise. The direction of the bearing will determine if we add the 10° or subtract the 10° to move counter clockwise.

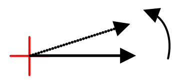
Original Bearing **Rotated Bearing** (10°)

(South) S 00°0’0” E + 10°



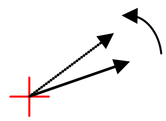
S 00°0’0” E
\+ 10°
S 10°0’0” E = S 10° E

(East) N 90°0’0” E - 10°



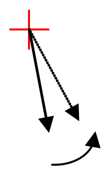
N 90°0’0” E
\- 10°
N 80°0’0” E = N 80° E

N 73°44’45” E - 10°



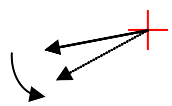
N 73°44’45” E
\- 10°
N 63°44’45” E = N 63°44’45” E

S 16°15’15” E + 10°



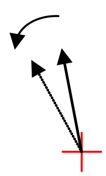
S 16°15’15” E
\+ 10°
S 26°15’15” E = S 26°15’15” E

S 80° W - 10°



S 80°0’0” W
\- 10°
S 70°0’0” W = S 70° W

N 16°15’15” W + 10°



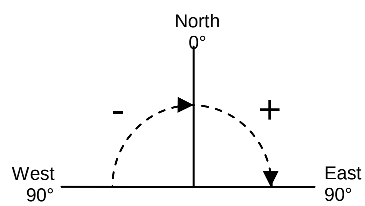
N 16°15’15” W
\+ 10°
N 26°15’15” W = N 26°15’15” W

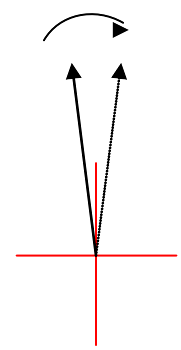
Rotated Legal Description (10°)

Part of the NW quarter of Section 23, Township 5 North, Range 2 West, Salt Lake Base and Meridian. Beginning at a point South 10° East 106.37 feet and North 80° East 72.48 feet from the Northwest corner of said section. Running thence North 63°44'45" East 75 feet; thence South 26°15'15" East 120.87 feet; thence South 70° West 75.45 feet; thence North 26°15'15" West 112.65 feet' to the point of beginning.

When you rotate a description and it crosses into a new quadrant, you have to switch from addition to subtraction or from subtraction to addition.

Example: We start with a bearing of North 02°00’00” West and we rotate it 5° clockwise.





We begin in the Northwest quadrant and so we begin by subtracting the rotation angle. However once we reach 0° we are headed directly North and in order to continuing to rotate clockwise we start to rotate into the Northeast quadrant where we add in order to rotate clockwise.

Then there are still 3° of rotation remaining of the original 5° rotation. So we now add the 3° to get the result North 03°00’00” East.

NOTE: *This basic math section shows only simple rotations to help you understand the concept of rotating bearings and descriptions. Actual rotations of legal descriptions to match the basis of bearing with the section line bearing is almost always rotated with more precision than a whole degree and includes rotating degrees minutes and seconds. If you need to rotate a description mathematically, each individual bearing should be converted to decimal degrees in order to perform the math functions and then be converted back into degrees minutes and seconds to draw that bearing.*