UTAH ASSOCIATION OF COUNTY RECORDER'S

CADASTRAL MAPPING – 2013
COURSE 1

Entry level mapping for Utah Recorder's Offices

APPLIED MATHEMATICS

TABLE OF CONTENTS:

- 1.2.7 Lettering Sheet Tools
  - a. Lettering Guide
  - b. Drafting Pencils
  - c. Drafting Lead
  - d. Paper or Mylar
  - e. 'T' Square
  - f. Masking Tape
  - g. Lead Sharpener
- 1.2.8 Lettering Sheets
- 1.2.15 Sheet Assignment
- 1.2.19 Line Weight

1.3 Chapter 3 - Significant Digits.

- 1.3.9 Rounding
- 1.3.21 Counting v. Measuring
- 1.3.22 Precision Indexes
- 1.3.33 One Digit Rule

1.4 Chapter 4 - Algebra.

- 1.4.5 The Term
- 1.4.6 Exponents and Square Root
  - 1.4.7 Beginning with Exponents
  - 1.4.8 Multiplication of Exponents
  - 1.4.9 Exponents in Division
  - 1.4.10 Power of Power
  - 1.4.11 Power of Zero, or Zero Power
  - 1.4.12 Negative Exponent
  - 1.4.13 Using the Square Root
  - 1.4.14 Understanding Axioms
  - 1.4.15 Understanding Transposing
  - 1.4.16 Eliminating Parentheses
  - 1.4.17 Factoring using F.O.I.L.
    - 1.4.18 Common Monomial
    - 1.4.19 Binomials
    - 1.4.20 Binomials and Trinomials
    - 1.4.21 Polynomial
  - 1.4.22 Fractional Equations
  - 1.4.24 Literal Equations and Formulas
  - 1.4.25 Quadratic Equation

1.5 Chapter 5 - HMS conversions.

- 1.5.2 Degrees, Minutes, and Seconds
- 1.5.13 HMS Format to HDD Format
- 1.5.15 HDD Format to HMS Format

1.6 Chapter 6 - Basic Geometry, Definitions:

- 1.6.4 Line
- 1.6.5 Point
- 1.6.6 Ray
- 1.6.7 Line Segment
- 1.6.8 Vertex
- 1.6.9 Angle
- 1.6.10 Types of Angles
  - a. Straight angle
  - b. Right angle
  - c. Acute angle
  - d. Obtuse angle
  - e. Complementary angle
  - f. Supplementary angle
  - g. Interior angle
  - h. Exterior angle
- 1.6.11 Parallel lines
- 1.6.12 Intersecting lines
- 1.6.13 Transversal
- 1.6.14 Plane
- 1.6.16 Geometric Construction:
- 1.6.22 Lines:
  - 1.6.23 Bisecting a line
  - 1.6.24 Parallel lines
  - 1.6.25 Perpendicular line thru a point on a line
  - 1.6.26 Line perpendicular to a line thru a point on a line
  - 1.6.27 Divide a line into equal segments
    - 1.6.28 Method One
    - 1.6.29 Method Two

1.6.30 Angles:

- 1.6.31 Bisecting an angle
- 1.6.32 Congruent angles
- 1.6.33 Creating 30°, 60°, and 45° angles
- 1.6.34 A 60° angle
- 1.6.35 A 30° angle
- 1.6.36 A 45° angle
- 1.6.37 Isosceles triangle
- 1.6.38 Equilateral triangle

1.6.40 Circles:

- 1.6.41 Center of a circle
- 1.6.42 Tangents to a circle
- 1.6.43 Three point arc

UTAH ASSOCIATION OF COUNTY RECORDER'S

CADASTRAL MAPPING COURSE 2

For mid-level mapping staff for Utah Recorder's Offices

APPLIED MAPPING

TABLE OF CONTENTS:

closing corners`

i. Magnetic deviation and the difference between theory and actual survey
ii. Mining claims should be a separate chapter (have Craig, Alan, and Chad work on this)
iii. What controls a boundary possibly part of chapter 8
 (1) plat or staking
  (a) Hasenyager IBLA
 (2) outline the process of federal patenting

iv. Water Boundaries separate chapter
v. Meander lines
vi. Three mile method
vii. Multiple corners separate chapter
    (1) Original
    (2) Obliterated
    (3) Lost
    (4) Common report
    (5) Use of Aerials???
    (6) Monument Tie Sheets
        (a) typical data found on sheet and how to utilize them
        (b) USNG monument numbering system

Need school trust lands and railroad lands discussed
Include a copy of the 1973 Sample Plat

Chapter 16 - STATE PLANE COORDINATES:

b. Latitude and longitudes
    i. include discussion on the accuracy meaning of decimal of seconds.
c. Lambert
d. Mercator
e. Zones
f. Grid to ground
    i. bearing sheets
    ii. Monument tie sheets
g. How does GIS tie into or use datums?
    i. Converting True Bearing and True Mean Bearing to grid - geodetic bearings to grid. Manual 2009 pg 29
    ii. For converting GLO/BLM plats to the county Cartesian system.

Chapter 17 - GPS (Global Positioning Systems):

h. Accuracies over history of technology
i. How GPS works?

Chapter 18 - DESCRIPTIONS:

8.01 Legal Descriptions
8.02 Ault v. Holden and Authority to Interpret Validity
8.03 Rules of Construction relating to Priority of Calls; Metes v. Bounds
reading and drafting expanded in the ownership plat section?
    Court Cases
    Weber co. Book 1172-203 & B 1152-311 examples
Nov. 2010 suggestion to discuss precedence of calls
    (1) Ambiguity - Patent or Latent, a classic definition of latent ambiguity is the argument between surveyors on the location of a boundary. Jeff lucas, Oct 2007 pob discussed this as an out come of Jensen v.

Bartlett, 286 P.2d 804, 806 (Utah 1955)

(2) Oregon statute regarding rules of construction. Oct 2007 POB (6) in particular references boundaries of lots shown on a plat are subordinate to the actions of the owners that seem to coincide with the survey and plat.

8.04 Treaty of Guadalupe Hidalgo and the U.S. - Mexican boarder

8.05 Aliquot Part Descriptions.

j. Basic
   - i. Aliquot descriptions stressing the comma and formatting issues
   - ii. Descriptions that assumed the size and bearings of the section to be that of the GLO plats
   - iii. Ambiguity of description
   - iv. Simultenaous vrs. Sequential conveyances
     Senior Title and Senior Surveys

k. Ones with curves
   - i. tangent
   - ii. non-tangent
   - iii. design elements and what criteria would usually be held

l. UDOT Descriptions
   - i. deeds
   - ii. right-of-way maps
   - iii. City streets i.e. Hall v. North Ogden City
   - iv. Dooly Block v. Salt Lake Rapid Transit

Descriptions with Curves:
   - (1) Highway - arch definition
   - (2) Railroad - chord definition

m. Spirals
   - i. Basic calculations relating to railroads
   - ii. identification of station point on curves (staking our a spiral)
   - iii. positioning a deed to the monumentation

n. Geometric construction
   - i. Midpoint of line
   - ii. Radius point of an arc
   - iii. Etc.
   - iv.

o. Railroads
   - i. valuation maps
   - ii. spiral calculations
   - iii. look at my railroad books for other information

p. Subdivision and Townsite plats
   - i. old
   - ii. new
   - iii. what holds plat or staking
   - iv. orientation

q. DEED ROTATION

i. Basis of Bearing see 5.13.18

(1) Definition - see Idaho Code and utah 17-23-17

ii. determining the basis of bearing in survey plats

(1) are there more than one basis? Usually not but how does a call to a road affect a survey bearing shown on the plat?

iii. multiple basis in single metes and bounds descriptions

(1) bounds calls affecting

(2) using adjacent parcel descriptions or common lines

iv. Resolving conflicting bearing bases related to adjacent parcels/subdivisions

r. REMAINDER PARCELS

i. Damage caused by writing

ii. Less and except

Chapter 19 - OWNERSHIP PLATS

i. Required Items by law

(1) **UCA 17-21-21. Ownership plats -- Use of geographic information systems or computer systems.**(1) The county recorder shall prepare and keep ownership plats drawn to a convenient scale, which show the record owners of each tract of land in the county, together with the dimensions of the tract.(2) The county recorder may not be required to:(a) show ownership of timeshare units or timeshare estates on ownership plats; or(b) show lot or unit ownership on subdivisions or condominium plats or other ownership plats if that information is available through computer systems or other indexes.(3) Nothing in this chapter precludes the use of geographic information systems or computer systems by the recorder if the systems include all of the information required by this section.

ii. Other standards

(1) Mapping and Parcel Identification Standards of Practice, Utah State Tax Division 8.1.1

(a) vesting document numbers and dates, page 8-8

(b) Standardized plat symbols, lines, and letters, page 8-8

iii. Parcel Numbers

iv. Suggested features/data

(1) IAAO 3.8 Map Products - Boundaries of all parcels, parcel identifiers, parcel dimensions and areas, **easements that influence value**, subdivision or plat boundaries, as well as block and lot numbers, boundaries and names of political subdivisions, boundaries and names of geographic subdivisions such as sections, districts, etc., locations and names of streets, highways, alleys, railroads, rivers, lakes, and other geographic features.

a. Base layer needs to be discussed:

i. Multiple mons

ii. Street monuments
iii. UDOT r/w mons
iv. County mons
v. Others
vi. Rivers
vii. Fences
viii. Pavement
ix. Topographic layers
x. R/w corridors
xi. Subdivision layers
xii. Parcel layers
xiii. Easement layers
xiv. Cemeteries

Chapter 20 - CONDOMINIUM OWNERSHIP ACT

Chapter 21 - ANNEXATIONS:

Chapter 22 - VACATING OF ROADS AND STREETS:
Hall v. North Ogden City, 109 Utah 325; 175 P.2d 703 (Utah 1946)

APPENDIX:
Appendix 1: Hall v. North Ogden City, 109 Utah 304; 166 P.2d 221, (Utah Supreme Court 1946)

Appendix 2: Salt Lake County v. Metro West Ready Mix, Inc., 2004 UT 23, 89 P.3d 155, (Utah Supreme Court 2004)

Appendix 3: Scott v. Hansen, 422 P.2d 525, 18 Utah 2d 303, (Utah Supreme Court 1966)

UTAH ASSOCIATION OF COUNTY RECORDER'S

CADASTRAL MAPPING COURSE 3

For experienced mapping staff for Utah Recorder's Offices

TITLE AND TAXATION

TABLE OF CONTENTS:

APPLIED MATHEMATICS
Course 1. Preface.

1.0 Preface:

1.0.1 **Historical Note from the 2006 Preface**: A note from Randy A. Covington, Utah County Recorder retired, and former Chairman of the Cadastral Committee for the Study Manual For Cadastral Mapping Technology, State of Utah 2006 Edition Utah Association of County Recorders.

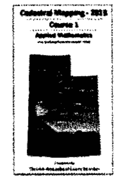
1.0.2 **Quoting from the 2006 Edition Preface:**

1.0.3 (Quote) "In 1974 I was working as a mapper for Washington County, in that year I took a trip around Utah to find out how mapping was done in other counties. My visit to eight counties revealed a wide variety of maps and mapping techniques. Some counties attempted to map as exactly as their equipment, calculators and survey monuments would allow. Other maps were drawn out of scale and were only valuable as rough references. Neatness in lettering ranged from very professional to rough hand lettering. Each county was doing the best it could with the monies available for equipment and mapping personnel.

1.0.4 "Shortly after that trip, I left both mapping and the State. When I returned in 1978 to Utah County I found that there had been a new development in mapping in the state. The State Tax Commission had realized the difficulty of assessing taxes using the counties' ownership maps that existed then state wide. They took the matter to the legislature to obtain funding for a state wide project to redraw ownership maps. A plat standards committee was formed and mapping standards were established by the committee to help fulfill the mandate of the legislature. $250,000 per year was appropriated to fund the map redraw project statewide. An amount of $50.00 was paid to the counties for each map redrawn to the newly established standards. Most counties took advantage of the funding and either completed or nearly completed the redrawing of their maps over the period that the redraw project was funded. This project greatly improved the quality of maps in the state."

1.0.5 "Many improvements were realized in the redrawn maps:

1. Maps were constructed on durable, reproducible medium - mainly mylar (but some linen).
2. Permanent features were lettered mechanically and in ink (ie., map headings,



subdivisions, street names, monuments, lakes, rivers, railroads, etc...).
3. With better drafting equipment, computers and calculators more accurate maps were made.
4. Standard symbols, line weights and scale were employed to produce maps that were more readable and easier to interpret.

1.0.6 "It should be mentioned that the plat standards committee established some standards which were taken directly from the IAAO (International Association of Assessing Officers) standards book. Many of those standards, which were created to accommodate the eastern states, do not have application to mapping in Utah, as for example, the use of aerial photos to determine property lines rather than deed records. Also some lines and symbols were required which would produce a topographic map and make it difficult to determine ownership boundary lines. Fortunately most counties ignored these problem requirements and produced maps that are clear and concise." (close quote)

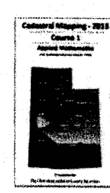
1.0.7 **Note from the 2013 Cadastral Mapping Committee:**

a. In paragraph 1.0.6 the quote from the 2006 manual indicates that Eastern States used aerial photos to produce their maps. The comment made is that "the use of aerial photos to **determine** property line rather than **deed records**." was the practice in other States and that the IAAO encouraged the use of these photos in the production of cadastral maps.

b. With the prolific use of GIS systems in the mapping world, which includes cadastral mapping of the County Recorders, the use of aerial photos has become a vital tool to produce maps that accurately represent the deed. While the use of aerial photography can greatly enhance the accuracy of mapping bounds calls to fences, roads, railroads, water ways and other physical features, the purpose of cadastral mapping is not to **determine** property lines.

c. This subject of how to properly interpret a legal description will be presented in Course 2 in much more detail. However, ...

d. To "determine" a property line is not the function of the cadastral mapping the determination of property lines is the regulated practice of Land Surveying. The use of the tools available to cadastral mappers and the requirement (whether by standards of the IAAO, the State, or simply the perception of the public) to produce the most accurate maps possible do require that many of the legal requirements of deed description interpretation be employed by the cadastral mapper.



e. This process also requires that the cadastral mapper have a significant knowledge of some aspects of surveying as it relates to the law of boundaries and how the law will interpret deed descriptions.

f. It is because of the need of our society to have accurate tax maps that this Course and the following Courses will teach cadastral mapping in a way that, when the principles taught are applied, the maps will be the most accurate possible without overstepping our bounds into the legal profession, the title industry, or the surveying profession.

1.0.8 **Continuing from the 2006 Preface:**

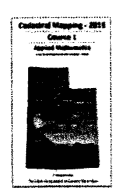
1.0.9 (Quote) "In 1984 State funding for the map redraw program ceased. Many counties had completed their redrawn maps and many other counties, seeing the value of the redrawn maps, continued the redraw projects at their own county's expense.

1.0.10 "In 1995, in an attempt to do away with inactive standing committees, the State Legislature repealed, along with others, the statute which created the state plat standards committee. Without a current committee, there is no mechanism to "fix" problems that exist with the original plat standards. There is no State funding as an incentive to meet the old state plat standards. Generally, Recorders in Utah are currently applying reasonable and workable mapping standards to their constructions of maps.

1.0.11 "Currently, a number of counties are using computers to assist them in constructing their ownership maps. At least four different computer mapping programs are being used at the present time. They all use coordinate geometry and the programs range from basic line and label mapping (C.A.D.) to sophisticated G.I.S. (Geographic Information Systems).

1.0.12 "As I discussed this new computer software dimensions in mapping with the other Recorders, we felt that there should be <u>two</u> levels of Certified Cadastral Mappers. Those who pass the basic course outlined in this manual and those who pass the basic course and also demonstrate computer mapping skills. Those who receive the Cadastral Mapping Certificate and have demonstrated the additional computer mapping skills will have the title "Computer Mapping Specialist" added to their "Certified Cadastral Mapper" certificate. This will reflect the enhanced skill level." (Close quote.)

1.0.13 It should also be recognized that the standards which were derived by the State committee are being used by the State Tax Commission and they encourage the implementation of those standards by the Recorders of the State. While it is recognized by



UACR that each county functions differently than their counterparts, the 2013 Cadastral Mapping Committee would recommend to the individual counties to help standardize the mapping processes by adopting this training and the standards proposed herein.

1.0.14 Mr. Covington indicated that the 2006 program had two levels of certification. It has been decided by the 2013 Committee that three levels should be employed to better address the specific areas that a cadastral mapper should be proficient in. Namely, Applied Mathematics, Applied Mapping, and Title and Taxation.

1.0.15 **2012 Mapping Poll:**

1.0.16 A poll of the County Recorder Offices in Utah was taken to determine the most prevalent type of drafting method currently in use. The results of that poll are as follows.

a. 40% use CAD Drafting,

b. 60% use Hand Drafting,

c. 70% use some type of ESRI product.

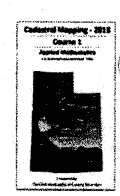
Not all counties use one type exclusively, 40% of the responding counties use one type exclusively. The fact that 70% of the respondents are using, either exclusively or migrating to, ESRI products is an indication that there is a need to teach a more detailed GIS course for the counties. This course is scheduled to be taught in Course 2. However, since so many are still using Hand and Cad drafting techniques we will be relating the material, as best possible, to each of the two types of drafting techniques.

1.0.17 **Continuing from the 2006 Preface:**

1.0.18 (Quote) "Those Recorders who are using computers to produce maps for their office will need to develop a manual on their specific computer mapping system. The Recorders will need to administer a test of their own making which their mappers must pass to demonstrate competency in the use of their mapping equipment." (Close quote.)

1.0.19 **Note from the 2013 Cadastral Mapping Committee:**

1.0.20 The statement regarding the production of specific mapping manuals for each of the individual counties in the State we do not believe to be necessary. The 2013 Manual should suffice for the uses and instructional materials relating to the most prevalent drafting methods being used. Each county already knows how to use their specific equipment. The goal with this course is to provide standard drafting techniques to help make drafting easier and more understandable.



1.0.21 The following information from the 2006 Manual indicates that the section on laws of the State have been updated and it should be noted that the 2013 Manual will also be updated to the current laws. Furthermore, it is the responsibility of each county to stay abreast of the laws which may affect the office of recorder and the mapping process. The Cadastral Committee encourages each county to help keep the Committee appraised of such changes so that the manuals and lectures can be updated.

1.0.22 The section on title searching will also be updated to incorporate some of the new electronic searching tools that are available and how to build a chain of title.

1.0.23 **Continuing from the 2006 Preface:**

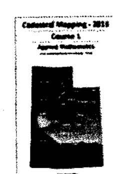
1.0.24 (Quote) "This section of this manual on real estate law has been updated to reflect current laws. The section on title searching and drafting will remain the same as in the past editions. The section on surveying and mapping has been updated and enlarged to include sections on G.P.S., G.I.S., metric, the division of a section of land and survey accuracy.

1.0.25 "The section on mathematics has been, by far, the most difficult discipline for most people to grasp. It has been enlarged to include a more gradual approach and incorporates step by step solutions to the math problems. The formula used to perform the intersection of two lines is quite complex and goes beyond the intent of this section which is to teach basic trigonometry and mathematic functions to solve mapping problems. The formula will be included to reference only.

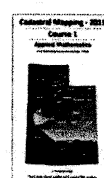
1.0.26 "I hope this new addition of the Study Manual for Cadastral Mapping Technology will be informative.
Randall A. Covington
Utah County Recorder
Chairman - Cadastral Committee" (Close quote.)

1.0.27 **Final Preface Note from the 2013 Cadastral Mapping Committee:**

1.0.28 Some of the assumptions relating to the study of "trigonometry and mathematic functions" where it is indicated that there are functions which were beyond the scope of the 2006 manual will be studied in the 2013 Courses. This course of study will expand on those principles and include the knowledge of how to do hand calculations for processes which may be done by computers or calculators. Where appropriate and possible discussions on how to do the same functions in the CAD or GIS systems will be discussed.



1.0.29 The reason to expand this subject is so that the drafter will have an understanding of how the geometry of the solution works so that when faced with a problem in a CAD system the mathematical or graphical solution to the problem will be easier. While some functions require more steps in the calculation process, such as a bearing-bearing intersection, the process can be described utilizing the basic algebraic and trigonometric functions that will be taught in Chapters 4 and 8.



INTRODUCTION:

1.0.30 **Introduction:** This introduction is also from the 2006 Edition of the manual.

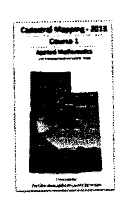
1.0.31 (Quote) "Data which a mapper must use to solve problems comes from a myriad of sources both new and old. The general need for the precise location of land did not exist until 30 – 35 years ago when the values of land began to increase at a rapid rate. At the present time most land values are 10 – 50 times higher than they were in the early 1960's. Careful licensing of land surveyors in addition to the high technology equipment, ie: G.P.S. equipment, electronic distance meters, computers, etc... developed for their use, has greatly increased the precision of the land survey. Many legal descriptions which appear on older documents and some newer documents were determined by "self surveys", (ie. Rough measurements of bearings and distances by untrained persons with improper equipment), "table top surveys", (descriptions made up by information from GLO maps, deeds or tax descriptions).

1.0.32 "How do mappers handle a set of data? How do they reconcile data from an old inaccurate description with data from a more precise description? How do they calculate acreage of irregular parcels? This section on mathematics will deal with these and other problems commonly encountered by the cadastral mapper.

1.0.33 "The old saying, "There are more ways than one to skin a cat," was never more applicable than it is in solving mathematics problems. Problems and their solutions presented in this section are based on the most concise approach. If the reader wishes to solve the problems with a different approach, that is entirely his/her option as long as sound principles are followed.

1.0.34 "Many cadastral mappers have found calculators, and particularly programmable calculators with surveying problems capabilities, to be most valuable in the day to day work routine. If these problems can be solved more efficiently through the use of a calculator, by all means use one. Some Recorder's office mapping departments have graphics computers with good coordinate geometry software which have the ability to solve problems by direct interaction with the drawing on the computer screen.

1.0.35 "It is recommended that before proceeding with the problems of this section the student obtain a calculator with trigonometry functions (ie.. One that will display Sine, Cosine, and Tangent functions). Also, it would be useful if the calculator has the functions which will convert degrees, minutes



and seconds (angle) to decimal degrees (also convert decimal degrees back to degrees, minutes an[d] seconds). ..." (Close quote.)

1.0.36 For the 2013 Course, the information recited above is still relevant and will be presented and discussed as the course progresses. The equipment will be discussed in more detail in Chapter 1. Some of the concepts mentioned will be discussed in greater detail in Course 2 when the subject of mapping is presented.

1.0.37 Again, from the 2006 introduction;

1.0.38 (Quote) "Most people working in a technical discipline have the support of a professional organization. That organization lends support to the individual by assisting in some or all of the following:

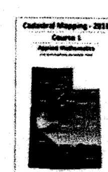
1. Provide training.
2. Establish standards for performance and certification.
3. Introduce new technologies through continuing education and perodic publications.
4. Provide assistance to help find employment.

1.0.39 "On June 26, 1981 the Utah Association of [County] Recorders met in Park City and determined that in order to meet the professional needs of Cadastral Mappers in their offices, the Recorders needed to create a professional training program. This was the first step in creating a professional organization. Over the years since then many people have worked to help produce and revise this manual, the examination and training associated with it.

1.0.40 "The following persons have worked on committee which have been instrumental in getting the manual where it is now: (Some individuals worked on more than one writing of the manual but they are only mentioned once.)

1.0.41 **"1982 - Original publication of the manual**

Jon Freston - Weber County Recorder
Dan Edwards - Private Consultant
Michael Gleed - Cache County Recorder
Glen Acomb - Chief Deputy Salt Lake County Recorder
Nina Reid - Utah County Recorder
Carol Dean Page - Davis County Recorder
Herb Bentley - Washington County Recorder
Berry Lund - Mapping Supervisor Salt Lake County



1.0.42 **"1989 Revision**
Doug Crofts - Weber County Recorder
Joe Dean Huber - Wasatch County Recorder

1.0.43 **"1997 Revision**
Dixie Swasey - Emery County Recorder
Russell Shirts - Washington County Recorder
Randy Simmons - Uintah County Recorder
Les Barker - Garfield County Recorder
Jim Ashauer - Davis County Recorder
Randall Covington - Utah County Recorder
Rodney Campbell - Assistant Utah County Recorder
Dixie Matheson - Iron County Recorder" (Close quote.)

1.0.44 The names of the 2006 committee were not included in the documentation.

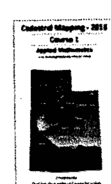
1.0.45 **2013 Revision – Applied Mathematics**
Chad Montgomery - Box Elder County Recorder
Vikki Barnett - Carbon County Recorder
Richard Maughan - Davis County Recorder
Dixie Swasey - Emery County Recorder
Connie Hansen - Millard County Recorder
Brenda Nelson - Morgan County Recorder
Alan Spriggs - Summit County Recorder
Ernest Rowley - Weber County Recorder/Surveyor

1.0.46 **Committee Report from the 2006 Edition:**

1.0.47 (Quote) "This committee was established by the Utah Association of County Recorders at the State Association of County Officials Convention at Park City on June 26, 1981. In June 1987 the committee met again to initiate needed changes. The 1989 revised edition resulted from that meeting." (Close quote)

1.0.48 Information regarding the issues of the Training Manual that were published between the 1989 and the 2006 edition was not available in the 2006 edition. The 2013 edition is the first major rework of the 2006 manual since it was first published.

1.0.49 **Committee Statement from the 2006 Edition:**



1.0.50 (Quote) "It is strongly recommended by this committee that the word plat as it applies to individuals, such as Platman, Platter, Plat Specialist, etc... be abolished, and in its place the title Cadastral Mapper be used. Upon certification, such individuals will be formally referred to as Certified Cadastral Mappers or C.C.M. this title more accurately describes the function done by these individuals. This committee also recommends the following:

a. **Committee Name**: Cadastral Mapping Certification Committee of the Utah Association of County Recorders.

b. **Function:**
1. Prepare the requirements for a certification program in Cadastral Mapping Technology.
2. Prepare a study manual for Cadastral Mapping Technology.
3. Organize and conduct training seminars for certification.
4. Administer the examination.
5. Plan workshops to introduce new technologies and advanced training."

1.0.51 **2013 Certification Requirements for Cadastral Mapping Technology:**

1.0.52 The 2006 manual contains an outline of the requirements and broad subject material that was to be taught for certification. This new 2013 Cadastral Mapping Manual (2013cmm) will provide a new set of requirements in Chapter 1 - Getting Started. The remainder of the material is outlined in the Table of Contents and the detailed manual.

1.0.53 Chapter 1 - Getting Started, is designed to function as a syllabus for the course. This will give you information on the subjects, learning outcomes, and other information related to types of equipment needed for the course. As a committee we wish each student of the program our best.

Respectfully,
Ernest D. Rowley, PLS, CFedS
Weber County Recorder/Surveyor
2013 Cadastral Mapping Committee Chair

APPLIED MATHEMATICS
Course 1. Chapter 1.

NOTE: In preparation for the first class you should read this chapter and come prepared with the basic hand drafting tools listed along with a sheet of paper to be used in class.

1.1 Getting Started Syllabus:

1.1.1 **Welcome** to the study of Cadastral Mapping for the use of The Utah Association of County Recorders. As Courses 1, 2, and 3 are presented a history, background, information, discussion, problems, and solutions will be presented relating to demonstrating the principles of cadastral mapping. Course 1 will focus on topics relating to Applied Mathematics, Course 2 will focus on topics relating to Applied Mapping, and Course 3 will focus on topics relating to Title and Taxation.

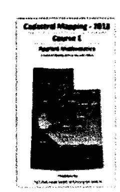
1.1.2 Course 1 topics are designed to give the student a basic understanding of principles of mathematics that will be used in Course 2. These topics range from basic principles of Mathematics and Algebra to a general understanding of Trigonometry and Geometry specifically the application of formulas associated with mapping. There will be discussions on board (hand) and CAD (computer) drafting techniques and how geometric construction relates to both methods of map creation.

1.1.3 Course 1 will also include a discussion on Basis of Bearing in Chapter 10. This discussion is critical for any mapper to understand especially the cadastral mapper. The discussion will provide the cadastral mapper with an understanding of how to properly implement the variety of bearing bases that recorded descriptions use and to create maps that reduce or eliminate artificial parcel conflicts related to the improper use of Basis of Bearings and Datums.

1.1.4 This work will include Appendix information on various subjects. In Course 1 the subjects will include a limited amount of Court Cases that demonstrate the law as it applies to principles discussed, conversion factors and constants which will be helpful to the student, and definitions of commonly used terms and other terms that are used in the profession of surveying and mapping, collectively Cartography.

1.1.5 **Course Outline.** This Program will be divided into 3 Courses. Each course will focus on skills that will pertain to Entry Level, Mid-level, and Experienced Cadastral Mappers. This is Course 1. In this course we will discuss some principles in depth and the others will be an introduction to particular subjects that will be built upon in subsequent courses.

1.1.6 In Course 1, Chapter 1 (this chapter) is to function similar to a college syllabus.



1.1.7 Chapter 2 will be presenting information from both the Hand Drafting skills and from Computer Aided Drafting skill sets. This is done because of the need for mappers to work effectively with both mediums. There are counties that maintain hard copy (mylar) plats as the primary or official source of the Ownership Plat[1] while other counties may use some type of electronic map as their official document. The Committee is not aware of any County using GIS as their official Ownership map.

1.1.8 Chapter 3 will discuss the basics associated with rounding numbers and what the term "Precision Index" means. This is necessary for the mapping student to understand the concept of "how close is close enough".

1.1.9 Chapter 4 is a beginner's discussion of Algebra. This is not meant to be an in-depth discussion on the subject. The material and course is designed to be a refresher for those that have had Algebra courses or to provide some basic skills for those students that have not had Algebra courses. All of the principles that will be taught will be able to be solved using the basic algebraic rules presented here.

1.1.10 Chapter 5 is where the concept of angles is introduced. The process of converting from Base 60 values to Base 10 values is presented along with the definitions of each.

1.1.11 Chapter 6 is where the basic principles of geometry are introduced for use in constructing polygons (parcels/traverses) and other shapes used to properly interpret a legal description and produce an accurate Ownership Plat.

1.1.12 Chapter 7 begins the study of bearing and azimuth calculations. The student should obtain a solid understanding of the concepts presented because they are the basic building blocks of deed descriptions and maps.

1.1.13 Chapter 8 is a beginner's discussion of Trigonometry. As with Algebra, this discussion is not meant to be an in-depth discussion or study of the subject. It should provide the knowledge and tools necessary for the mapper to use basic principles of trigonometry to perform specific mapping functions even for someone that has not been previously exposed to the subject.

1.1.14 Chapter 9 will be the introduction subject for Datums. The concept of Datums is developed by an understanding of coordinate systems and how to work with them. Details with respect to Datums will be built upon in Course 2 but the subject needs to be introduced in a basic manner.

---

[1]. The Ownership Plat will be discussed in detail in Course 2. This document is required by law to be produced by the County Recorder which shows the parcels of land and the Ownership of those parcels as they exist in the County.

1.1.15 Chapter 10 will introduce a concept that is critical for the mapper to understand if maps are to reflect correct geographic data; "What is North?". A solid understanding of this subject is essential to properly map descriptions.

1.1.16 Chapter 11 is the study of the Circle, Arc, and Curve. This is one of the geometric shapes that was not discussed in Chapter 6 - Basic Geometry. The reason is that the subject is fairly extensive. The concept of geometry associated with the Simple Horizontal Curve is all that will be given at this point. Additional material relating to other types of curves will be discussed in Course 2.

1.1.17 Chapter 12 will be the study of the parcel or the traverse to a surveyor. The basics relating to the drafting of a parcel will be discussed in connection with the Metes of a description. Legal descriptions contain several parts, two of which are 1) the Metes and 2) the Bounds. The bounds of a description will be discussed in Course 2.

1.1.18 Chapter 13 begins the discussion of calculating areas of parcels or traverses. This discussion will focus on methods in calculating the area of a parcel for tax purposes.

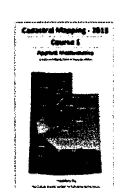
1.1.19 Chapter 14 will be a short basic discussion on the metric system. Because there was a time when the Utah Department of Transportation (UDOT) required surveyors and engineers to use the metric system in all their road work there will be deeds and highway maps prepared in the International System of Measurements (SI). The information presented will help the student to understand metric conversions to be able to properly map such data. A discussion on the difference between the U.S. Survey Foot and the International Foot will also be given because there was a time when Utah law mandated the use of the International Foot and some descriptions may be written in those units.

1.1.20 **The Appendix Material.** Contained in this section is material that is included to support concepts being presented which will aid the student in expanding their knowledge from outside sources, particularly the courts. The student is encouraged to spend time in the study of the information provided. The Appendix Material will not be limited in use to only Course 1. This material will be universal to each Course and will be added to as the Courses are developed and taught. The appendix material to date is as follows:

a. Appendix 1: This area will be populated with selected Court Cases that have relevance to the topics discussed in this training.

b. Appendix 2a: This is a copy of the standards for cadastral mapping that the State Tax Commission would like the counties to use.

c. Appendix 2b: Standard Abbreviations. In the Tax Commission standards are Standard Abbreviations that are allowed by Utah Code (See UCA 29-2-1335) for use in tax



proceedings. When mapping is done and an abbreviation is being used for any of the terms defined in this section the maps should also use these same abbreviations. Appendix 2b is a copy of pages 10 and 11 of the Tax Commission standards book and can be used as a quick listing of the abbreviations so that you can copy the page and have it handy when doing updating on the maps.

d. Appendix 2c: This section is the IAAO standards for digital mapping. Some of the information and recommendations in this document are not applicable to Utah but when there are some of their standards that would make sense for use in Utah the training will make mention of the standard and where to find it.

e. Appendix 3: This is another sheet that can be copied for use in map updating to use as a quick reference on lettering height.

f. Appendix 4: The conversion factors contained in this section may not be comprehensive of the factors that would be used by a cadastral mapper. If there are factors that you use which are not included in this material let the committee chair know and the list will be updated.

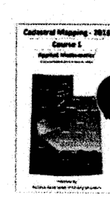
g. Appendix 5: Currently this appendix is the Definitions section where there are many terms associated with mapping, title, and surveying that you can use to help you in your reading. This list is not complete and as the Courses are further developed the Definitions Appendix will be updated and re-distributed.

h. The Index: Although an index is not part of the appendix section it will be part of the manual immediately after the appendix. The index will be added to and revised as the Courses are developed.

1.1.21 **Text.** The basic text for these courses will be the manuals provided by the Utah Association of County Recorders (UACR) and should provide all the study material that you will need to successfully complete this program. Other text materials that you wish to use to supplement these manuals can be used at your own discretion. Supplemental material and handouts, when used, will be provided as the courses progress.

1.1.22 **Equipment needs.** Because this is an applied mathematics course we will begin the discussion with an introduction to some basic calculator functions and usage. This is being done because the course is designed for you to use a scientific calculator in the process of completing the work, examples, and assignments.

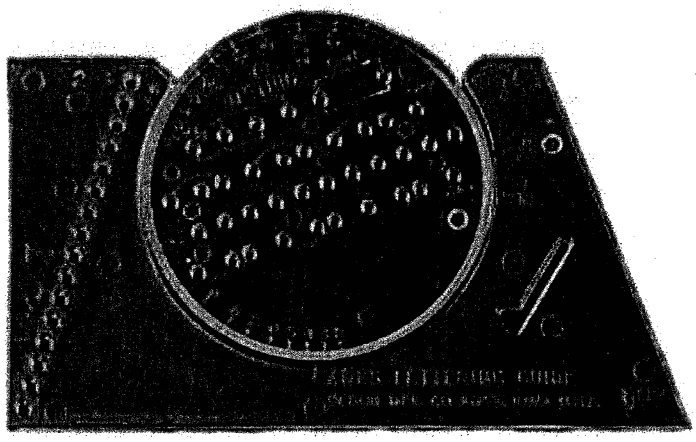
1.1.23 Although the course is not recommending a particular type of **calculator**, the discussion will revolve around the scientific calculator that uses RPN (Reverse Polish Notation) as the means of evaluating expressions. Your calculator should have the following minimum



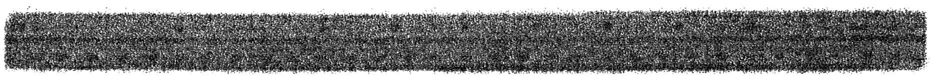
capabilities; displaying 9 or 10 decimal place accuracy, HMS conversions, three trigonometric functions (the sine, cosine, and tangent) and their inverse functions, squaring and the square root function, and the value of PI (π).

1.1.24 For some of the hand drafting exercises you may need some very basic **board drafting equipment**. For instance,

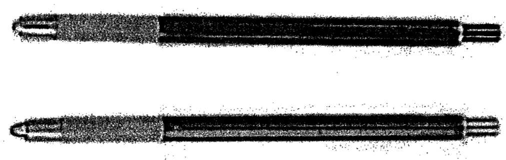
a. a lettering guide,



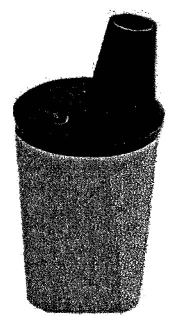
b. scale,



c. drafting pencil and lead,



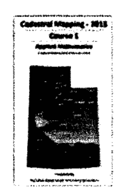
d. pencil sharpener,



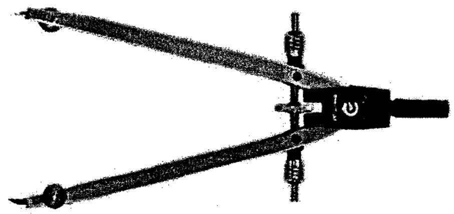
e. 'T' square,

f. erasers,

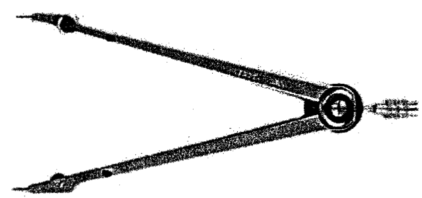
g. triangles (a 30°-60° triangle and a 45° triangle at a minimum),



h. compass,



i. dividers[1], and



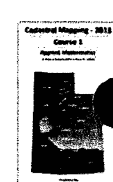
j. masking tape.

k. See the Tools subject in Chapter 2 for more detail on the type of equipment and their use.

1.1.25 Other equipment or tools may be added as the course progresses.

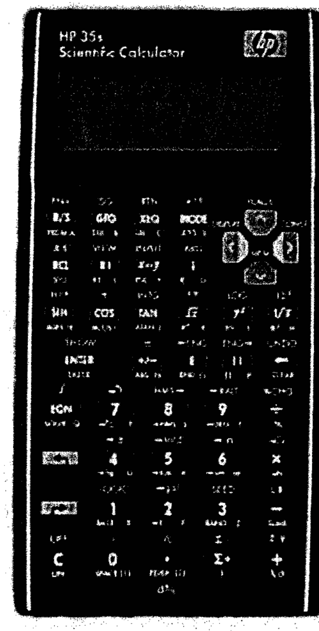
1.1.26 **Quizzes and Exams.** Instructions relating to quizzes and exams will be given at the time they are assigned. In general, all quizzes and exams will be given as open book, open note, open calculator, however, computer technology will only be allowed if specifically stated by the instructor for specific testing purposes. You may also use other books and reference materials that you have used in your study. Most exams will be given as a take home exam with a due date.

[1] See the Definitions Appendix for an explanation of these tools.



1.1.27 With this introduction it is time to begin the subject material.

1.1.28 **Calculator Operation.** This subject will be presented for the use of RPN (Reverse Polish Notation) calculators. If you are not familiar with this type of operating system you can learn about them as we go or you can do a web search using the search term "Reverse Polish Notation". You will probably need to spell it out to get the correct search results.



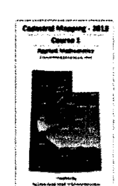
1.1.29 Essentially, the RPN notation is the opposite of Polish Notation. Polish Notation in mathematics is shown in the following equation, 4 + 3 = 7. This would be evaluated in a calculator using polish notation as "four plus three" then evaluate the expression using the equal key. This is the typical manner in which most math courses are taught in the grade school levels, K-12.

1.1.30 In engineering sciences the RPN style of notation is introduced because of the power and flexibility to evaluate complex equations. One of the "power" capabilities of RPN is the Stack. The concept and use will be explained as the discussion progresses. Suffice it to say, that the Stack is a means of placing intermediate results of terms in a location where the arithmetic operators can be used in correct math hierarchy (Multiplication and division before addition and subtraction and the evaluation of parentheses).

1.1.31 **The Enter Key.** It should be noted that an RPN calculator has no equals (=) key or button. Instead the calculator is provided with an "Enter" key. The Enter key is used to place values on the Stack. To evaluate the equation, 4 + 3 = 7, the following key strokes are used, four, Enter, three, plus. These key strokes will give the result of 7 without having to press an equals key.

1.1.32 The calculator will display the information showing the value 4 on the Stack by pressing the 4 key and pressing Enter. This places in the first stack register, which is known as the 'x' register, the number 4. Then when the 3 key is pressed the 3 is held in a location known as the command line. This command line is not a Stack location but is a location where the value is waiting for another key to be pressed giving the calculator direction on what to do with the values in the 'x' register and the command line.

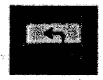
1.1.33 If the Enter key is pressed then the 3 is placed in the 'x' register and the 4 is moved to the 'y' register. The two values are now waiting for some operation to be executed. At this point by pressing the '+' key the calculator performs the addition with the values which are in the 'y' and 'x' registers, in that order.



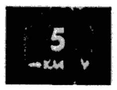
1.1.34 **Sample Key Strokes.** The RPN key strokes to solve the following equations are provided as a sample to help the user understand how the Stack works and how to evaluate an equation using the proper math hierarchy.

a. Area of a circle. $A = \pi r^2$

If the radius of the circle is known to be 500 units then the area would be calculated with the following RPN key strokes.

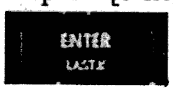
Step 1 - [gold shift key]  [π key] 

this action will place the value of PI in the 'x' register, then

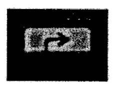
Step 2 – [5 key] 

[0 key] 

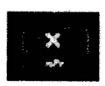
[0 key] 

[Enter key] 

pressing the Enter key at this point places the value of 500 in the 'x' register and moves the value of π to the 'y' register in the stack.

Step 3 - [blue shift key]  [x^2 key]  this will replace the value of 500 with the value 250,000.00 (which is the 500 squared) in the 'x' register.

From the equation we see the πr^2 is a term and that the value of PI and the radius squared are to be multiplied, therefore,

Step 4 - [x key] 

which is the multiplication key. The calculator will perform the multiplication with the values in the 'x' and 'y' registers and the 'x' register will now show the value of 785,398.163398 which is the Area in square units.

b. Law of Cosine. $c^2 = a^2 + b^2 - 2ab \cos C$

This equation will calculate a side of an oblique triangle and the details on how this equation is used and functions will be presented in Chapter 8 Trigonometry. For now the key strokes of how to solve such an equation will be demonstrated.

If the following values are assigned to the variables of the equation; a = 100, b = 250, and C = 30°10'25", the key strokes to solve this problem are as follows.

$c^2 = 100^2 + 250^2 - 2{*}100{*}250{*}\cos 30°10'25''$

Step 1 - [1 key] [0 key] [0 key] [Enter key] this will place the value of 100 in the 'x' register.

Step 2 - [blue shift key] [x^2 key] which will square 100 and place the value of 10,000.00 in the 'x' register.

Step 3 - [2 key] [5 key] [0 key] [Enter key] this will place the value of 250 in the 'x' register and move the value of 10,000.00 to the 'y' register.

Step 4 - [blue shift key] [x^2 key] which will square 250 and place the value of 62,500.00 in the 'x' register leaving 10,000.00 in the 'y' register.

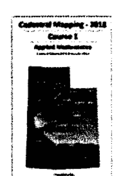
Step 5 - [+ key] executing the addition function at this point will add the values which are in the 'x' and 'y' registers and display on the 'x' register the value of 72,500.00.

Because math hierarchy requires that multiplications and divisions occur before additions and subtractions the next term must be evaluated prior to using the '-' operator of the equation. Therefore, the next step is,

Step 6 - [2 key]]Enter key] This will place the value of 2 in the 'x' register moving 72,500.00 to the 'y' register.

Notice at this point that the values visible on the display of the calculator are both '2', this is because the bottom line is showing what is in the "command line" of the calculator and the top number is the 'x' register.

Step 7 - [1 key] [0 key] [0 key] [x key] these key strokes will place the value of 100 in the command line and the [x key] will perform a multiplication between the value of 100 and the value 2. The result will display 200 on the calculator.



Note that in this circumstance the calculator is showing the 'x' register on the bottom of the screen and the 'y' register on the top of the screen.

Step 8 - [2 key] [5 key] [0 key] [x key] these key strokes will place the value of 250 in the command line and the [x key] will perform a multiplication between the value of 250 and the value 200. The result will display 50,000.00 on the 'x' register.

Step 9 - [3 key] [0 key] [. key] [1 key] [0 key] [2 key] [5 key] [gold shift key] [HMS ⇨] this series of key strokes will place the value of 30.1025 on the command line and convert the value into decimal degrees. The screen should now show 30.173611111 on the bottom 'x' register and 50,000.00 on the top 'y' register.

Because the value of 30.1025 as keyed on the calculator is in base 60 and not base 10 a conversion to base 10 is necessary. This function and process will be discussed in more detail in Chapter 5 HMS Conversions. ***Important***, when keying in an angle in degrees-minutes-seconds format you must convert the value to decimal degrees **without** using the [Enter key]. If not then the calculator will place 30.1025 in the 'y' register and the calculations will be out of sequence for finishing the problem with the Stack.

Note, [HMS⇨], the shifted function of the [8 key],. will convert the value into base 10 or the decimal equivalent of the angular value which again displays 30.173611111 in the 'x' register.

Step 10 - [cos key] this will convert the decimal degree angle into the ratio associated with the trigonometric function of the Cosine. The value of 0.864506388 is now shown in the 'x' register and the 'y' register is displaying the value of 50,000.00.

This function and the other trigonometric functions will be discussed in detail in Chapter 8.

Step 11 - [x key] 

this will perform a multiplication with the two values which are in the 'x' and 'y' registers and display in the 'x' register the value 43,225.3193841 and showing the value of 72,500 in the 'y' register.

Step 12 - [- key] 

this will perform a subtraction between the values in the 'x' and 'y' registers and display in the 'x' register the value 29,274.6806159.

These steps almost complete the problem. The equation noted above shows that the left side of the = sign is being squared and to isolate the valve of 'c' each side would be square rooted, canceling the square from the left side and requiring that the square root be applied to the right side of the equation.

$c^2 = 29{,}274.6806159$

$c = \sqrt{29{,}274.6806159}$

So to complete the problem the final key stroke is,

Step 13 - [square root key] 

this will result in the solution of the equation presented and gives the distance of side 'c' to be 171.098452991.

c. Right Triangle Solution. Tan θ = x/y

This equation will calculate an angle of a right triangle (which will be discussed in detail in Chapter 8). If the opposite side is 20 units and the adjacent side is 600 units the problem will be solved with the following key strokes. Prior to making the solution steps the equation will need to be solved to isolate θ. This is done in the following equation which will be solved in the Steps.

$$\theta = \mathrm{Tan}^{-1} \frac{20}{600}$$

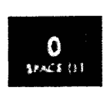
Step 1 - [2 key] 

[0 key] 

[Enter key] 

this will place on the stack and in the 'x' register the value of 20.

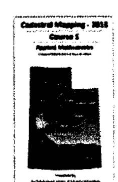
Step 2 - [6 key] 

[0 key] 

[0 key] 

[÷ key] 

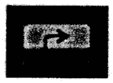
this will place the value of 600 in the command line and the execution of the [÷ key]



will perform a division of the values in the 'x' register and the command line resulting in the value of 0.033333333 shown on the display in the 'x' register.

Step 3 - [blue shift key]  [ATAN key] 

which will display the decimal equivalent of the angular value for θ of 1.909152433.

Step 4 - [blue shift key]  [⇨ HMS] 

will convert the value from base 10 to base 60 or into the degree-minute-second format. The value of 1.543294876 is now displayed on the screen in the 'x' register.

This value is read as 1°54'32.94876".

d. Quadratic Equation.

Try this equation without key stroke prompting.

Solve the following a quadratic equation by formula.

$3x^2 + 7x + 2 = 0$

The quadratic formula is:

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

Fill in the blanks as follows and solve:

$$x = \frac{-7 \pm \sqrt{7^2 - 4*3*2}}{2*3}$$

NOTE: One thing that you will need to know is how to treat the ± part of the equation. This requires two separate solutions, one for the + and another for the – part of the equation. Therefore, the two equations to be solved are:

$$x = \frac{-7 + \sqrt{7^2 - 4*3*2}}{2*3} \qquad x = \frac{-7 - \sqrt{7^2 - 4*3*2}}{2*3}$$

Working each problem separately will produce the following results:

x = -0.33333 & x = -2.00

So how did you do?

The details of the is problem with be discussed in the lecture part of the Course. The equation and it's use will not be discussed until much later in the program.

1.1.35 Try one more...

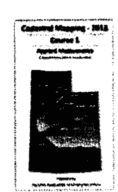
a. **Polygon Equation.**

$$(n-2)180 = \sum \theta$$

In this case n=8.

$\sum \theta = 1080°$

1.1.36 The polygon equation and it's use will also be discussed in more detail when the traverse is presented.



APPLIED MATHEMATICS
Course 1. Chapter 2.

1.2 Basic Drafting:

1.2.1. **Introduction.** While the technology of drafting has moved into the world of computers instead of the board drafting processes (or hand drafting) the need for the cadastral mapper to have and maintain some basic board drafting skills has not completely faded. This section will focus on skills that will primarily relate to functions that may still be used even in offices which use CAD or GIS as their primary drafting tool.

1.2.2. In a straw poll the question was posed to Recorder's regarding what type of method is used to produce the Ownership Plat. Of those responding it was found that most use more than one method to produce the plat. There are 3 basic methods used by County's; Hand Drafting, CAD Drafting, and GIS or ESRI products. The results of the poll are given in 1.0.15 Preface.

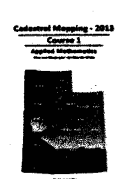
1.2.3. So, - what does the information reveal. It seems to indicate that Hand Drafting techniques are used by virtually all in some form or another which means that spending time on techniques used by pencil and compass will not be wasted. Where skills relating to hand drafting are used many times to update the hard copy Ownership plats the skill is still needed. More than this though is the fact that the techniques of geometric construction taught using traditional (hand) drafting is that computer drafting becomes easier if an understanding of those methods are solid.

1.2.4. As the course progresses there will be drafting assignments which will be required to be completed whether the student is using hand drafting equipment or using CAD drafting systems. So as this chapter progresses both techniques will be presented.

1.2.5. Because every student will need some of the basic hand drafting skills even if the primary medium being used in their offices is CAD there will be assignments relating to hand drafting which will be required of all students.

1.2.6. In Chapter 1 it was indicated that you will need to have the following tools; a lettering guide, scale, drafting pencil, pencil sharpener, 'T' square, erasers, triangles (a 30°-60° triangle and a 45° triangle), compass, and dividers. The basic use of these tools will be demonstrated and assignments given to help the student build confidence in their use and how to translate what is learned into the electronic world.

1.2.7. The first assignment will be that of doing Lettering Sheets and again is for all students. The tools used to do lettering sheets will be shown and graphically demonstrated so that you will be able to begin to practice doing tool guided lettering. This assignment will be an ongoing assignment - meaning that until the instructor grades the students sheets and indicates that the sheets meet desired legibility of print - Lettering Sheets will continue to be turned in at the beginning of each class session.



1.2.8. **Lettering Sheet Tools**. The tools used to accomplish this task are, the lettering guide, drafting pencil (two types of lead, a hard lead for guide lines and a soft lead for the text), paper or mylar, and some type of 'T' square and masking tape. The 'T' square can be any type of straight edge that will accomplish the process of drafting the guide lines and then using the lettering guide to guide the pencil strokes.

a. Lettering Guide. This tool is designed to help the drafter create "lined paper" so that lettering on paper or mylar can be done with the help of guide lines (the use of the terms paper and mylar will be interchangeable in the material unless a specific property relating to each is necessary to explain). This is accomplished by rotating the circular part of the tool and selecting the proper holes to use with the pencil. The sides of the tool are also designed to provide aid to the pencil strokes that would be vertical lines or lines that are italicized vertical lines. The use of the lettering guide will be demonstrated later in the discussion.

b. Drafting Pencils. The type of pencil used in drafting is not the typical pencil which is a wood casing around a graphite center. Drafting pencils are also known as a mechanical pencil. They are designed with clutches (similar to a bird claw) that extend or reach out of the pencil when the top button is pressed to grab the lead. This means that one pencil can be used with several different types of lead, graphite based (for paper products) or plastic based (for mylar or film products).

c. Drafting Lead. It will be necessary for the student to have two different 'hardness' of lead. If the primary medium is paper then a the two types of lead will be designated as 2H for the guide lines and HB for the lettering. If the primary medium is mylar the two types of lead will be a blue colored lead for guide lines and a plastic lead for lettering.

d. Paper or Mylar. You will need to use paper or mylar made for the drafting process. The paper is not typing paper or another paper typically used in colleges or business. The paper that you will need is called Velum. This Velum can be purchased in drafting or engineering supply companies and possibly in a college book store. The Velum may be able to be found in 8.5 x 11 sheet pads. Mylar practice sheets may be more expensive and difficult to find.

e. 'T' Square. The purpose of this tool is to be able to create parallel lines from top to bottom of the page. The 'T' Square can be used on most any desk or table that has a straight side or edge that the 'T' Square can be used with. For this drafting tool there may be alternatives to accomplish the same thing that a 'T' Square is used for. If you have drafting machines available they may work better. There is also a device known as a rolling ruler which may also work.

f. Masking Tape. This type of tape is used because it does not stick to the Velum or mylar sheet like other tapes would. This prevents tearing when the sheet has been completed and removed from the table.

g. Lead Sharpener. This device is used to sharpen the special leads for the mechanical pencil. The use of the sharpener will depend on what type you have and it would be suggested that you review the documentation of the sharpener to know how to sharpen the lead.

1.2.9. **Lettering Sheets**. To setup the sheet for practice, first, find a table and chair that you will be comfortable sitting at and can lean over in a way that your arms and shoulders are not uncomfortable. A comfortable location can make the difference in a good lettering sheet and one that is not so good. The table will need to have a straight side that the 'T' Square can be used on.

1.2.10. After the setup and you begin to do the sheet, you will need to pay attention to how fast you proceed to write. Most people have a tendency to speed up about half way through the sheet and when that happens the latter part of the sheet is not as good as the beginning. Learning to be consistent in the speed in which you letter will be a valuable skill to producing consistent lettering from the beginning of the work day to the end of the work day.

1.2.11. After finding a comfortable table and chair, using the 'T' Square, align the page on the table so that the top or bottom of the sheet are parallel with the 'T' Square. Once aligned use the masking tape to secure the paper to the table by placing a small strip of tape at each of the four corners of the sheet. You will find that having all corners taped down will keep the tools from snagging on the corners and tearing or shifting the sheet.

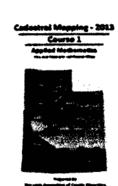
1.2.12. Sharpening the lead is the next setup step. This may seem obvious but keeping a proper point on the lead is critical in being able to produce a consistent line width. Drawing lines and forming letters require a different type of pencil point. To draw lines a regular point is used and can be used for some types of lettering, however, to do most lettering a 'chisel' point is used.

1.2.13. When drawing the guide lines there is a pencil rolling technique that is used to help keep the lead from developing a 'chisel' point. The mechanical pencil has a metal diamond pattern grip where your fingers grasp the pencil. This surface is designed to give the fingers a positive grip on the pencil so that the rolling can occur in a smooth and consistent manner. The pencil should be gripped with the index finger and thumb and it may rest in the crotch of the hand. Gripping and resting the pencil in this manner will cause the lead to wear at a consistent angle and when rolled at a consistent rate, the lead will maintain a sharp point longer than is it is not rolled.

1.2.14. The 'chiseling' of the lead point is done to add width to certain strokes of the letter. It may give the lettering a more artistic and pleasing look. Generally, when composing the letter the vertical strokes of the letter are the widest and the chisel would be placed on the paper in a manner that will accomplish the width for the vertical strokes. To get a chisel on the lead a small piece fine grit sand paper will work.[1] Instead of sharpening to a point use the sand paper to sand the end at an approximate 30° angle creating a chisel.

---

[1]. It is important that the type of sand paper be a fine enough grit to create a smooth surface on the lead. Otherwise the chisel will not be able to make a pleasing line.



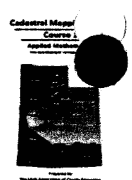
1.2.15. Now that the chair and table are selected, the paper taped so that it is parallel to the 'T' Square, and a 2H lead sharpened to a point along with the HB lead sanded to a chisel, it is now time to figure out how to use the lettering guide and setup the sheet.

1.2.16. **Line Weight.** Line weight was important to be able to produce maps that are clear and understandable. Line weight is shown as varying width of a line. Traditionally, important subjects are shown in thicker/heavier lines, those that are less important are thinner/lighter lines. In the cadastral map line weight is used to show the difference between parcels, easements, road rights-of-way, map border, titles, City boundaries and other tax boundaries, etc.

1.2.17. From the Historical review in the Preface chapter we learn that in the early 1980's the State provided funding for map standardization and part of that project was the establishment of drafting standards. The IAAO[1] also provides recommendations for drafting standards to be established by the State Associations. In this standard they give guide lines on the need for Electronic Standards associated with CAD systems.

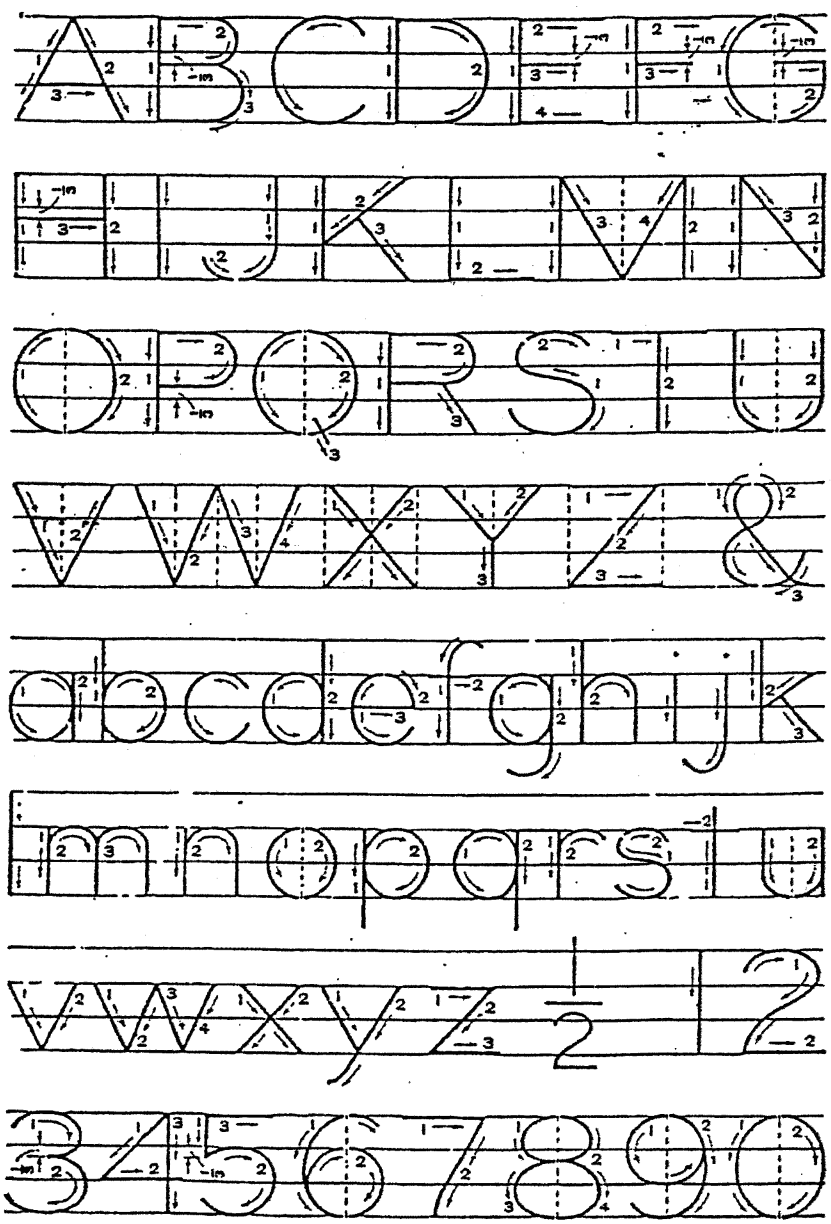
1.2.18. The State Tax Commission has a document titled Mapping and Parcel Identification Standards of Practice, Rev. July 2010 which we will be examining.

[1]. International Association of Assessing Officers.

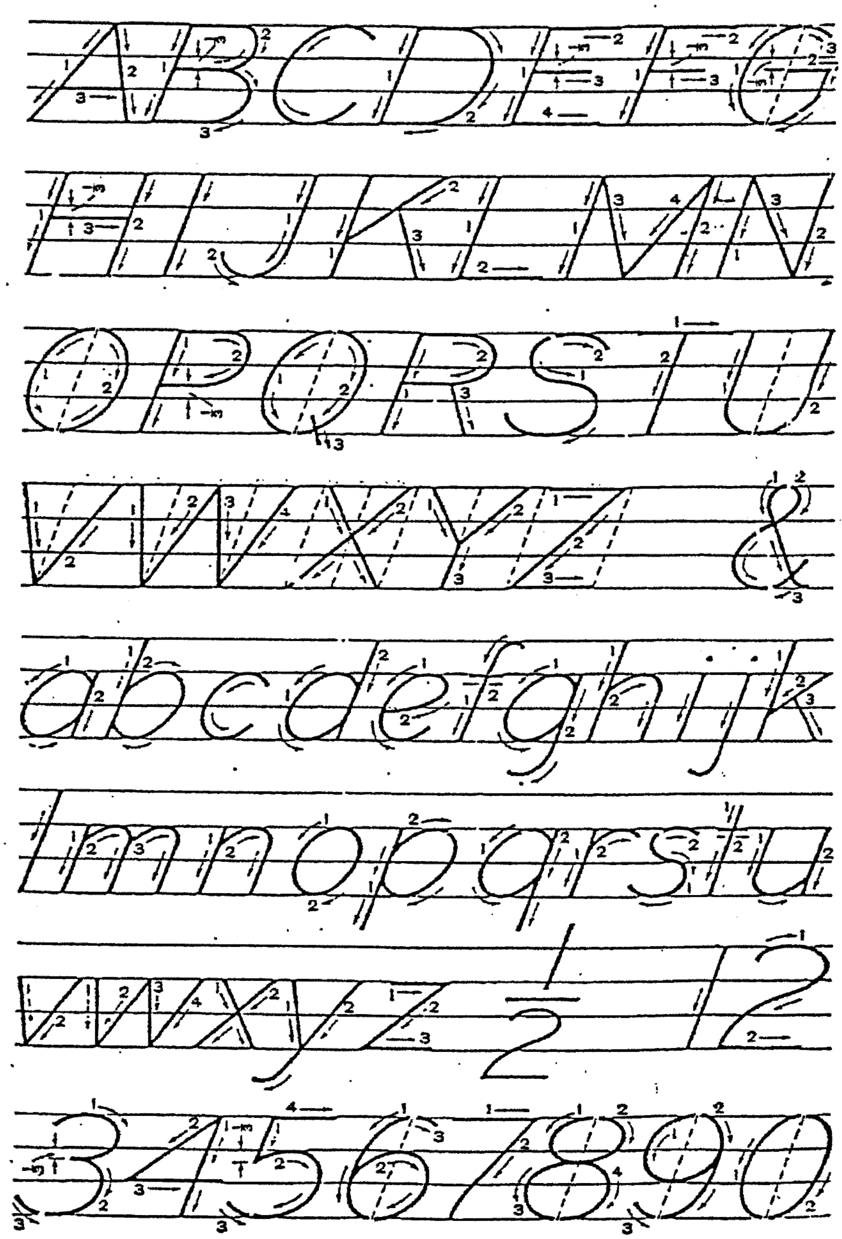


1.2.19. Lettering:

1.2.20. Vertical Single-Stroke Lettering: Single-stroke lettering does not mean that the entire letter is formed without lifting the pencil off the page. It means a line is made with one stroke. The directions of the stroke are indicated by the small arrows by the letters and the order in which the strokes are made are shown in the example below.

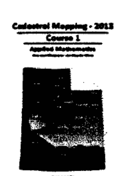


1.2.21. Inclined Single-Stroke Lettering: This type of lettering looks like italicized text.



1.2.22. **Sheet Assignment.** Find a book that you like and find a chapter or page which you will enjoy copying. Draw guide lines that are 0.1 inch letter height and line spacing of 0.1 inch. The guide lines can be drafted quickly and easily using an Ames Lettering Guide. Again, the instructions that are provided with the Ames Lettering Guide are the best source for learning all the uses of the tool.

1.2.23. Start by setting the lettering guide to the proper guide number so that guide lines will be 0.1 inch apart. The numbers on the disk represent 1/32nd of an inch, so the number 3 represents 3/32, 4 represents 4/32, etc. Rotating the circular disk so that the frame index mark is half way between the 3 and 4 on the disk sets the guide to approximately 0.1 inch. The lines are then traced on to the sheet using the middle set of holes as shown using the hard lead (2H) and light pressure on the pencil and paper.



APPLIED MATHEMATICS
Course 1. Chapter 3.

1.3 Significant Digits:

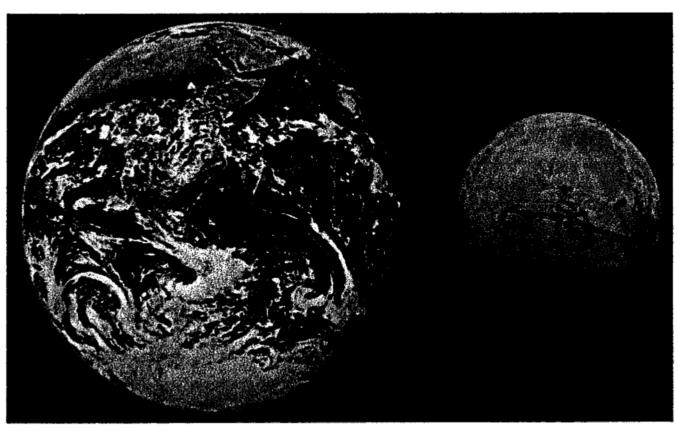
1.3.1 One of the basic functions that the cadastral mapper is required to perform, perhaps on a daily basis, is that of calculating the area of a parcel for tax purposes. This is required because much of the time deed descriptions leave off the acreage of the parcel. The discussion relating to this subject will culminate on calculating area, the use of tolerance, and some general Rules of Thumb, relating to how area could be identified in the tax system.

1.3.2 All mapping processes require some type of calculation. Whether it is to determine missing legs of a polygon, missing elements of a curve, the length of a line, the rotation to apply to a description, or calculating area. Rounding calculations may cause serious errors in the results.

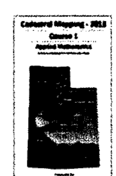
1.3.3 This discussion could be summarized as, DON'T ROUND! Although there is an appropriate time to round, it is not when the calculation process is being worked through. This simple step cannot be over emphasized.

1.3.4 Inappropriate rounding in calculations will produce errors in the results. Some of these results can be of sufficient size to create mapping problems. This is especially the case where a rounded calculation result is carried forward and used in another calculation process. Doing this will degrade the result of the next calculation and ultimately degrade the final outcome and cause errors in the work. Errors that could be significant.

1.3.5 Having indicated that rounding should not be done there is, as with all things, an exception to this principle. The process of rounding appropriately encompasses more than whether to drop or round up when examining numbers. Rounding is part of a process known as Significant Digits. Before discussing Significant Digits the proper use of rounding needs to be discussed.



1.3.6 Rounding takes place in many aspects of life. Many times we use rounding as a form of estimating quantity. For instance, the number of fans in attendance to a sporting event could be estimated and the estimated value could be rounded to a specific value, say, there are 35,000 people in attendance. The value could be viewed as being rounded to the nearest 1,000 spectators. The distance to Mars



could be estimated as being 352,000,000 miles from earth and probably represents a rounding to the nearest million miles.[1]

1.3.7 The width and length of a room may be estimated to the nearest foot. In these estimates we have been looking at values that are expressing quantities to the left of the decimal place and the trailing zeros have been retained to give an indication of the magnitude of the number. When rounding is being utilized with numbers that have digits to the right of the decimal point, then there are rules associated with rounding which in essence will cause some of the digits to be dropped.

1.3.8 The principle of rounding is utilized in everyday life. Furthermore, in grade school we learned the basics of how to round by dropping or adding a set of numbers. Generally, our first exposure to the principle of rounding would teach the first two rules, the third rule is one that is derived from statistical analysis and is used in measurement technology not generally taught in K-12 education.[2]

1.3.9 **ROUNDING:**

1.3.10 When the number to be rounded is **less than 5**, no change will be made in the number retained. Thus, rounding this value to the nearest one hundredth - 102.563 where the "3" is less than 5 the rounded value would become 102.56.

1.3.11 When the number to be rounded is **greater than 5**, the number preceding it is increased by a value of one (1). Thus, rounding this value to the nearest one hundredth - 102.567 where the 7 is greater than 5, the 6 would be increased by 1 and the rounded value would become 102.57.

1.3.12 **Statistically speaking**, when the digit to be rounded is 5, the number preceding it;

a. is increased by a value of one (1) **if it is odd**, for example rounding to tenths, 10.35 becomes 10.4, because the "3" which precedes the "5" is odd and is rounded up, but

b. remains unchanged **if it is even,** for example again rounding to tenths, 10.65 would become 10.6 because the "6" which precedes the "5" is even and remains the same.

---

1. Just a point of information, the distance to Mars is not a constant and varies due to the orbits of the Earth and Mars around the Sun. The average distance from the Sun is roughly 230 million km. The Earth/Mars image is a size comparison created by NASA. NASA copyright policy states that "NASA material is not protected by copyright unless noted".

2. The term K-12 makes reference to Kindergarten through the 12th Grade in State educational systems. It should be noted that there may be some particular courses in the Junior and Senior High School grades that may teach the 3rd principle of rounding but the practice is not prevalent.

1.3.13 The third rule is one that the student of measurement technology needs to understand. This includes the cadastral mapper because deed descriptions are a result of measurements. Therefore, the proper mapping of measurements should follow all three of these rules including those of Significant Digits.

1.3.14 The theory of significant digits is based in the principle that a displayed (drafted or mapped) value which represents a measurement (calculated or directly observed), will not have greater accuracy than any value of least accuracy involved in the displayed value.

1.3.15 What this means is that when calculating the area of a parcel with rectangular dimensions of 100.21ft by 95.3ft the calculated result would be (length times width) 100.21 * 95.3 = 9,550.013 Sq.ft. The result, according to the theory of significant digits, would only be noted as 9,550 Sq.ft. This is because 95.3 is the value of least accuracy and contains only 3 significant digits and the rounded result of 9550[1] only contains 3 significant digits. Let's see how this works by examining the rules associated with identifying significant digits.

1.3.16 There are 3 basic rules to identify significant digits. They are;

a. Leading zeros **are not** significant. For instance, 0.0000723 the zeros are not significant, only the 7, 2, and 3 are significant. So there are 3 significant digits in this value, the other zeros are place holders.

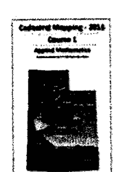
b. Ending zeros **are not** significant unless they follow a decimal point. For instance, 352,000,000 the zeros are not significant, only the 3, 5, and 2, indicating there are only 3 significant digits in this value. In the case where the value 110.00 has zeros to the right of a decimal point the zeros become significant because they are indicating the accuracy of the value. Therefore, 110.00 has 5 significant digits.

c. Zeros that occur between other digits in a value **are** significant. For instance, 50,908 the zeros are significant and this value contains 5 significant digits. The value of 11.0000723 contains 9 significant digits, but notice that when examining the value of 0.0000723 in rule 1 the zeros were **not** significant.

Example 1:

(a)	936.7	has 4 significant digits.
(b)	807	has 3 significant digits.
(c)	240	has 2 significant digits.
(d)	800	has 1 significant digit.
(e)	0.0009	has 1 significant digit.

[1]. This value only has 3 significant digits because the number is shown to have no decimal point. Therefore, the zero is not significant. Had the value been written with a decimal point then the zero would be significant and the value would have 4 significant digits. It is displayed this way so that the principle can be discussed and built on.



(f) 0.054 has 2 significant digits.
(g) 0.0070 has 2 significant digits.
(h) 0.0408 has 3 significant digits.
(i) 24.80 has 4 significant digits.

Exercise 1:

Round off the following values to 4 significant digits.

(a) 469,353
(b) 25,365
(c) 568.954
(d) 0.070065
(e) 7.0055
(f) 0.05235
(g) 864,250
(h) 54,364

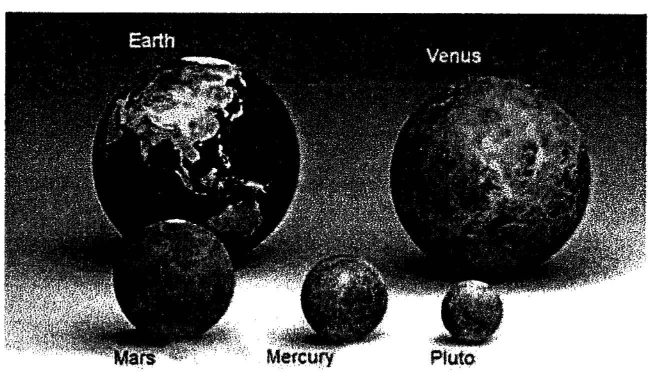
1.3.17 The theory of significant digits also has an exception which is dependent on measured values. The exception states that when the accuracy of the measurement tool is known the value may be displayed to the accuracy of the measurement tool. This is assuming that the measurement tool is being utilized by one trained and competent in measurement technology (e.g. the land surveyor or their qualified staff). *This exception fails when the measurements are made by a layperson.*

1.3.18 In our previous example of calculating the area of a parcel with dimensions of 100.21ft by 95.3ft the result of the calculation was 9,550.013 Sq.ft. and we determined that the value should be displayed to 3 significant digits because 95.3 only has 3 significant digits and 100.21 has 5 significant digits.

1.3.19 If we knew that the measurement equipment used was capable of one hundredth foot accuracy (0.01ft), the exception would tell us that the value should be displayed as 9,550.01 Sq.ft. (being rounded to the accuracy of the measurement tool). However, if the tool used is only accurate to the tenth of a foot (0.1ft) then the value would be displayed as 9,550.0 Sq.ft.

1.3.20 Why would the cadastral mapper need to know this theory? Because the theory of significant digits and the exception related to measurement technology has a direct bearing on the accuracy of a deed description and how that deed description should be properly mapped.

1.3.21 **Counting v. Measuring.**
Explaining the difference between counting and measurements will help to understand this principle a little more. Counting is a function that has no remainder and is not an estimate. Counting is a precise function.

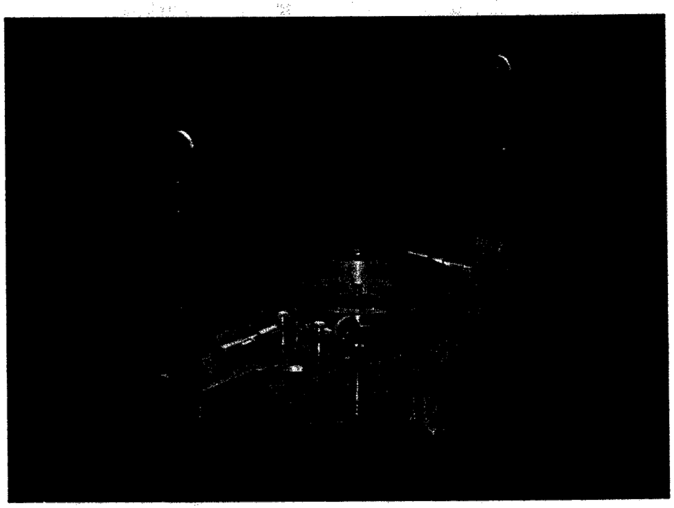


For instance, how many planets are there in the picture? By counting we can determine that there are 5 planets in the photo.[1] You can do the same counting exercise by counting the number of people in a room, the number of windows in a building, or other object counting exercises.

1.3.22 **Precision Indexes.** Measuring an object, especially a property line, is not an exact counting function. Any measurement contains a precision index or measurement error. Simply put, they are not exact. No two individuals can measure a line and arrive at the same precise value. Every measurement has an un-resolvable error contained within it. In surveying, measurement error is unavoidable and cannot be eliminated, however, a mistake or blunder is unacceptable and must be avoided or eliminated.[2]

1.3.23 Measurement error is a result of three main categories which affect all measurements. One, Instrumental, Two, Personal, and Three, Natural. While this is not a discussion on measurement science an understanding of the first may help the student understand why measurements made by two different surveyors can vary.

1.3.24 To make a comparison of technology relating to angular and distance measurements, first compare these two types of angle measurement tools.



Peep-site Transit – this instrument was used to survey Ogden City.

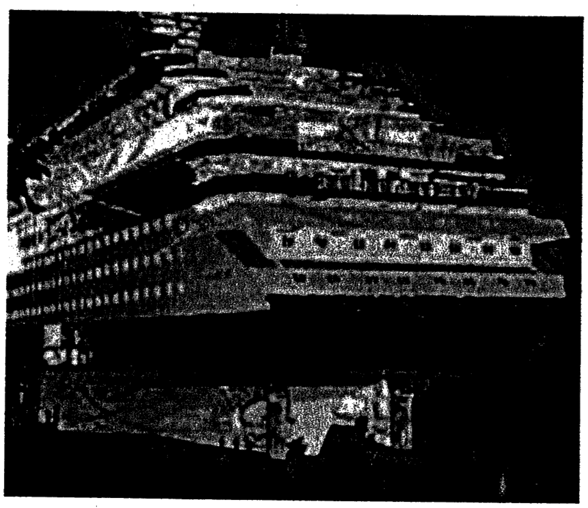
Digital Scanner – collects millions of data points per second.

[1]. There are 5 if Pluto is still considered a planet. "Originally classified as the ninth planet from the Sun, Pluto was recategorized as a dwarf planet and plutoid due to the discovery that it is one of several large bodies within the newly charted Kuiper belt." Credit: Wikipedia on Pluto.

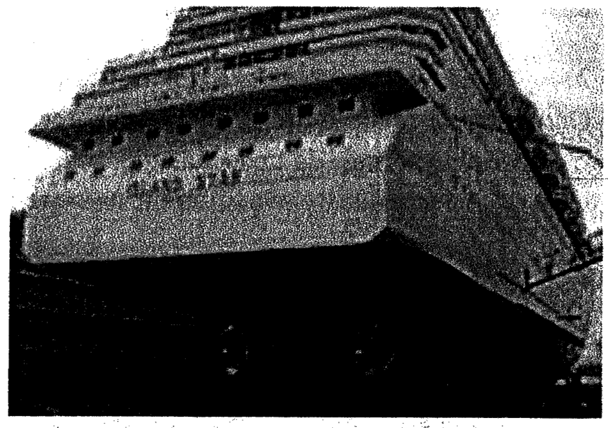
[2]. The discussion of measurement error and mistakes or blunders is beyond the scope of this work. This is a principle of surveying science, however, it is helpful for the cadastral mapper to understand that measurements are not exact. This concept should help the cadastral mapper understand why measurements of the same line of a parcel can differ from person to person and surveyor to surveyor.

1.3.25 The Transit is the typical type of instrument used in the original GLO surveys of the PLSS. The instrument was only capable of measuring an angle to the nearest degree of arc, however, a skilled Instrumentman may have been able to estimate increments of 15 minutes of arc.

1.3.26 The Digital Scanner is some of the newest technology today and produces a three dimensional scan of anything in view of the instrument. An example of a scan is shown below.



Digital scan of a ship.[2]



The ship that was scanned.

1.3.27 The ability to more precisely measure today from what was capable a hundred years ago has changed in a dramatic way. The ability of the GLO surveyor using a Peep-site Transit to measure a section line was far less than it is today with GPS and Scanning technology.

1.3.28 When a layperson makes their own measurements and records those measurements in a land description they are most likely misrepresenting the true length of the line measured. Such line may be longer or shorter than that which was noted and reported in the description. This would be especially true in the case where the individual making the measurement laid the tape measure so that it is flat on the ground.

1.3.29 If the ground has any elevation change the measurement will represent a SLOPE distance and not a HORIZONTAL distance. All measurements made by the land surveyor are reduced to the horizontal component. No land description, plat, or mapping product prepared by a surveyor are slope lengths. If slope lengths were used in the descriptions – no description would be able to close.

[2]. These images of the ship and the scan were obtained from the website of Arc Surveying & Mapping, Inc. Copyright © 2013. All Rights Reserved.

1.3.30 This reduction is made so that two dimensional coordinates (x, y or Northing and Easting) can be used and plain trigonometry and plain geometry can be used in the calculation processes. The other reason all measurements are reduced to their horizontal equilivant is so that the ground distortion is removed so that land shapes and areas can be more accurately represented on a flat sheet of paper.

1.3.31 I the situation where the layperson has made measurements they less likely to know this distance reduction needs to be made and as a result their measurements will contain distortion that when placed on the flat sheet of paper or coordinate system the metes of the description will contain significant measurement error. Such information would be a more or less distance or angle (bearing or azimuth) regardless of whether the written distance stated so or not. Consider the following example.

640 Ac Reg. Sec

Example 2:

Ex.2.1. John is patented regular Section 26.

All federal GLO and BLM Section survey measurements are noted as the horizontal component of the line.

Ex.2.2. John sells to Tom the northeast quarter of the section which they have surveyed by a land surveyor and they fence the property as surveyed.

Ex.2.3. Tom sells to his son Kevin a building lot in the southwest corner of Tom's property and they prepare a description of the lot from measurements they made. They used the existing fences John and Tom built in accordance to their "survey" a few years prior as the beginning of the description.

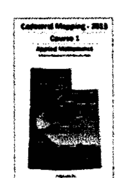
Ex.2.4. The description they prepared read as follows:

Beginning at the Southwest corner of the Northeast Quarter of Section 26, ... some base and meridian; Thence North 150 feet; thence East 291 feet; thence South 150 feet; thence West 291 feet to the place of beginning. Containing 1 acre.

Ex.2.5. Tom and Kevin fence the new building lot as **they** measured it.

Ex.2.6. **Now let's ask the question, how much land was sold?** This discussion is *not* going to delve into legal questions of ownership nor what constitutes the boundaries, it is simply going to evaluate the description with respect to the theory of significant digits and rounding. Principles of mathematics, statistics, and counting vrs. measuring will also be discussed.

Ex.2.7. The fact that the land was "surveyed" by the owners brings the question of how much land was actually sold. The law states that what the owners actually did on the



ground is what was actually conveyed, despite what they may have written in a description. The effect of the law on descriptions will be discussed more fully in Course 2 so for now we will only examine the description as written.

Ex.2.8. So going back to the question of how much land was sold, let's add, how much land was being sold **as described**? The simple response would be one acre, that is what the description states. Because the description includes the acreage there is no reason for any further evaluation.

Ex.2.9. The number of digits being displayed in the description can be important with respect to an interpretation of the writing especially if the description was written by a surveyor. For instance the description stated that there is 1 acre, not 1.0 nor 1.00 acres, just 1 acre being sold. In **measurement science** it is understood that the value being displayed has meaning.

Ex.2.10. The problem with descriptions is that many are written by individuals that have no concept of measurement science. Leaving the cadastral mapper with a dilemma in being able to properly interpret the description and not assign to precise of an accuracy to descriptions that are not prepared by surveyors. Resulting in conflicts that present themselves as overlaps and gaps.

Ex.2.11. So in evaluating this number as a **measured** value by a surveyor the 1 has inherent with the value a tolerance or range. In this case it would be 0.5 acres plus or minus and we could write the value as 1 ± 0.5 acres. So we have a range between 0.5 and 1.5 acres. Had this description been written by a surveyor, the digits displayed in this value as written may be an indication of the precision of the surveyors work[3], but because in our example it was written by the land owners we will not discuss the subject of precision here.

Ex.2.12. When we see from the description that the distances or lengths of the sides are to the nearest foot indicating that the area is ± 0.5 acres seems to be quite unreasonable. So let's see what the **metes** of the description tell us about the area.

Ex.2.13. We can see from the bearings and distances contained therein, that the property is a rectangle having sides of 150 feet and 291 feet. Therefore, using the equation for calculating area (Length times Width) we have 150*291=43,650 Sq.ft. This is 90 sq.ft. more than 1 acre (that being 43,560 sq.ft.). Converting sq.ft. into acres we would have 1.002066116 acres.[4]

Ex.2.14. If we now examine the acres with respect to significant digits, the 150 has 2 significant digits and 291 contain 3 significant digits. So rounding to 2 or 3 significant digits we would have1.0 or 1.00 acres. Although this use of significant digits is correct,

[3]. There is an exception to this evaluation even when the description is prepared by a land surveyor. The exception would be related to what the standard of practice is for final survey documents.

[4]. T his value is displayed to the 9^{th} decimal place and should not be understood that 9 places would be all that was necessary to utilize in the calculation process. This is being used as an example of significant digits only. The concept of how many decimal places is required in the calculation processes will be discussed in more detail later.

showing area to the tenth or hundredth of an acre may give a false representation of accuracy.

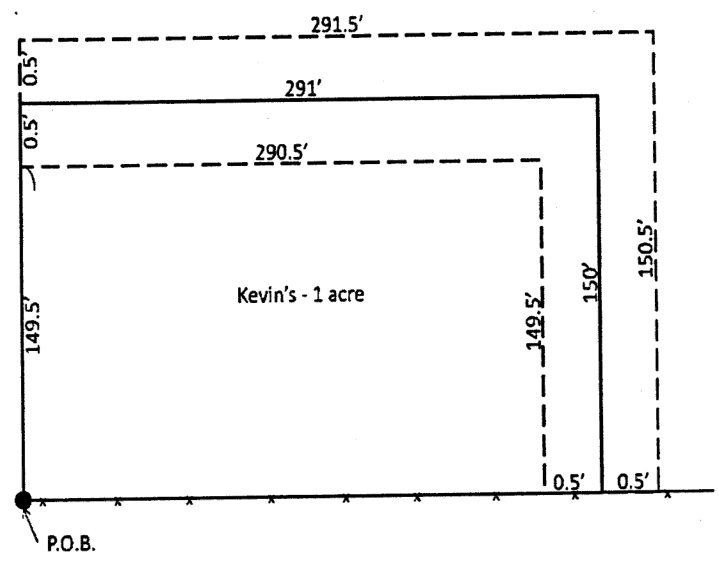
Ex.2.15. By showing the area as 1.00 acre we are saying that the area is accurate to plus or minus five thousandth of an acre (0.005 ± acre). In other words, the area could be written as 1.00 ± 0.005 acre. This would give a tolerance or range between 0.995 acre and 1.005 acre.

Ex.2.16. Notice that 1.005 is now larger than the calculated value of 1.002066116 acres. To complete our evaluation we see the difference in area is now approximately 435.6 sq.ft. which is much greater than the 90 sq.ft. originally started with. Using the theory of significant digits in this example, the area has been degraded in accuracy by about 20 percent.

Ex.2.17. This discrepancy between the 1 acre and the calculated area of 1.002066116 acres requires further evaluation to be made to understand what this description may have intended to convey.

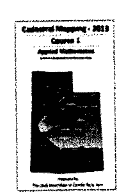
Ex.2.18. The evaluation of area that we have done to this point is assuming that the 150 and 291 foot distances are precise measurements. Because we know that they are not precise another evaluation can be made with respect to the measurements as written.

Ex.2.19. We begin by understanding the same principle of tolerance or range that we examined with the 1 acre. That is, the two distances are shown only to the nearest foot. Using the principle of tolerance we would say that the 150 feet is actually 150 ±0.5 feet and the 291 feet is actually 291 ±0.5 feet.



Ex.2.20. Thus, we have a range for one side of the property of 149.5 to 150.5 feet and another range for the other side of the property of 290.5 to 291.5 feet. If we then calculate the area for the two ranges we have 149.5*290.5=43,429.75 sq.ft. and 150.5*291.5=43,870.75 sq.ft. which is a difference of 441 sq.ft.

1.3.32 The reason for going through this exercise is to show that just based on the numbers contained in this simple description and using the principle of significant digits and measurement science, the differing results make it difficult for the cadastral mapper to know what the area should be for the tax role.



Because this description contains a Patent Ambiguity[5] the only way to determine what was actually intended to be conveyed is to survey the property. Remember, they (the grantor and grantee) fenced the property and the law recognizes the fence as the true intent of the conveyance and not the writing.[6]

1.3.33 Furthermore, it is important that the cadastral mapper understand that in processing these documents the precise writing of a description should be retained in the tax record. No alteration is acceptable because the intent and meaning of the description can be changed by something as simple as changing the area from 1 Acre to 1.0 Acres, adding or subtracting accuracy to bearings, or changing a distance from 150 to 150.0 feet, or leaving out a call for fencing or other bounds call.

1.3.34 It can be seen from this evaluation that small, seemingly unimportant, changes can make a significant difference in how the description is to be examined, surveyed or mapped.

1.3.35 So how is the cadastral mapper to know the accuracy of a description? Without knowing the type of measurement instrument used, the people making the measurement (were they surveyors or not), and the physical conditions that the measurements were made under, I don't believe that a cadastral mapper can know the accuracy of a description because the true area is a process of surveying.

1.3.36 Since the goal in this discussion is to obtain a value that can be used for tax assessment. It may not be necessary to go beyond the deed as written but to continue the evaluation with one more example to see how the type of measurements can change the entire meaning of a description, let's add one more circumstance to this example.

1.3.37 Let's say that when the 150 feet was measured by Tom and Kevin that they paced the distance. Pacing is a legitimate form of measurement and can give approximate results depending on the skill of the individuals doing the pacing.

1.3.38 Now the question is how accurate could one or both of them pace a distance? Where they indicated by the writing that the measurement was made to the nearest foot, perhaps the truth is that their pacing was only to the nearest 5 or 10 feet over the total length. Now the intent of what was actually conveyed may become a little more difficult to determine.

1.3.39 Even if we were too assume that their pace was fairly consistent, what was the actual length of their pace. Was it consistently 6 inches short? Or was it consistently 6 inches

[5]. Ambiguity's come in two types, Latent and Patent. Without getting to a detailed discussion of the law suffice it to say that Latent ambiguity is a mistake or discrepancy between what is written and what is actually found on the ground, Patent ambiguity is a mistake that is visible or found in the writing of the deed.

[6]. In the limited circumstance being described the fences become the boundaries of the property but it should be noted that not all situations are this simple. Not all fences are property boundaries and not all deeds describe the boundaries.

long? This would take the evaluation a completely different direction and we would evaluate the lengths with respect to a Systematic Error.[4] A completely different evaluation could be made if their pace was erratic.

1.3.40 We could say that each pace was short by 0.5 feet and that they believed that their pacing was precisely 3 feet long, each step. They then stepped off 50 paces. What they believed to be a distance of 150 feet was actually short by the 6 inch error per pace. Then we can see that the distance they believed to be 150 feet was actually only 125 feet (50 * 0.5 = 25 and 150 – 25 = 125).

1.3.41 As can be seen from the evaluations that we have done, if the description contains the area of the parcel then the value written in the deed should be utilized as the taxable area[5] and not a calculated value from the metes of a description because the two will probably contradict each other.

1.3.42 However, what if the description does not have an area listed? A calculation is then necessary to identify the taxable area. To begin the discussion on the process of calculating these areas a discussion related to the One Digit Rule should be made.

1.3.43 **One Extra Digit Rule.**

1.3.44 The essence of the one digit rule is to prevent rounding errors in calculated values. As previously discussed the easiest way to prevent rounding errors is to simply not round in the calculation process. However, the use of the One Extra Digit Rule will also help in knowing if you are using enough digits in the calculation.

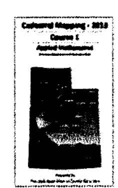
1.3.45 The rule is simple. If you need to have accuracy in a measured distance of hundredths of a foot all the values used in a calculation need to be carried to the thousandth of a foot plus one extra digit (meaning the ten thousandths). This includes the use of constants. Let's examine how this works.

metes - numerical call
& bounds - to the object called
↳ * takes precedence over metes * unless it is unknown or unidentifiable

4. Systematic Errors may be present in any measured quantity. A professional surveyor will have measurement techniques and processes in place to eliminate (or at least minimize) the effects of systematic errors. A discussion on this principle is beyond the scope of this work.

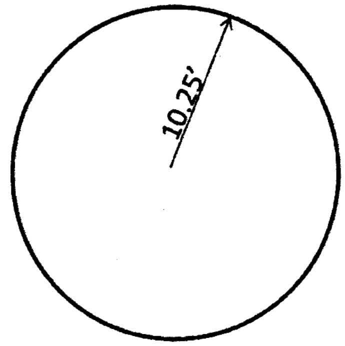
5. Having stated this, the area should always be checked to see that the value contained in the deed is not in gross error. If there is a mistake made in the description that value should be corrected and an appropriate note placed in the tax record. But before this correction can be made a thorough evaluation must be made in relation to tolerances. The discussion on tolerances is made later in the chapter.

Don't Round your Calculations!!



1.000 AC = 43560 ft sq.

1.3.46 When calculating the circumference of a circle the value of π need only be noted to the 4[rd] decimal place if accuracy is required to the hundredth foot.



Calculate the Circumference

Cir = 2 π r for a circle having a radius of 10.25 feet the equation can be calculated as -

Cir = 2 π 10.25

1.3.47 To evaluate the effect of rounding and the one extra digit rule let's start by using a value of 3.141592654 for π.

Cir = 2 * 3.141592654 * 10.25

Cir = 64.402649399

Cir = 64.40 rounded to hundredths.

1.3.48 Rounding the value of π to **4 decimal places** results in the following...

Cir = 2 * 3.1416 * 10.25

Cir = 64.4028

Cir = 64.40 rounded to hundredths.

1.3.49 Truncating the value of π to **3 decimal places** results in the following...

Cir = 2 * 3.141 * 10.25

Cir = 64.3905

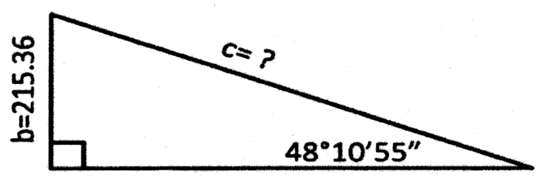
Cir = 64.39 rounded to hundredths.

1.3.50 It can be seen from the example that to prevent rounding in the 2[nd] decimal place the values used in the calculation process need to be carried to the 4[th] decimal place. So to maintain accuracy in the hundredth place the thousandths place need to be prevented from unnecessary rounding so that the 3[rd] place will not adversely affect the 2[nd] place. To accomplish this One Extra Digit must be utilized that being the 4[th] decimal place.

1.3.51 Let's examine this concept when working with angular values. When calculating angles the calculator input is made by replacing the degree symbol with the decimal point. An angle of 48°10'55" is keyed into the calculator as 48.1055. Notice that the value now has a

minimum of 4 decimal places to display the seconds of an angle. Let's solve the following problem...

$$\text{Sin } 48°10'55" = \frac{215.36}{c}$$



Rearrange the equation to solve for 'c'.

$$c = \frac{215.36}{\text{Sin } 48°10'55"}$$

Convert the angle from base 60 to base 10 (this concept will be developed more fully in a later section).

$$c = \frac{215.36}{\text{Sin } 48.181944444}$$

Apply the sin function to the decimal degree value which results in the following fraction, then complete the evaluation of the fraction.

$$c = \frac{215.36}{0.745265919}$$

c = **288.970680801**

1.3.52 So far the processes applied in the calculation have been carried out to the 9th decimal place and results in a value for 'c' of **288.970680801** units. Let's examine the same calculation if rounding is incorporated to the 4th decimal place.

$$c = \frac{215.36}{\text{Sin } 48°10'55"}$$

Convert the angle from base 60 to base 10.

$$c = \frac{215.36}{\text{Sin } 48.1819}$$

Apply the sin function to the decimal degree value which results in the following fraction then complete the equation.

$$c = \frac{215.36}{0.7453}$$

c = **288.9575**

1.3.53 The result this time for 'c' is **288.9575**, notice that the result is different in the 5^{th} decimal place. When calculating distances that we are only required to have an accuracy to the nearest hundredth (1/100) foot accuracy in the calculation process where angles and their trigonometric functions are involved the values cannot be rounded to the 2^{nd} or even the 4^{th} decimal place.

1.3.54 Working this same problem and rounding the values to the 5^{th} decimal place will result in 'c' equaling 288.9729 which demonstrates that 5 decimal places, at a minimum, is required to maintain accuracy in the hundredth place in this calculation sequence.

1.3.55 What this also demonstrates is that to maintain accuracy in the 4^{th} decimal place it is required to use values greater than 5 decimals. This would be necessary when the calculated distance may be required to be used to calculate an angle and maintain accuracy to the second of arc. So working this same problem and rounding the values to the 6^{th} decimal place will result in 'c' equaling 288.9706494 showing that accuracy is maintained to the 4^{th} decimal.

1.3.56 While this appears to be breaking the One Extra Digit Rule the reason that additional accuracy is required in this process is because there is more than one calculation being made with rounded values. The more steps in a calculation that are required to obtain a final result, the greater the number of decimal places are required to maintain accuracy especially in angular values.

1.3.57 When the cadastral mapper is required to calculate an area based on a description for taxing purposes, the tax record provided to the Assessor and the public should indicate that the area value was calculated by the County Recorder's Office and that it is based solely on the information provided in the deed description and that the values may have been rounded.

1.3.58 By doing this, the Recorder's Office will not be indicating false accuracy that may be misused or misunderstood. The notation will also provide the land owner notice that, should they disagree with the value, they know what the originating source was. Thus, giving them the opportunity to take some affirmative action to correct or update the value.

1.3.59 This notation will also provide the County with the knowledge that the area was calculated by the recorder's office and if the land owner could demonstrate that the calculation was

either incorrect or that the land owner could produce a survey of the property that shows what the area should be the taxable area could be updated.

1.3.60 *A note in regard to this updating process.* If there is a value to be changed in the tax data base there should be a method of tracking why the value was changed. This could be done in the form of a recorded affidavit signed by the protestor (owner), a signed statement to be included in the "Out Card" file[7], or some other method that your county has to keep track of changes that affect the valuation of parcels.

1.3.61 The use of significant digits will help in being able to know what accuracy to display an area that has to be calculated for the tax roll. Although, there are exceptions to every rule some judgment may need to be exercised.

1.3.62 So to complete the discussion on *Kevin's acre* assuming that the deed does not have the acreage included.

150 * 291 = 43650 sqft which is

1.0020661166 acres.

It may be a good rule of thumb that when parcels are an acre or less in size that the area be shown to the nearest square foot. And when the parcel is over an acre the area could be noted as acres.

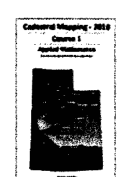
1.3.63 One of the principles associated with precision indexes is that the result of a calculation process cannot have greater accuracy than the original values. With the principle of significant digits the value 150 has 2 significant digits and 291 has 3 significant digits, therefore, the final value for the area in acres should only retain 2 or 3 significant digits. So1.0 or 1.00 acres would be the value used for the tax roll.

1.3.64 However, lets assume that the values were displayed to the hundredth foot as in 150.00 feet and 291.00 feet. Each value has 5 significant digits and if we followed the rules of significant digits the acreage would be displayed as 1.0021 acres and may result in a greater impression of accuracy than originally intended - or does it?

1.3.65 Using the area to the 4th decimal place and making the reverse calculation for sq ft we get.

1.0021 * 43560 = 43651.476 sqft. This produces an error of 1.476 sqft in the reverse calculation.

7. The term "Out Card" is used in the Weber County Recorder's Office to identify the old manual Tax Card system of parcel identification. The system was used to track changes associated with the property including such information as the Areas, Parcel Numbering, Description, Ownership, Mailing address, etc. Today changes are made electronically but the office still keeps a paper record of the "Out Card" transactions so that there is a written history of what was changed and why the change was made.



1.3.66 **Zoning Issues:**

1.3.67 There is another complication that should be taken into account when deciding to what accuracy to display the taxable area. That may be the zoning of a parcel. Many times a building permit may be approved or denied based on what our records show with respect to the area of a parcel. If the deed does not have the area noted and the county is required to calculate the area it should be done as precise as possible.

1.3.68 To see how this works, some zones have a 1 acre minimum requirement, some may only require 40,000 sqft, and yet others may have much larger areas such as 3 acres, 5 acres or even 40 acres. Improper rounding or displaying without the proper accuracy may cause difficulty for an owner in obtaining a building permit. This is because the planning offices use the areas shown on the Ownership plats as a rigid value in determining the conforming status of a parcel.

1.3.69 Therefore, if the county is making the area calculation and has not noted the source of the area is it possible that people are being denied permits when they are entitled to receive them. The grandfathering of parcels should take this into account, giving the owner the opportunity to provide the county with an area, not just for taxing but also for zoning issues. While the County Recorder has no control over zoning or development rights it is something that the recorder can provide information with respect to the source of an area.

1.3.70 **But how can the benefit of the doubt be given to the public?**

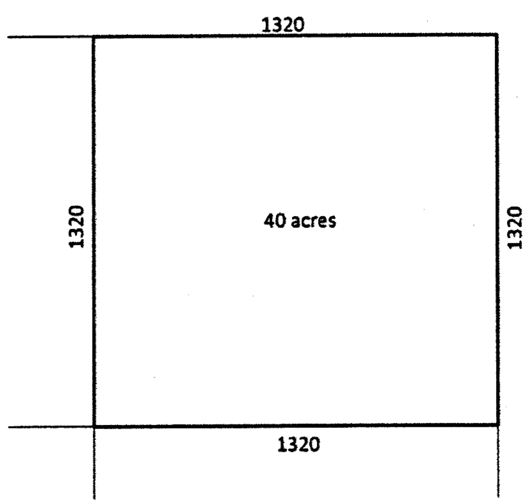
1.3.71 We have seen from our examples that there can be several hundred square feet of difference in the area of a parcel depending on what tolerance method of evaluation we utilize to obtain an area. It would be good for the mapper to know what the zoning size is that a parcel resides in so that if the calculation of area is below that minimum size by a slight amount the same type of evaluation that we have demonstrated can be made to see which process may be used to prevent the Taxable Area from indicating that a parcel violates the minimum size requirements. Thus, causing a zoning violation based on our calculation.

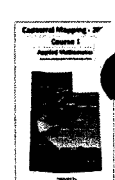
1.3.72 **Exceptions:**

1.3.73 As with all things there are exceptions to the rules. In dealing with taxable areas there could also be another evaluation with respect to large farms or ranch parcels that have to be calculated.

1.3.74 The size of the parcel can also affect the amount of error in the lengths of lines of the parcel. Using a 40 acre parcel as an example we can demonstrate this concept. In general terms a 40 acre

The NE ¼ of the NE ¼ of Section 10, ...





parcel is thought of as a quarter of a quarter Section. Based on a standard Section the dimensions of the tract would be a square each side being 1320 feet in length. Using this assumption we can evaluate the effect of size and length error. We are also going to assume that the tract is a standard 40 acre tract.[8]

1.3.75 The mathematics to calculate the area of this parcel is quite simple.

1320 * 1320 = 1742400 ft sq

1742400 / 43560 = 40 acres

1.3.76 When the Patent from the Federal Government was issued, the area for this tract included a notation of size, that being the same 40 acres that the approved GLO plat map indicated for this parcel. What is important for the cadastral mapper to understand on this principle is the intent of the government with respect to the size and shape of the parcel. This subject will be dealt with in more detail in Course 2, however, to give a brief understanding of this concept – the Federal Government was in the process of disposing of the Public Lands as rapidly as possible to help pay off Federal debt.

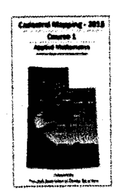
1.3.77 This need meant that as long as there was value paid for this land the Federal Government did not care if the property conveyed was greater or smaller than what the record indicated. Furthermore, the government would not come back and try to collect more money should it be found that the parcel was greater than stated, nor would they attempt to refund money if the parcel was found to be smaller. The State Tax Commission operates on the same principle with one exception, the citizen can protest the size of the parcel and thus contest the value.

1.3.78 The federal government did not give the citizen the opportunity to protest the area. In fact, because there was no protest ability there was an un-stated practice by the government to have the lengths of the Section lines measured long so that the government could not be as easily accused of shorting someone of land.

1.3.79 So what does that have to do with size related to taxable acreage for assessment purposes? If you were to ask any surveyor if they have ever found a section to be a square mile (which would make the 40 acre tract a square with sides of 1320 feet) the response without exception would be that they have not ever found one to be such. This means that when the record is showing a square section and square tracts of standard size, the truth is that they are not that size and shape on the ground or the title.

---

[8]. This evaluation is being done in regard to several assumptions each of which are not found in the real world. There is no such thing as a "Standard" section. The Section, by law, is defined as that which the GLO surveyor laid out and marked on the ground, not what was reported in the notes and platted on the plat document used in the patenting process. The technology of the day when Sections were being surveyed and monumented were not what we would consider highly accurate tools.



1.3.80 They are always something different than what is reported in the Patents and on the GLO plats. This fact was fully known to the government and in reality the system was designed to accommodate this fact. Without getting into the surveying and legal reasons for this situation let's return to a discussion on an evaluation of area.

1.3.81 We could then say that because we know the area is not as the GLO reported it to be the size of the tract could be larger or smaller than the documents represent. In evaluating this situation we could assume that the distances along the sides were accurate to the nearest foot. This would give us the evaluation process that we did with Kevin's acre. Where the range would be 1320 ± 0.5 feet. This means that:

1319.5 * 1319.5 = 1741080.25 ft sq

1741080.25 / 43560 = 39.96970 acres

AND:

1320.5 * 1320.5 = 1743720.25 ft sq

1743720.25 / 43560 = 40.03031 acres

1.3.82 Or would the evaluation be more appropriate in examining the area which was described as 40 acres. This indicates a range as well and would be 40 ± 0.5 acre. That means that:

39.5 * 43560 = 1720620 ft sq

$\sqrt{1720620}$ = 1311.72406 feet on each side.

40.5 * 43560 = 1764180 ft sq

$\sqrt{1764180}$ = 1328.22 feet on each side.

1.3.83 Another examination and probably the correct one to use in evaluating the area of a deed that uses an aliquot part description is to evaluate the distance tolerance in terms of chains. This indicates that the sides would be 20 chains plus or minus half a chain. That means that the tolerance for the sides would be:

19.5 * 19.5 = 380.25 sq chains which equals 38.025 acres.

AND

20.5 * 20.5 = 420.25 sq chains which equals 42.025 acres.

1.3.84 Using the Area Tolerance with respect to chain measurements reveals a tolerance of 38.025 to 42.025 acres and gives a latitude of 4 acres.

1.3.85 Evaluating the distance component with significant digits the value of 1320 feet has 3 significant digits. This may indicate that the distance is only accurate to the nearest 10 feet. Again, without attempting to discuss the science of surveying measurement, suffice it to say that the original GLO surveyor may have been lucky to obtain accuracies to the nearest 10 feet or the nearest ½ chain.

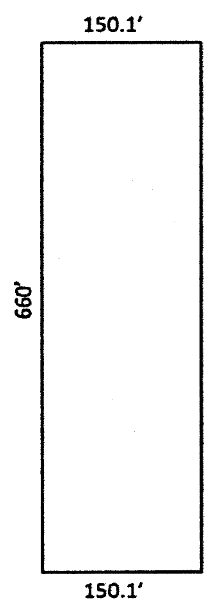
1.3.86 You can see from this evaluation that statistically a 40 acre parcel may be greater or smaller than 40 acres by as much as 4 acres. There is another complication in this process of area calculation of irregular tracts which has to do with parcels that contain Closing Errors. Closing errors in descriptions will be dealt with in Course 2.

1.3.87 Now to the real reason for going through this evaluation is to demonstrate that without a competent and accurate survey of the land the actual area of a parcel is only a BEST GUESS. That relying on assumed measurements from records may give a false value for area. To further develop this subject complete the following exercises.

1.3.88 Exercise 2:

1.3.89 Calculate the possible areas for the Tract A using Significant Digits and tolerances.

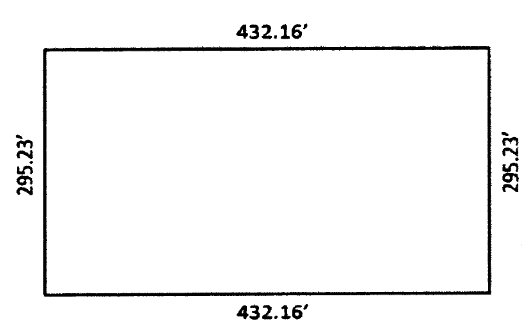
1.3.90 Hint. Remember that according to the theory of Significant Digits the result should not have more significant digits than the value with the least number of significant digits which was used in the calculation process.

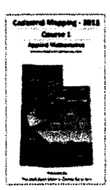


Tract A

1.3.91 Exercise 3:

1.3.92 Calculate the possible areas for the tract A using Significant Digits and tolerances.

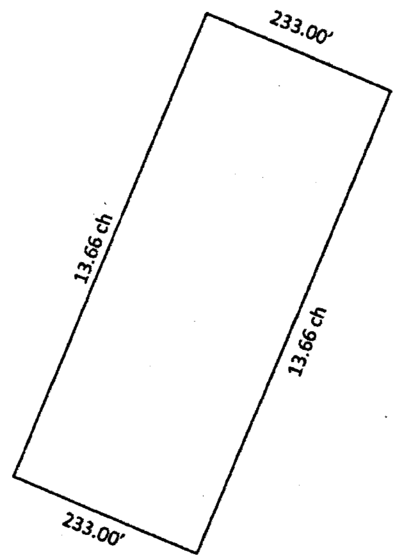




1.3.93 After completing the exercises then return to the following paragraph.

1.3.94 As long as there are not a mixing of feet and Chains the calculated results can be fairly straight forward. So looking at the situation where a parcel is described using feet and chains let's see how to go about calculating the area of the parcel.

1.3.95 **Mixing of units of measurement.**



1.3.96 In this problem the side lengths recorded are 13.66 chains and the ends are 233.00 feet in length. Calculating acres using the information given we would convert chains to feet (13.66 * 66 = 901.56') and multiply this value with 233.00 feet resulting in 210063.48 sq ft or 4.82239 acres. This gives a value for the parcel and a place with which to use as a base line in evaluating tolerances.

1.3.97 To see why making the calculation in this manner can be an issue let's examine the chain units.

1.3.98 The chain is noted to the hundredth of a chain. There are 100 links in a chain. Each link is equal to 0.66 feet. So the accuracy of the chain value is to the nearest 2/3 foot or about 8 inches.

1.3.99 Because the two distances started as different units it may not appropriate to simply multiply the length by the width to get the area. To correctly evaluate this problem use the principle of significant digits and tolerances. In this case each distance will have different tolerance ranges. For instance;

1.3.100 Because the chain value is noted to plus or minus 1/100th chain the tolerance would be found to be plus or minus 0.005 chain. Therefore, the range is 13.655 chains and 13.665 chains. Converting these ranges into foot equivalents the values would be 901.23 feet and 901.89 feet.

1.3.101 The width of the parcel being 233.00 feet indicates that the tolerance is plus or minus 0.005 foot. (Note that the two are vastly different since 0.005 chain is 8 inches and 0.005 feet is about 1/16th inch.) The distance would have a range of 232.995 feet and 233.005 feet. Calculating the area from the two tolerance ranges:

901.23 * 232.995 = 209982.08385 sq ft or 4.82052 acres.

And:

901.89 * 233.005 = 210144.87945 sq ft or 4.82426 acres.

It must be noted that tolerances of 0.005 feet in land surveying can not be achieved. However, for the discussion the use of the mathematical principles are appropriate. The subject of surveying measurement accuracy will be developed in a later discussion.

CONCLUSION:

1.3.102 What the last 3 exercises have shown is that all area calculations contain error and there is a statistical means of evaluating that error through the use of tolerances. This analysis in each case revealed a range associated with the area. It is easy to analyze with a parcel that is a square or rectangle and derive a range related to tolerance. It becomes more difficult to make the same analysis of multi-sided parcels.[9]

1.3.103 The purpose of this discussion is to help the student understand that areas, no matter how precisely they are calculated, contain error. And when discussing areas used in taxation the purpose is to give the Assessor a basis to determine value in which taxes are levied. The goal of the mapper should be to provide an area, when none is in the deed, that is reasonable for the process of assessment. And if possible, demonstrate to planners that a ridged adherence to areas calculated by the recorders office is not reasonable nor in the best interest of the public.

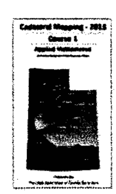
1.3.104 **Self Reporting System:**

1.3.105 The tax system is a self reporting system. Meaning, with respect to real property, to have a claim to land that is more easily proved a system of recording deeds has been established. The claim of title through documents that have been changed or are fraudulent is less likely once recorded. The document then becomes evidence of the writing and intent as of the date of the recording.

1.3.106 The reason our system has been designed as it has is so that the public has fewer means to dispute taxes based on the size of the parcel. This was deliberate and intentional in the design of the system of land registration because it was recognized (as it is today) that all measurements contain some amount of error. This is true regardless of whether the land owner, surveyor, or someone else performs the measurements. Recognizing this fact the system was established so that the owner would declare the size of the parcel in the deed and subsequently record that with the government.

1.3.107 This system helps to prevent these types of disputes yet there are still deeds that fail to declare the area and in those cases the recorder is obligated to establish some value for

[9]. A simple method for evaluating an irregular parcel would be to assign tolerance to each distance in the parcel and then evaluate the area based on those tolerances.



taxation. In doing this the public has the ability to dispute the area but should do it is a way that the recorder can rely on.

1.3.108 When disputes arise over calculated areas the public should be informed that the calculated value was based on the document that they placed in the record and that to change the calculated area they should have a survey done that can accurately identify their ownership and parcel size then have the survey filed with the County Surveyor and record a new deed with the surveyed information with the County Recorder.

1.3.109 Many times the land owner will not want to pay for a survey to get this information and want to know if they have to have the survey. I always respond to this question that there is no law requiring that they have a surveyor do the work for them but if they have someone else (meaning one that is not a surveyor) do work on their property to identify their property lines the person doing that work, even if it is the land owner, could be charged with a violation of law and cited for surveying without a license.[10]

1.3.110 Without having to delve into the science of surveying measurement and the theory of Error Analysis, we could develop some rules of thumb that may help simplify the tax area calculations.

1.3.111 **Rules for your Thumb:**

a. If the parcel is 1 acre or smaller the size should be displayed in square feet and rounded to the nearest square foot.

b. If the parcel is greater than 1 acre the size should be displayed in acres. The use of significant digits should be employed to determine how accurate the value is to be displayed. However,

c. If the parcel is greater than 40 acres the size could be rounded to the nearest acre.

d. If the deed has no area and the office has to calculation one there should be placed in the tax record a note or notice letting the public know that the area was derived by the county.

e. In the case of aliquot part descriptions, using the record acreage from the GLO plats would be appropriate unless the parcel has been recently surveyed and the survey demonstrates that the area on the GLO plats is in error.

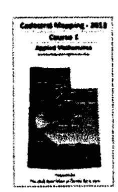
10 . The reason is that a property line is not a boundary of exclusive ownership. Every boundary line has at least two owners. The State has recognized that it is in the best interest of the public to have a dis-interested third party identify the boundary. This is done to minimize land disputes.

1.3.112 Each office of recorder has their own policies and should this information be helpful in this regard perhaps some of the Rules of Thumb could become part of your policies. As with any policy, always check with your elected official before implementing changes to how your office does it's business.

US CODE
43.

Title starts w/ Patent
(federal gov)

*What AC on the deed
use acres +
Don't change



SOLUTIONS:

Exercise 1: Solutions for 1.3.16 Exercise 1:

(a) 469,400
(b) 25,360
(c) 569.0
(d) 0.07006
(e) 7.006
(f) 0.05235
(g) 864,200
(h) 54,360

Exercise 2: Solutions for 1.3.75 Exercise 2:

Solution 1. Calculate as written.

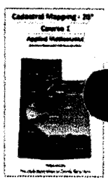
1. The two distances are; 660 ft and 150.1 ft.
2. 660 has 2 significant digits and 150.1 has 4 significant digits.
3. Since this is a rectangle, multiply the sides to obtain the area as displayed.
   660 * 150.1 = 99066.0 ft sq
4. Since the result is greater than 43560 ft sq calculate the area in acres.
   99066 / 43560 = 2.27424242424 acres
5. Since the least number of significant digits is 2 the area could be displayed as **2.3 acres.**

Solution 2. Calculate using tolerances.

1. The two distances are; 660 and 150.1 feet.
2. 660 has 2 significant digits and 150.1 has 4 significant digits.
3. Evaluating the distances relating to tolerances,
4. 660 is shown to the nearest 10 feet based on the number of significant digits.
5. This gives a tolerance of plus or minus 5 feet.
6. Therefore, the range for the sides are 655 and 665 feet.
7. 150.1 is shown to the nearest tenth (0.1) feet.
8. This gives a tolerance of plus or minus 0.05 feet.
9. Therefore, the range of the top and bottom are 150.05 and 150.15 feet.
10. Multiplying the two different ranges.
    655 * 150.05 = 98282.75 ft sq
    98282.75 / 43560 = 2.25626147842 acres
    And:
    665 * 150.15 = 99849.75 ft sq
    99849.75 / 43560 = 2.29223484848 acres
11. Rounding to the least significant digits for both areas the values could be displayed as **2.3 acres or 2.3 acres.**

Exercise 3: Solutions for 1.3.78 Exercise 3:

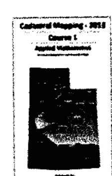
Solution 1. Calculate as written.



1. The two distances are; 295.23ft and 432.16 ft.
2. Each distance has 5 significant digits.
3. Since this is a rectangle, multiply the sides to obtain the area as displayed.
   295.23 * 432.16 = 127586.596800 ft sq
4. Since the result is greater than 43560 ft sq calculate the area in acres.
   127586.596800 / 43560 = 2.92898523416 ft sq
5. Rounding to 5 significant digits the area can be displayed as **2.9290 acres**.

Solution 2. Calculate using tolerance.

1. The two distances are; 295.23ft and 432.16 ft.
2. Each distance has 5 significant digits.
3. Evaluate the distances relating to tolerances,
4. Because each distance is noted to the hundredth of a foot the distances indicate that they are plus or minus 0.005 feet. (Technically this is unrealistic to expect in measurement science but that will be discussed in Course 2.)
5. Multiply using the tolerances.
   295.225 * 432.155 = 127582.959875 sq ft
   And;
   295.235 * 432.165 = 127590.233775 sq ft
6. Converting square feet to acres.
   127582.959875 / 43560 = 2.92890174185 acres
   And;
   127590.233775 / 43560 = 2.92906872762 acres
7. Rounding to significant digits the area can be displayed as either **2.9289 acres and 2.9291 acres.**
8. If the values were rounded to the 3rd decimal place then they would be virtually the same.



L*W = AREA

APPLIED MATHEMATICS:
Course 1. Chapter 4.

1.4 Algebra:

1.4.1 To begin with, this subject is not going to be an in depth study of the principles of Algebra. To do so would require several semesters of college level course work. In this training we will only be covering enough of the subject so that the cadastral mapper can effectively make the necessary calculations relating to the accurate mapping of parcels, roads, highways, railroads, subdivisions, easements, and other boundaries as may be required by the job, project or description at hand.

1.4.2 That being said, the list of items that can be part of a cadastral map as indicated above is only a partial list of the information that is required on a good tax map. As we proceed into this discussion we will utilize as many real world examples and situations as possible. The hope is that we cover the bulk of the variety and type of information that could be found on the Ownership plat.

1.4.3 Most of the rules of Algebra that we will be discussing will focus on the rearrangement of equations with the goal of solving for an unknown variable. **There are a few basic rules that should be committed to memory** when performing Algebraic rearrangement and will be necessary to understand to calculate geometric representations of mapping data.

1.4.4 **LITERAL NUMBERS.**
A literal number is a Letter or Symbol that represents any value or coefficient.

1.4.5 Multiplication as used in the discussion of mathematical equations throughout this work may be expressed using the following notation conventions.

a. Multiplication can be expressed using the multiplication sign (the asterisk, *) where the equation could read, x * y = z, and is read x times y equals z,

b. or using parenthesis the equation could read, x (y) = z, and is read x times y equals z,

c. or an expression having only variables may be expressed simply as, xy = z, and is read x times y equals z.

1.4.6 The use of the 'x' to represent a multiplication is avoided to prevent confusion with the variable 'x'.

1.4.7 **THE TERM.**
Identification of the **TERM** is important in properly doing algebraic rearrangement.

a. The word TERM is used in mathematics as a means of identifying a single number (or coefficient), single variable, the product of numbers (or coefficients), or the product of variables. Each term is separated by an addition (+) or subtraction (-) symbol.

b. For instance the expression, 2 + 2 - 3 = 1 contains 4 terms. The 2 is one term, the second 2 is another term, the 3 is the third term, and the final term is the 1. Three terms are on the left side of the equation and 1 term is on the right side of the equation.

c. Terms can be a combination of real numbers and variables. Consider the following, 5 + 3y + xyz contains 3 terms. The 5 is one term, the 3y is another term, and the third term is the xyz.

d. It is also important to understand the difference between Similar (Like) and Dissimilar (Unlike) Terms. Like terms can be combined but Unlike terms cannot be combined. A Like term is one in which **the variables and powers are the same**, however, the coefficients do not have to be the same.

e. For instance the expression $6x^2 + 3x^2$ are like terms because the 'x' is squared and is in both terms. This expression could be combined the result being $9x^2$. The expression $8xy - 8x^2y$ are dissimilar terms (unlike terms) because the 'x' is not squared in both terms and cannot be combined.

1.4.8 **EXPONENTS and SQUARE ROOT** is used in, but not limited to, the process in calculating areas of parcels or tracts that have a curved boundary.

1.4.9 **Beginning with Exponents** it should be understood that the expression x^2 means any value of x times the same value of x, as in:

$$x * x = x^2$$

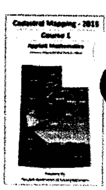
a. the result being read as 'x' squared. Likewise the following expression is true

$$x * x * x * x = x^4$$

b. the result being read as 'x' raised to the fourth power or simply 'x' to the 4th. Note: the '*' symbol in these expressions represents the multiplication symbol.

c. The exponent is the power to which a value is to be raised thus when we say y raised to the fifth power we are saying that the value of y has an exponent of five, which is noted as y^5 and is the same as

$$y * y * y * y * y = y^5.$$



d. These expressions are true because the variable whether stated or not has an implied exponent of 1 (one). For example: $z^1 * z^1 * z^1 = z^3$. The same is true with coefficients.

1.4.10 **Multiplication of Exponents** having the same base (or variable). When variables with exponents are included in a multiplication problem the exponents are added, for example:

Multiplication of Exponents

$$x^2 * x^4 = x^{2+4} = x^6$$

*Add

a. which is the same a stating

$$(x * x)\,(x * x * x * x) = x^6.$$

b. This principle of multiplication works with **Negative Exponents** having the same base as well. In the expression

$$y^3 * y^{-2} = y,$$

c. which is the same as saying that

$$(y^1 * y^1 * y^1)\,(y^{-1} * y^{-1}) = y$$

d. and can be written as

$$y^{3-2} = y.$$

e. Using coefficients (real numbers) in expressions with exponents we see the following is true.

Division of Exponents

$$3^2 * 5^4 = 9 * 625 = 5{,}625$$

1.4.11 Using **Exponents in Division** having the same base the exponents are subtracted, for example:

*Subtract

$$\frac{x^5}{x^3} = x^{5-3} = x^2 \quad \text{or} \quad \frac{\cancel{x} * \cancel{x} * \cancel{x} * x * x}{\cancel{x} * \cancel{x} * \cancel{x}} = x * x = x^2$$

a. where a value divided by itself equals 1, effectively canceling itself in division. Therefore, using the principle of **Negative Exponents** in division results in the following:

$a^2 \div a^{-3}$ same as $\frac{a^2}{a^{-3}}$

$a^{2-(-3)} = a^{2+3} = a^5$

$$\frac{c^2}{c^{-3}} = c^{2-(-3)} = c^{2+3} = c^5 \quad \text{and} \quad \frac{z^3}{z^5} = z^{3-5} = z^{-2}$$

b. The next 3 principles of exponents will probably not be utilized much, if at all, in solving the types of problems that the cadastral mapper may encounter but should be discussed for a complete discussion of the subject of exponents. First, powers of powers, second, power of zero, and the negative exponent.

1.4.10 **Power of Power.** Expressions written as $(y^2)^3$ read as y squared cubed, or y squared raised to the 3rd power and is known as the power of a power and is evaluated as follows:

$$(y^2)^3 = (y * y)\,(y * y)\,(y * y) = y^{2*3} = y^6$$

a. and could be read as y squared, times y squared, times y squared equals y to the sixth power. When powers of powers are evaluated the exponents are added. Two other examples of how this works are:

$$(2x^3y^2)^2 = (2x^3y^2)\,(2x^3y^2) = 4x^{3*2}y^{2*2} = 4x^6y^4$$

b. and

$$\frac{x^2\,(y^3)^2}{(xy^2)^2} = \frac{x^2 * y^{3*2}}{x^{1*2} * y^{2*2}} = \left(\frac{x^2 * y^6}{x * y^4}\right) = x^{2-1} * y^{6-4} = xy^2$$

x y2

1.4.11 **Power of Zero, or Zero Power.** Variables which have a power or exponent of zero (0) are equal to one (1). To demonstrate how this is true consider the following example:

$$\frac{z^3}{z^3} = z^{3-3} = z^0$$

or

$$\frac{z^3}{z^3} = \frac{z * z * z}{z * z * z} = 1$$

if we look at this in another way using real numbers instead of variables we see that

$$\frac{2^3}{2^3} = \frac{2*2*2}{2*2*2} = \frac{8}{8} = 1$$

1.4.12 **Negative Exponent.** When evaluating a negative exponent the following is a true statement.

$$\frac{z^3}{z^5} = z^{-2} = \frac{1}{z^2}$$

a. Thus, when evaluating a negative exponent the expression is equal to 1 (one) divided by the coefficient or variable having the same exponent with the sign of the exponent changed. Some additional examples using real numbers are:

$$5^{-1} = \frac{1}{5} \quad \text{also} \quad 6^{-4} = \frac{1}{6^4}$$

1.4.13 **Using the Square Root** function. Doing a square root by hand will not be taught in this section, the use of a calculator with the square root function will be necessary. If, however, the student would like to learn to do a square root by hand you can find the process detailed in the second edition of the text "Land Surveyors Reference Manual" by Andrew L. Harbin, page 1-11.[1]

a. A square root is essentially the reverse process of squaring. For instance, five squared equals twenty five [$5^2 = 25$] and the square root of twenty five equals five [$\sqrt{25} = 5$].

b. To see how this works in the rearrangement of an algebraic equation we will need to discuss some additional principles relating to the solving of equations known as Axioms. At the appropriate place we will introduce the use of squaring and taking the square root in solving algebraic equations.

1.4.14 **Understanding AXIOMS**. Axioms are basically rules used to rearrange an equation so that the result or rearrangement does not change the truth of the statement. Axioms may be grouped into 3 basic rules that should be memorized.

a. **One**: The same value may be added or subtracted from EACH side of an equation without changing the truth of the equation.

b. **Two**: The same value, which is not zero (0), may be multiplied or divided from EACH side of an equation without changing the truth of the equation.

c. **Three**: Applying the square root to EACH side of an equation can be done without changing the truth of the equation.

1.4.15 **Rule one** may be demonstrated in the following manner.

1 . Newer editions of the text do not contain the section on performing a hand calculated square root.

a. With an initial equation of

$$x + y = z$$

b. if another variable, t, is added to each side of the equation the result would be

$$x + y + t = z + t$$

c. to prove this is true, substitute the following values for each variable where $x = 5$, $y = 3$, $t = 2$, and $z = 8$. Is the resulting equation true?

$$5 + 3 + 2 = 8 + 2$$

d. Yes, because

$$10 = 10$$

1.4.16 **Another example** of how this works with subtraction would be to add to each side a negative y as in

$$x + y = z$$

then subtracting y

$$x + y - y = z - y$$

a. to prove this is true, substitute the following values for each variable where $x = 5$, $y = 3$, and $z = 8$. Is the resulting equation true?

$$5 + 3 - 3 = 8 - 3$$

b. Yes, because

$$5 = 5$$

c. Notice that the negative y is included on EACH side of the equation. When this equation is further evaluated we can see that y minus y on the left side of the equation will cancel (or equal zero) resulting in an equation before substitution of

$$x = z - y$$

d. Therefore, these two equations, $x + y = z$ and $x = z - y$ **are true statements** or equal statements. If we say that z is unknown in both equations, how can we find the value of z in the first equation if we know $x = 5$, and $y = 2$. So

$$5 + 2 = z$$

e. evaluating the equation we get

$$7 = z$$

f. and to use the same values for x and y in the second equation we see that

$$5 = z - 2$$

g. To find out what the value of z equals we need to rearrange the equation so that z is on one side alone, with no other numbers (coefficients) or variables. This is stated as "solving the equation for z". This can be done by removing the negative 2 from the right hand side of the equation which is done by adding a positive 2 to each side of the equation.

$$2 + 5 = z - 2 + 2$$

h. from which we can see that the positive and negative 2 on the right hand side of the equation a (-2) + 2 = 0, so the 2's will cancel one another resulting in

$$2 + 5 = z$$

i. finishing the evaluation we get,

$$7 = z$$

j. To prove this result we need to substitute 7 for z in the original equation

$$2 + 5 = 7$$

which is a true statement.

1.4.17 Let's say that we do not know the value for x in the first equation but we do know the value for y (being 2) and z (being 7). The equation, $x + y = z$, then needs to be "solved for x".

$$x + 2 = 7$$

a. The solution can be found by adding negative two (-2) to each side of the equation in this manner.

$$x + 2 - 2 = 7 - 2$$

b. On the left side of the equation 2 - 2 = 0, so the 2 is canceled resulting in the following equation

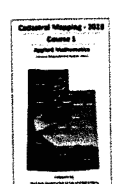
$$x = 7 - 2$$

c. finishing the evaluation

$$x = 5.$$

d. To prove this to be true we can compare the original equation to see that x did in fact equal 5.

1.4.18 **Rule two** may be demonstrated in the following manner. Using the equation x times y equals z



$x * y = z$ (which can be written as $xy = z$)

a. If we know $x = 5$ and $y = 2$ we also know from examination that $z = 10$. If we substitute the values for x and y and evaluate the equation for z we see that,

$$5*2 = 10$$

b. If we say that we know the values for y and z but we do not know the value of x we can "*solve the equation for x*" by multiplying each side of the equation by the reciprocal of y in the following manner:

$$xy = z$$

thus

$$x*y*\frac{1}{y} = z*\frac{1}{y}$$

c. From which we can combine the equations to read as:

$$x*\frac{y}{y} = \frac{z}{y}$$

d. from which the fraction of y/y will cancel and the resulting equation will be

$$x = \frac{z}{y}$$

e. If we substitute the values of $y = 2$ and $z = 7$ the result is:

$$x = \frac{7}{2}$$

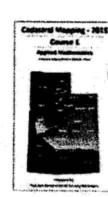
f. finishing the evaluation we see that

$$x = 3.5$$

g. which can be proven by substituting 3.5 into the original equation

$3.5 * 2 = 7$ is a true statement.

1.4.19 The same can be shown to be true when working with an equation which contains a fraction as in



$$4 = \frac{x}{5}$$

a. To solve we will multiply each Term in the equation by the reciprocal of one-fifth (1/5) which is five (5), which can also be written as 5/1, as follows

$$\frac{5}{1} * 4 = \frac{x}{5} * \frac{5}{1}$$

b. then,

$$5 * 4 = \frac{x*5}{5}$$

c. where the 5/5 will cancel resulting in

$$5 * 4 = x$$

or

$$20 = x$$

d. and when substituted into the original equation we can see that the result is a true statement.

1.4.20 **Rule three** can be demonstrated in the following manner. If we say that

$$5 = \sqrt{x}$$

a. then the value of x can be found by taking the square (which is essentially the reciprocal of the square root function) of each term on both sides of the equation as follows,

$$5^2 = (\sqrt{x})^2$$

b. in this equation the square root and the square on the right side of the equation will cancel one another resulting in the following equation,

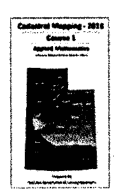
$$5^2 = x$$

c. the solution of which is x = 25.

1.4.21 The reverse of this process is also true as seen in the following evaluation. If,

$$\sqrt{25} = y$$

a. then,



$$(\sqrt{25})^2 = y^2$$

b. In this case the left side of the equation the square root will cancel the square resulting in

$$25 = y^2$$

c. the solution of this equation is y = 5.

1.4.22 In the equation

$$x = \sqrt{25}$$

a. the result is ± 5, and is read Plus or Minus Five.

b. This is because the solution of a problem involving a square root is both plus and minus containing two solutions positive five (+ 5) and negative five (- 5).

1.4.23 **Understanding TRANSPOSING.** Understanding transposing is easier when Axiom One, discussed above, is understood. Transposing is moving one Term from one side of an equation to the other side of the equation by changing the sign of the Term. Essentially, this accomplishes the same result as using the Axiom and can be demonstrated in the following manner by solving the equation for y.

$$8 + y = 13$$

then,

$$y = 13 - 8$$

a. In this solution 8 was moved to the other side of the equation by changing the sign from positive to negative. Another example is to solve the following equation for x

$$x - 4 = 10$$

then,

$$x = 10 + 4$$

b. In this solution the negative four (-4) was moved to the other side of the equation by changing the sign from negative to positive. Another example of how transposing is used to solve equations can be demonstrated by solving the following equation for x.

$$4x - 4 = x + 14$$

Transposing -4;	$4x = x + 14 + 4$
Transposing x;	$4x - x = 14 + 4$
Combining like terms;	$3x = 18$
Multiplying by the reciprocal of 3;	$\frac{1}{3} * 3x = 18 * \frac{1}{3}$
	$\frac{3x}{3} = \frac{18}{3}$
Canceling the 3's on the left side, then dividing the fraction on the right side.	$x = 6$

1.4.24 **Eliminating PARENTHESES.** The elimination of the parentheses in an equation is governed by two basic rules.

a. **Rule One**, if the sign preceding parentheses or brackets is positive they may be removed without changing the sign of the terms within the parentheses or brackets. For example,

$$x + (3y - 5)$$

may be written as

$$x + 3y - 5$$

or

$$a + (-2b + 7)$$

may be written as

$$a - 2b + 7$$

b. **Rule Two**, if the sign preceding the parentheses or brackets is negative they may be removed by changing the sign of *each term* within the parentheses or brackets. For example:

$$x - (3y - 5)$$

may be written as

$$x - 3y + 5$$

or

$$a - (-2b + 7$$

may be written as

a + 2b - 7

1.4.25 These two rules follow the principles of factoring to remove parentheses because they can be rewritten as

x + 1(3y - 5)

a. where the one (1) is implied. The parentheses are removed by factoring the term 1(3y - 5) by multiplying each term in the parentheses by the expression outside of the parentheses. For example, one times three y and one times negative five and is written

x + 1*3y + 1*(-5)

b. which may be written

x + 3y -5

1.4.26 The rule associated with changing the sign of each term can be demonstrated using the same principle, for instance.

x - (3y - 5)

again the term in the parenthesis has an implied 1 as in,

x - 1(3y - 5)

factoring results in,

x + (– 1)*3y – (-1)*5

performing the multiplications then results in,

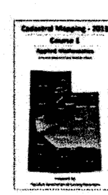
x – 3y + 5

1.4.27 The principles of factoring Monomials and Polynomials are explained using the F.O.I.L. method of reduction.

1.4.28 **FACTORING using F.O.I.L.** One means of reduction of an equation is used to remove or eliminate parentheses or brackets and is shown in the following examples is factoring using the F.O.I.L. method. The acronym F.O.I.L. makes reference to First, Outside, Inside, Last.

1.4.29 Factoring Monomials has briefly been discussed in rule two for eliminating parentheses or brackets. This discussion will give some examples of how to F.O.I.L. combinations of monomials, binomials, trinomials, and polynomials.

a. A Monomial is a single Term expression which may or may not contain parentheses or brackets. The following are examples of monomials:



i. x^2

ii. $8x$

iii. $-3y^5$

and even,

iv. $a^2c(8 - 2b)$

which is also known as a Common Monomial.

b. In the expression

$3y + x$

there are two Terms and is also known as a Binomial.

c. If an expression contains three Terms the expression is considered to be a Trinomial such as

$4a - 3b + c$

1.4.30 Both of the expressions $3y + x$ and $4a - 3b + c$ are considered Polynomials. Polynomials are distinguished by the use of addition, subtraction, multiplication and non-negative integer exponents.

a. Where $2x^2 - 6y + z$ is a Trinomial and $2x^2 - 6/y + z^{3/2}$ is not because the second term is a fraction and the third term is a non-integer exponent.

1.4.31 Factoring a **Common Monomial** using the F.O.I.L. method is performed by multiplying each expression outside of the parentheses with each term within the parentheses as shown in

$8x(2x^2 - 6y + z)$ is a Common Monomial.

To start,

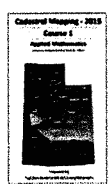
$8x(2x^2 - 6y + z)$

$8x * 2x^2$

Next multiply 8x by the term -6y,

$8x(2x^2 - 6y + z)$

$8x * 2x^2 + 8x * (-6y)$



Finally, multiply 8x by the term z,

$8x(2x^2 - 6y + z)$

$8x * 2x^2 + 8x * (-6y) + 8x * z$

The resulting equation can now have like terms combined.

$16x^3 + (-48xy) + 8xz$

or

$16x^3 - 48xy + 8xz$

1.4.32 FOIL'ing two **Binomials** is accomplished by multiplying each term within the first parentheses with each term within the second parentheses. For instance,

$(x + 4)(x + 5)$

Beginning by multiplying the first term in each parenthesis,

$(x + 4)(x + 5)$

$x * x$

Next multiply the first term in the first parenthesis by the second term in the second parenthesis,

$(x + 4)(x + 5)$

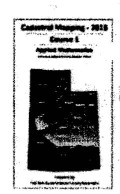
$x * x + x * 5$

Next multiply the second term in the first parenthesis by the first term in the second parenthesis,

$(x + 4)(x + 5)$

$x * x + x * 5 + 4 * x$

Next multiply the second term in the first parenthesis by the second term in the second



parenthesis,

$$(x + 4)(x + 5)$$

$$x * x + x * 5 + 4 * x + 4 * 5$$

Complete by combining like terms.

$$x^2 + 5x + 4x + 20$$

$$x^2 + 9x + 20$$

1.4.33 FOIL'ing a **Binomial** and a **Trinomial** is accomplished in much the same manner by multiplying each term within the first parentheses with each term within the second parentheses. For instance;

$$(3xy + y)(x^2 - y + z)$$

$$3xy * x^2 + 3xy * y + 3xy * z + y * x^2 + y * y + y * z$$

$3x^3y + 3xy^2 + 3xyz + x^2y + y^2 + yz$ (There are no like terms left to combine.)

1.4.34 FOIL'ing a **Polynomial** of the type shown.

$$2(x + 3)(x - 4) = 6 + 2x(x - 5)$$

a. The steps to solve this equation is to F.O.I.L. the parenthesis which results in the following equation;

$$2(x^2 - 4x + 3x - 12) = 6 + 2x^2 - 10x$$

Then finish F.O.I.L.'ing the left side of the equation which finishes eliminating the parenthesis,

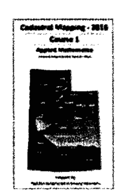
$$2x^2 - 8x + 6x - 24 = 6 + 2x^2 - 10x$$

This has been rearranged to see like terms for combining.

$$2x^2 - 2x^2 - 8x + 10x + 6x = 6 + 24$$

Which results in,

$$8x = 30$$



To finish the solution of the equation solve for 'x'.

$$x = \frac{30}{8} \quad \text{or, } x = 3\,\frac{3}{4}$$

1.4.35 Working with **FRACTIONAL EQUATIONS** is similar to non-fractional equations in that the Axioms already discussed apply in the same manner. The goal when faced with a fractional equation is to eliminate the denominators if possible. To illustrate the similarity consider the following example.

$$\frac{3x}{4} + 5 = 2x - \frac{1}{3}$$

1.4.36 Then, eliminating the denominators can be done in a single step by multiplying EACH TERM in the equation by the reciprocal of one fourth (1/4) and one third (1/3) as follows,

$$\frac{3}{1} * \frac{4}{1} * \frac{3x}{4} + 5 * \frac{3}{1} * \frac{4}{1} = 2x * \frac{3}{1} * \frac{4}{1} - \frac{1}{3} * \frac{3}{1} * \frac{4}{1}$$

a. Then perform the multiplications,

$$\frac{3 * 4 * 3x}{4} + 5 * 3 * 4 = 2x * 3 * 4 - \frac{1 * 3 * 4}{3}$$

b. The 4/4 cancels from the left side and the 3/3 cancels from the right side and combining like terms results in the following equation.

$$9x + 60 = 24x - 4$$

c. Now that the fractions are removed the equation can be solved for x as follows,

i. Transpose to place like terms on the same side of the equation,

ii. $9x - 24x = -4 + (-60)$

iii. Combine like terms,

$$-15x = -64$$

iv. Multiplying by the reciprocal of -15,

$$\frac{-15x}{-15} = \frac{-64}{-15}$$

v. Resolve the fractions,

$$x = 4.2666666...$$

vi. This problem could have been also solved by using the lowest common denominator, however, the solution shown above accomplished the task without having to determine the lowest common denominator.

1.4.37 **LITERAL EQUATIONS and FORMULAS** are the most common condition that will require solutions to complete a given task in cadastral mapping. For instance, the equation shown below is used to solve for Fahrenheit when the temperature in Celsius is known. But what if Fahrenheit is known and we need to know the Celsius equivalent, then the equation needs to be rearranged and solved for C.

$$F = \frac{9}{5} C + 32$$

Solve

$$F - 32 = \frac{9}{5} C$$

$$\frac{5}{1} * (F - 32) = \frac{\cancel{5}}{1} * \frac{9}{\cancel{5}} C$$

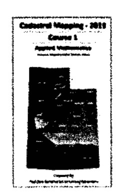
$$5(F-32) = 9C$$

$$\frac{1}{9} * 5(F - 32) = 9C * \frac{1}{9}$$

$$\frac{5(F - 32)}{9} = C \text{ or, } \frac{5}{9} (F-32) = C$$

1.4.38 A **QUADRATIC EQUATION** is a non-linear equation. Identifying a quadratic equation is the first step in being able to solve them. The general form of a quadratic equation is in the form,

$$Ax^2 + Bx + C = 0 \text{ (also known as a complete quadratic).}$$



1.4.39 A quadratic equation in general form contains three terms, one containing a variable (x in this case) that is squared, a second containing the same variable (again this is the x variable in the middle term) which is not squared. And the third is a coefficient. In the general form the C term represents a coefficient as do the A and B.

1.4.40 A quadratic equation has two roots (the word "quadratic" means two roots), one is positive and the other negative. The following equation is known as a Pure Quadratic,

$3x^2 - 12 = 0$ (pure quadratic).

1.4.41 Arranging quadratic equations in general form will also help to identify the proper coefficients to substitute into the formula for solving a quadratic equation which is discussed later.

1.4.42 Consider the following equation,

$x^2 + 3 = 4x$

Arranging in general form,

$x^2 - 4x + 3 = 0$, which is incomplete quadratic.

Arranging in the form which would make this equation a complete quadratic would result in,

$x^2 + (-4x) + 3 = 0$, complete quadratic.

Examining another equation,

$x^2 = 7$

Arranging in general form,

$x^2 - 7 = 0$, pure quadratic

a. Notice that in the arrangement for general form above there is only two terms, the Bx does not show in this equation. This is because the B is zero in this equation. Pure quadratic equations can be solved as follows,

$x^2 - 7 = 0$

$x^2 = 7$

$x = \sqrt{7}$, solution.

b. Quadratic Equations that are in general form, and are not pure quadratic equations, can be solved using special formula.

1.4.30 **Solving a Quadratic Equation** can be done utilizing this formula.
The quadratic formula is

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

1.4.31 This formula has two solutions due to the square root (also known as the radical) which gives two solutions to the problem one being positive (+) and the other negative (-). The problem is worked twice, once for the + and another for the -. This actually results in two formulas, each having to be solved.

$$x = \frac{-B + \sqrt{B^2 - 4AC}}{2A} \quad \text{and} \quad x = \frac{-B - \sqrt{B^2 - 4AC}}{2A}$$

a. To illustrate this principle, solve the equation
$3x^2 = 4x + 2$

b. First arrange in general form.
$3x^2 - 4x - 2 = 0$

c. Use each equation shown above (the positive equation and the negative equation) then solve the first equation.

$$x = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$$

d. Fill in the information needed from the coefficients of the equation where A = 3, B = (-4), and C = (-2). Begin by solving the terms in the square root.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4\,(3)\,(-2)}}{2\,(3)}$$

$$x = \frac{4 \pm \sqrt{16 - (-24)}}{6}$$

$$x = \frac{4 \pm \sqrt{40}}{6}$$

e. From here the equation needs to be evaluated by solving the **positive solution** formula.

$$x = \frac{4 + \sqrt{40}}{6}$$

$$x = \frac{10.324555320}{6}$$

$x = 1.720759220$, positive solution.

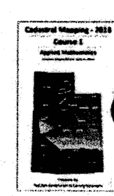
f. Now complete the problem by solving the **negative solution** formula.

$$x = \frac{4 - \sqrt{40}}{6}$$

$$x = \frac{-2.32455532}{6}$$

$x = -0.387425887$, negative solution.

g. From the solutions there are two results for the equations, $x = 1.720759220$ and $x = -0.387425887$.



APPLIED MATHEMATICS:
Course 1. Chapter 5.

1.5 HMS Conversions

1.5.1. The ability to quickly and accurately convert angular values from the Degrees, Minutes, Seconds format (sometimes referred to as Hours, Minutes format) into Decimal Degree format (or Decimal Hour format) is important to the cadastral mapper. While modern engineering calculators have the ability to make these calculations with one or two key strokes, the student should be able to make these calculations by hand.

1.5.2. Our number system is based on tens. Calculators use 10 as the numerical base as does mathematics. When utilizing angles denoted in the English unit of Degrees-Minutes-Seconds (DMS or HMS which I call base 60 values), the angle expressed as HMS must be converted into the base units of 10 for the calculator to properly use the value.

1.5.3. When utilizing measurements of angles there are several types of graduations used. Two types of angular measure are the Radian (metric measures) and the Degree (english measures). In this discussion we will focus on the English measure, Degrees, however, an understanding of radians and how to convert them to degrees should be presented.

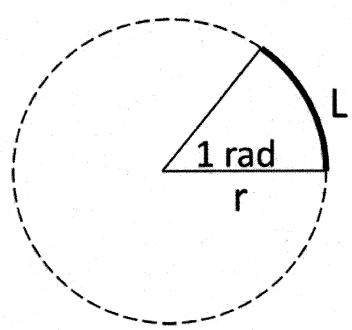
1.5.4. The reason for being able to convert radian to degree values is because of the fact that there has been a time that the Utah Department of Transportation (UDOT) was requiring all engineers, surveyors, and contractors to use the metric system of measure on their highway projects. Many of these projects required additional right of way width and the descriptions that were used to purchase those rights of way were written in metric units.

1.5.5. So what is the Radian?

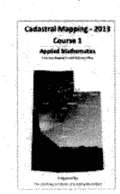
1.5.6. The abbreviation for radian is simply “rad”. And 1 rad in degrees (°) is equal to approximately 57°17’44.806247”.

1.5.7. The precise conversion of 1 rad to degrees is:

$$1 \text{ rad} = \frac{180^\circ}{\pi}$$

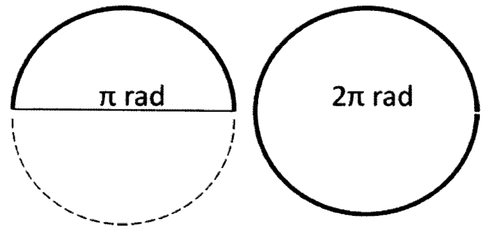


1.5.8. The definition of radian relates to the circle in the following way. From the figure at the right, the radius of the circle is equal to the bold arc shown as L and represents the length of the arc. Therefore, when the radius of a circle (r) and



the length of an arc of the circle (L) are equal the associated angle is 1 radian (1 rad).

1.5.9. Therefore, one half of a circle is π radians (π rad) and one full circle is equal to 2π radians (2π rad). Therefore, 180° equals π rad, and 360° equals 2π rad.

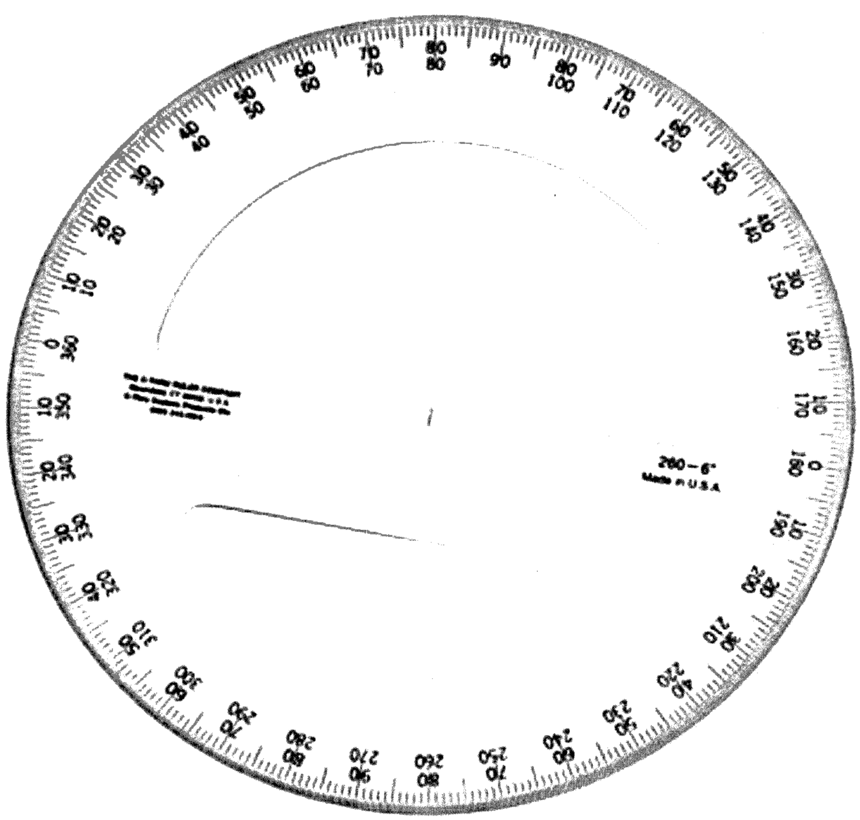


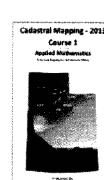
1.5.10. Converting degrees into radian is accomplished by the following equation, where n is the number of degrees;

$$\frac{n^\circ\pi}{180^\circ} \qquad \text{For example: } n = 30^\circ \qquad \frac{30^\circ\pi}{180^\circ} = \frac{\pi}{6} \text{ rad} = 0.52359877560 \text{ rad}$$

1.5.11. Therefore, a definition of radian could be stated as, an angle in which the length of an arc of a circle is equal to the radius of the circle. Note: radian measure is strictly in decimal form or base ten.

1.5.12. **Degrees, Minutes, and Seconds.** It is now necessary to understand how a circle is graduated into increments of equal size in the English system. These increments are known as Degrees, Minutes, and Seconds or simply referred to as Degrees and abbreviated as HMS herein.





There are other methods for dividing a circle into equal increments, the radian having already been discussed, but the others[1] will not be discussed in this manual.

1.5.13. One complete circle is divided into 360 equal segments called the Degree. Therefore, one degree is one three-hundred-sixtieth (1/360) of a circle. Most drafting protractors show these increments. The protractor shown is one that is graduated into Degrees. Around the outside of the circle are 360 individual graduation marks. For each 5 Degrees the graduation mark is slightly longer than the others.

1.5.14. This protractor has two different graduation systems labeled for each 10 Degrees. The labeling closes to the outside is divided into 4 quadrants, each having 90 Degrees to the quadrant. The inside labeling begins at the zero label (shown with a zero on the outside labeling and 360 on the inside labeling) and increasing in a clockwise direction ending with the label 360 Degrees.

1.5.15. The inside labeling represents how Azimuths are used in surveying and the outside labeling is done in the manner in which Bearings are defined by surveying. Azimuths and Bearings will be discussed in more detail in this manual. In mathematics and trigonometric texts angle measure is done counter-clockwise. Azimuths as related to surveying and cadastral mapping are the clockwise angle.

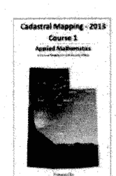
1.5.16. The Degree is further divided into smaller equal divisions known as Minutes. There are 60 Minutes in one Degree (which are not shown on the protractor, to do so would require a single degree space to divided into 60 equal spaces). Therefore, the Minute is 1/60th of a Degree.

1.5.17. The Minute is further divided into smaller equal divisions knows as Seconds. There are 60 Seconds in one Minute. The Second is then divided into decimal Seconds (using the base of 10) to obtain even smaller angular divisions and greater measurement precision. The Second is then 1/60th of a Minute.

1.5.18. When writing **Degrees, Minutes, and Seconds symbols** are used to indicate each division grouping. The Degree is represented by the Degree symbol (°), for instance 10 Degrees is noted as 10°. The superscript small zero is the symbol used to represent Degrees.

1.5.19. Minutes are written using a symbol typically recognized by the layperson as the foot symbol or the apostrophe. You may have seen descriptions that were written with a bearing being noted as N 25D 45 feet 35 inches W. It is obvious that the person that wrote the bearing did not know what the common symbology is for Degrees, Minutes, and Seconds and the

1 . Additional types of angular measure are the **Furman**, which is a unit of angular measure equal to 1/65,536 of a circle; the Binary degree, Binary radian, or "**Brad**", is a unit of angular measure related to 1/256 of a circle; the "**Mil**" or Angular Mil is used by many military organizations and refers to the one thousandth part, rounded it is approximately 1/6400 of a circle.



description should be carefully evaluated.

1.5.20. When an angular value of 10 Degrees and 25 Minutes is written using angular symbols it is noted as 10° 25'. The apostrophe (') in this circumstance is known as the Minute symbol.

1.5.21. Seconds are written using a symbol typically recognized by the layperson as the inch symbol or the quotation mark ("). When an angular value of 10 Degrees, 25 Minutes, and 34 Seconds is written using angular symbols it is noted as 10° 25' 34".

1.5.22. If a value required a finer or more precise measurement the Seconds would be further divided into decimal seconds and written as 10° 25' 34.83". When angular values are being written the Degree can be any value between zero (0) and 359°, the Minutes are always between zero (0) and 59', and Seconds are always between zero (0) and 59" to the left of the decimal point but the decimal of seconds could reach any accuracy value needed to the right of the decimal point. So an angle of 359° 59' 59.99999" indicates it to be less than 360° by 0.00001 second. Why would you want an angle to that accuracy in Seconds? The answer to that will be discussed when Latitudes and Longitudes are explained in the chapter on State Plane Coordinates in Course 2.

1.5.23. Now that the concept of angular representation has been made it is time to discuss converting HMS (Hours Minutes Seconds) format into HDD (Decimal Hours) format. The reason that the use of the term "Hour" is incorporated here instead of Degree is that many modern calculators use the Time functions to calculate angles. These calculators label the function using the HMS or the HDD abbreviations.

1.5.24. As stated before the reason for making the conversion is simple. The modern calculator is built on the decimal system where the base value is ten (Base 10). The calculator does not evaluate expressions in Base 60. Angular values are typically expressed in Base 60 format, e.g. 48° 15' 36" is Base 60. Converting this same angular value into a decimal of a degree the result would be 48.260000°.

1.5.25. To illustrate the concept of Base 60 and Base 10 consider the following angular value

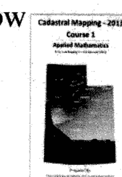
88° 30'

a) We know that 30 Minutes is half of a Degree so identifying the HDD of this angle is done by examination, the result being

88.5°

b) The two values are equivalent.

1.5.26. **HMS Format to HDD Format.** The above example is simple to evaluate, however, how



do we convert 48°15'36" from HMS format to HDD format? The first part of the process is to divide the Seconds by 60.

57° 52' 36''

$$\frac{36 \text{ Seconds}}{60 \text{ Seconds/Minute}} = 0.6 \text{ Minute}$$

36''/60 = .6

a) Next the 0.6' is added to the Minutes resulting in decimal Minutes of 15.6'. The decimal Minutes is then divided by 60.

$$\frac{15.6 \text{ Minutes}}{60 \text{ Minutes/Degree}} = 0.26 \text{ Degree}$$

b) Finally, the 0.26° is added to the Degrees resulting in decimal Degrees of 48.26°. This completes the conversion of HMS format into HDD format.

1.5.27. You can find out how to use the Time function of your calculator to convert from HMS format to HDD format in the owners manual of the calculator or refer to Chapter 1 on calculator use.

1.5.28. **HDD Format to HMS Format.** It is also necessary to reverse the process and convert from HDD format into HMS format. All calculated results using a calculator are displayed in HDD format. To do this we start by subtracting the Degrees as follows;

48.26° - **48°** = 0.26°   57.8766° - 57 = .8766

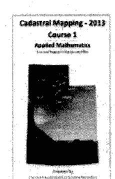
a) The next step is to multiply the decimal Degree by 60.

0.26 * 60 = 15.6 Minutes   .8766 * 60 = 52.6

b) Making this multiplication converts decimal Degrees into Minutes and decimal Minutes. Now subtract the Minutes and multiply the result by 60.

(15.6' - **15'**) * 60 = **36** Seconds

c) Having completed these calculations we can assemble the individual results into HMS format where we started with **48°**, multiplying decimal Degrees by 60 and obtained **15'** (Minutes), and finished by multiplying the decimal Minutes to obtained **36"** (Seconds), resulting in **48°15'36"**, thus reversing the process and converting from Decimal Degrees to Hours (Degrees), Minutes, and Seconds.



1.5.29. Examining another example, convert the following from HMS format to HDD format and back from HDD format to HMS format.

HMS – HDD
88°30' BASE 60

76° 24' 15"

30÷60=.50

15" / 60" = 0.25'

88.50° BASE 10

24' + 0.25' = 24.25'

? 88.50° - 88

24.25' / 60' = 0.4041666666...°

.50× 60 = 30.

76° + 0.4041666666...° = 76.40416666666...°

a) thus 76°24'15" is equal to 76.4041 and two-thirds degrees.

35°15 BASE 60

76.4041666666...°

15÷60=.25

Reversing the process…

.25 * 60 = 15.

(76.4041666666...° - **76°**) = 0.4041666666...°

5° 48 BASE 60

0.4041666666...° * 60' = 24.25'

48÷60=.80

24.25' - **24'** = 0.25'

5.80 BASE 10

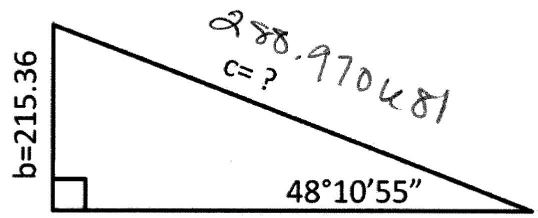
0.25' * 60" = **15"**

.80× 60 = 48.

b) then assemble the individual parts resulting in 76°24'15".

1.5.30. Using the HMS DDS conversion in a mathematical problem. The following example will use principles of trigonometry, however, it is not the purpose to explain how the solution is derived through trigonometry only to demonstrate the use of the conversion functions.

$$\text{Sin } 48°10'55" = \frac{215.36}{c}$$

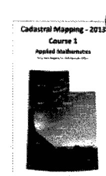


Rearrange the equation to solve for 'c'.

$$c = \frac{215.36}{\text{Sin } 48°10'55"}$$

Convert the angle from base 60 to base 10.

55 ÷ 60 = .916667



$$c = \frac{215.36}{\text{Sin } 48.181944444°}$$

Apply the sin function to the decimal degree value which results in the following fraction.

$$c = \frac{215.36}{0.745265919}$$

Then divide.

$$c = 288.970680801$$

Note: The result is a linear distance and is correct leaving it in base 10.

1.5.31. Adding and Subtracting Angles.

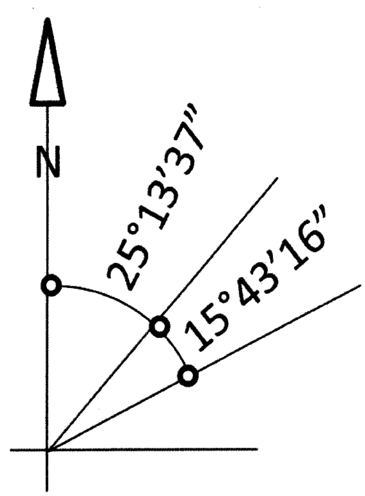
1.5.32. One of the most basic functions in drafting deed descriptions is the principle of adding and subtracting bearings. Adding and subtracting angles is used in working with bearings when rotating parcels is required. Rotating descriptions will be discussed in more detail in course 2.

1.5.33. There are two different methods to add or subtract angles. One is performing the functions with the angles still in base 60, the other is to perform the functions with the angles converted to base 10.

1.5.34. Let's look at performing addition and subtraction with angles in base 60.

1.5.35. Add the following two angles.

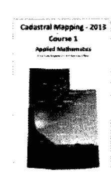
$$\begin{array}{r} 25°13'37" \\ \underline{+15°43'16"} \end{array}$$



1.5.36. The process begins by adding the seconds.

$$\begin{array}{r} 37" \\ \underline{+16"} \\ 53" \end{array}$$

1.5.37. Since 37" plus 16" equals 53" and 53" is less than 60" there is no additional calculation. The next part of the process is to add the minutes.



35°10'59"
35.183056
7°25'43"
7.428611
42.611667
42°36'42"

13’
+43’
56’

1.5.38. Again, there is no additional calculation in this addition because 56 minutes is less than 60 minutes. Next add the degrees.

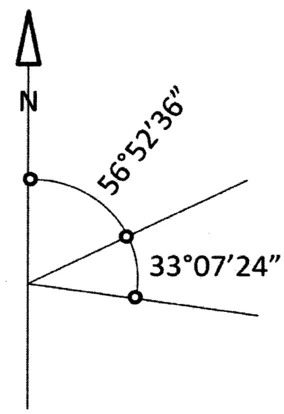
25°
+15°
40°

1.5.39. The result of the addition is

25°13’37”
+15°43’16”
40°56’53”

1.5.40. Let’s do another addition. This time we are going to add two angles that will require carrying.

56°52’36”
+33°07’24”



1.5.41. Again, we start the process by adding the seconds.

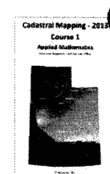
36”
+24”
60” = 1'

1.5.42. Because the addition resulted in precisely 60 seconds, which equals 1 minute, 1 minute is to be carried and the seconds will equal zero. So the equation will look like this.

1’
56°52’36”
+33°07’24”
00”

1.5.43. Next add the minutes.

1’
52’
+07’
60’ = 1°



1.5.44. Again, this addition resulted in precisely 60 minutes, which equals 1 degree. The 1 degree is to be carried and the minutes will result in zero, so the equation looks like this.

1°
56°52'36"
+33°07'24"
00'00"

1.5.45. Finally, add the degrees which results in the final value which is…

56°52'36"
+33°07'24"
90°00'00"

1.5.46. Let's do another addition.

6°55'46"
+31°17'36"

1.5.47. Again, we start the process by adding the seconds.

46"
+36"
82"

1.5.48. Because 82" is greater than 60" we subtract 60 from 82 equaling 22". We carry the 60" that was subtracted from the 82" as 1', the equation then looks like this.

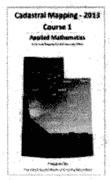
1'
6°55'46"
+31°17'36"
22"

1.5.49. Next add the minutes.

1'
55'
+17'
73'

1.5.50. Because 73' is greater than 60' we subtract 60 from 73 equaling 13'. We carry the 60' that was subtracted from the 73' as 1 degree, the equation then looks like this.

1°
6°55'46"
+31°17'36"
13'22"



1.5.51. The adding the degrees results in the equation looking like this.

$$\begin{array}{r} 6°55'46" \\ \underline{+31°17'36"} \\ 38°13'22" \end{array}$$

1.5.52. **Subtraction of angles** may require borrowing. Let's subtract the following two angles.

$$\begin{array}{r} 90°00'00" \\ \underline{-33°07'24"} \end{array}$$

1.5.53. Start by subtracting the seconds, however, because the 00" is less than the 24" there is a need to borrow from the minutes and because the minutes are also zero the borrowing will have to begin from the degrees. Therefore, borrowing 1 degree is equal to 60 minutes and borrowing one of the 60 minutes which equals 60 seconds will result in the following equation.

$$\begin{array}{r} 89°59'60" \\ \underline{-\ 33°07'24"} \end{array}$$

1.5.54. Subtracting the seconds.

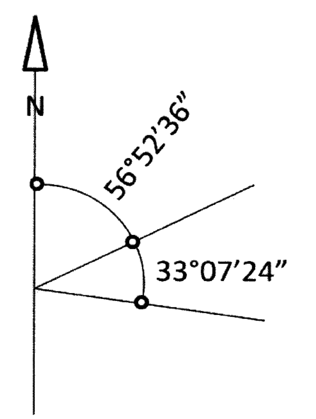
$$\begin{array}{r} 60" \\ \underline{-24"} \\ 36" \end{array}$$

1.5.55. Next subtract the minutes. Because 59' is greater than 07' there is no need to borrow from the degrees.

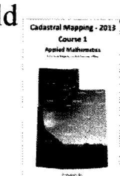
$$\begin{array}{r} 59' \\ \underline{-07'} \\ 52' \end{array}$$

1.5.56. Finish the subtraction with the degrees and the equation result is.

$$\begin{array}{r} 89°59'60" \\ \underline{-\ 33°07'24"} \\ 56°52'36" \end{array}$$

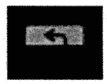
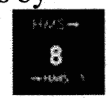


1.5.57. The **second method** for adding and subtracting angles is to convert the angles into decimal equivalents. Using the same angles that we have done previously the problem would



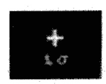
be approached using your calculator[2] as follows.

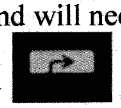
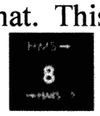
25°13'37"
+15°43'16"

1.5.58. Begin by placing the first angle on the calculator in the command line. The display will read 25.1337 with the angle in base 60. Next convert the angle to decimal degrees by pressing the left shift key  and then the pressing the HMS to Decimal key. 

1.5.59. The display should then contain 25.2269444444 which is now in the 'x' register. Now place the next angle in the command line by entering 15.4316 and converting the base 60 angle into decimal degrees (base 10). The same key strokes described for the first angle is used to make this conversion that being – press the left shift key and then press the HMS to Decimal key.

1.5.60. There will now be two values on the display. The first one (top one which is the 'y' register) will be 25.2269444444 and the second (bottom one which is the 'x' register) will be 15.721111111 and can now be added. Notice that in this calculation we have not used the Enter Key.

1.5.61. To complete the addition simply press the addition key. 

This will perform the addition of the two values in the 'x' and 'y' registers and place in the 'x' register the result of 40.948055556. This value is still in base 10 or decimal degree format and will need to be converted into HMS format. This is done by pressing the right shift key  and then the Decimal to HMS key 

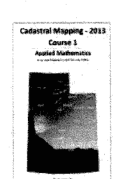
where the value now being displayed is 40.565300000. This is now in base 60 and is read as 40 degrees 56 minutes 53 seconds. This completes the problem.

25°13'37"
+15°43'16"
40°56'53"

1.5.62. One more thing that should be pointed out relating to the conversion of angles from base 10 to base 60, notice how the result of the addition resulted in a value of 40.948055556. At a glance it can be observed that this value is not in base 60 format because the first two numbers right of the decimal are a 9 and 4. We know that if the angle was base 60 the

---

2 . The key strokes and graphics are demonstrated using the HP35. Other calculators may vary on their use of the shift keys and placement of the conversion functions.



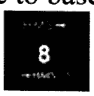
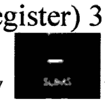
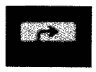
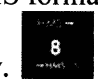
minutes cannot be greater than 59 so the fact that there is a 94 in the first two positions we know that this value is in base 10.

1.5.63. Being able to observe this fact is helpful but may not be true in all cases, for instance, if we converted the following angle 36°25'30" into decimal degrees results in a base 10 value of 36.4250 and the value would not be readily observed as a decimal degree number.

1.5.64. Now let's do a **subtraction problem** using the calculator. Using the same equation as before we begin by entering the first angle into the command line.

90°00'00"
<u>-33°07'24"</u>

1.5.65. Because the first angle is 90°00'00" it is only necessary to key 90 and press the Enter Key. This will place 90° in the 'x' register. Because the minutes and seconds are zero there is no need to convert the angle into decimal degrees.

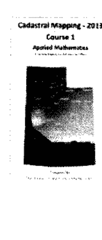
1.5.66. Next place the second angle in the command line as 33.0724 and convert the base 60 angle to base 10 by pressing the left shift key  and then pressing the HMS to Decimal key.  The calculator will now be displaying in the top line ('y' register) 90.000000000 and in the bottom line ('x' register) 33.123333333 and to complete the subtraction simply press the subtraction key  which will result in the bottom line ('x' register) with a value of 56.876666667. As you can observe the angle is in base 10 so to complete the subtraction the result will need to be converted into HMS format by pressing the right shift key  and then pressing the Decimal to HMS key. 

1.5.67. The result will then be displayed in the 'x' register (the bottom line) as 56.523600000 and is in base 60, so the result of the subtraction is...

90°00'00"
<u>-33°07'24"</u>
56°52'36"

1.5.68. It should be noted that not all calculations will result in angles that are exactly to the second of arc. Most often there will be decimals of seconds being expressed so in a situation where this would be the case the resulting angle may look something like this in the 'x' register of the calculator.

78.265416876



1.5.69. This angle would be written in base 60 as 78°26'54.16876". Notice how the seconds are 54 and then there is a decimal point. The digits to the right of this decimal point represent decimals of a second and are interpreted as a base 10 value.

1.5.70. As we begin to get into more detailed calculations this concept will become more familiar.

1.5.71. The following practice problems should be worked through so that you become comfortable in using the calculator to perform the addition and subtraction problems so when we study bearings and azimuths the process of adding and subtracting angles is second nature.

1.5.72. Practice Problems:

1.5.73. Perform the following **additions.**

a) 85°15’36”
+16°52’43” — Solution : 102°08’19”

b) 15°41’23”
+36°27’00” — Solution : 52°08’23”

c) 30°39’36”
+52°18’53” — Solution : 82°58’29”

d) 42°56’02”
+72°47’42” — Solution : 115°43’44”

e) 111°36’06”
+65°49’58” — Solution : 177°20’58”

f) 36°38’18”
+66°15’50” — Solution : 102°54’08”

g) 68°55’17”
+10°28’09” — Solution : 79°23’26”

1.5.74. Perform the following **subtractions.**

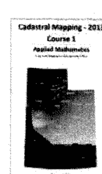
a) 83°33’31”
- 10°28’09” — Solution : 73°05’22”

b) 187°38’15”
- 83°38’49” — Solution : 103°59’26”

c) 304°39’45”
- 39°21’23” — Solution : 265°18’22”

d) 226°12’53”
- 113°17’43” — Solution : 112°55’10”

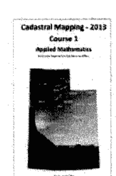
e) 29°21’22”
- 11°52’43” — Solution : 17°28’39”



f) 197°00’36”
   <u>- 52°35’22”</u>   Solution : 144°25’14”

g) 180°00’00”
   <u>-90°35’46”</u>   Solution : 89°24’14”

1.5.75.



APPLIED MATHEMATICS:
Course 1. Chapter 6.

1.6. Basic Geometry:

1.6.1. Geometry is not a static science. The principles of geometry are constantly progressing and the methods of learning the principles of geometry are constantly changing in our educational system. One of the shifts in education has been to move away from using Constructs to teach the principles of geometry. These principles, that of Constructs, will be the focus of this chapter. It is the principles of constructs that the mapper and drafter must understand to be able to properly map Traverses (polygons or parcels) especially when data is limited.

1.6.2. Geometry may be defined as the study of Angles, Lines, Points, Surfaces, and Solids. The study that we will be making related to principles of geometry will be limited to the Euclidean plane. Euclidean space can be defined as a flat plane. Plane trigonometry is a result of Euclidean planes and was developed by a Greek mathematician Euclid. Plane trigonometry is an important part of geometry.

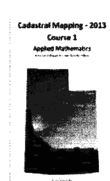
1.6.3. Using geometric principles the mathematical functions of Addition, Subtraction, Multiplication, Division, and obtaining Square Root are possible. The focus of this discussion will revolve around solving geometric problems, not necessarily how to add, subtract, multiply, divide or apply the square root.

1.6.4. Johann Heinrich Lambert a Swiss mathematician, physicist, and astronomer, born August 26, 1728 and died September 25, 1777. Lambert studied light intensity and was the first to introduce hyperbolic functions into trigonometry. Also, he made conjectures regarding non-Euclidean space. What then is Euclidean space?

1.6.5. "One way to think of the Euclidean plane is as a set of points satisfying certain relationships, expressible in terms of distance and angle. For example, there are two fundamental operations on the plane.

1.6.6. "One is *translation*, which means a shifting of the plane so that every point is shifted in the same direction and by the same distance.

1.6.7. "The other is *rotation* about a fixed point in the plane, in which every point in the plane turns about that fixed point through the same angle. One of the basic tenets of Euclidean



geometry is that two figures of the plane should be considered equivalent if one can be transformed into the other by some sequence of translations, rotations, and reflections."[1]

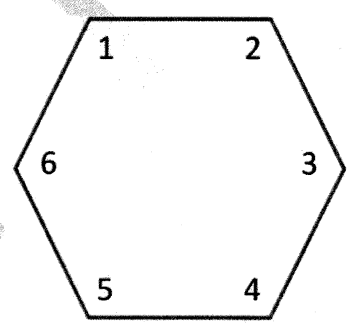
1.6.8. The principles of Rotation and Translation will be discussed in greater detail in Course 2. Lambert is also credited with the first proof that PI (π) is irrational in 1761.

1.6.9. Perhaps in more simple terms – Euclidean space can be defined as a flat plane. Plane trigonometry is a result of Euclidean planes and was developed by a Greek mathematician Euclid. Basic Geometry is part of the study of Plane surfaces.

1.6.10. The Polygon.

1.6.11. The polygon is the basic shape related to a land description, most often called the Parcel or traverse. Understand some basic aspect of the polygon will help in relating to the closure error of a land description. This principle is the evaluation of the following formula.

$(n-2)\ 180 = \Sigma\ IA$



1.6.12. Where

a. n = the number of interior angles, (TRAVERSE)

b. Σ IA = Summated Interior Angles or the total of all the interior angles when added will equal this value in degrees.

c. The polygon at the right has 6 interior angles, therefore,

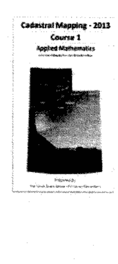
$(6-2)\ 180 = \Sigma\ IA$

$(4)\ 180 = 720°$

1.6.13. A natural question with this result would be related to the size of the angle being 720° when a circle contains 360°. The fact is that One Circle contains 360° but when circle is examined as an angle in rotation and then consider that the angle begins at 0° and rotates in a clock wise direction one full circle this will equal 360°. However, if the angle of rotation is continued around the circle, past 360° and continue to rotate the angle past this point around another full circle the number of degrees will equal 720°. The principle of doing this can also be demonstrated through the use of the following diagram.

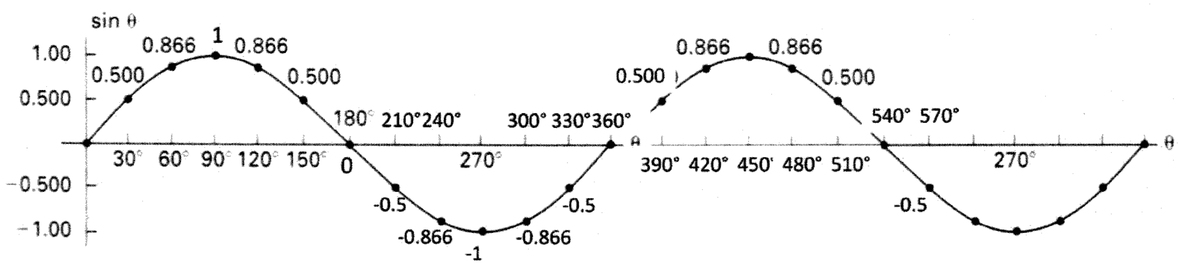
1.6.14. The Sine Wave.

[1] Wikipedia, The Free Encyclopedia on Euclidean planes.



1.6.15. The sine wave is developed by evaluating the Sine of an angle. For instance, the following wave is plotted by calculating the sine for the following angles. Beginning at zero, 0°, 30°, 60°, 90° - notice that when the sine of these angles are evaluated the result is a positive value equaling 0 (zero), 0.500, 0.866, and 1 which plots the first part of the curve.

SINE WAVE

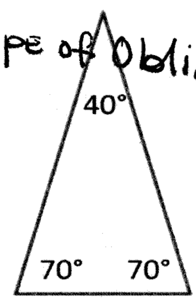


1.6.16. Continuing the calculations for 120°, 150°, 180° equals 0.866, 0.500, and 0. If the calculations are continued the resulting values will be as shown in the figure above.

1.6.17. Isosceles a type of Oblique Triangle.

1.6.18. A very simple definition of an oblique triangle is a triangle that has no right (90°) angles. In this discussion we need to identify the Isosceles triangle because it will be used in a type of solution later. The geometry of an isosceles triangle is shown in the figure. Notice that there are two angles of the triangle that are equal in size. This is what makes the triangle an isosceles.

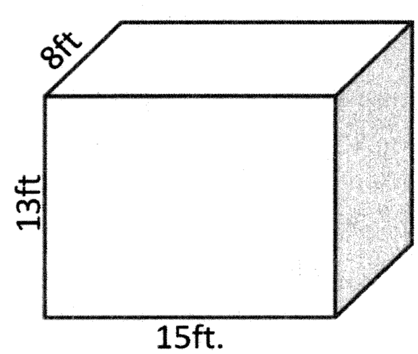
type of Oblique



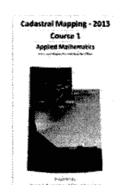
1.6.19. Oblique triangles will be further defined and discussed in the Chapter on Trigonometry. For now keep in mind that the isosceles triangle is one that two angles are the same value or size.

1.6.20. Dimensional Equations.

1.6.21. The subject of Dimensional Equations have been briefly discussed in the Chapter on Algebra. Here it will be mentioned again simply to refresh your mind on the concept that a dimensional equation need not be setup and evaluated as long as the correct conversion factors are known and used in an evaluation of a problem.



1.6.22. But to see how a dimensional equation works let's examine the following problem. How many cubic yards are contained in the shape? To set up a dimensional equation we would create a fraction as follows:



US ~~Foot~~ Survey Feet
↳ 3.2808333333

$$\frac{(15\ \text{ft})\ (13\ \text{ft})\ (8\ \text{ft})}{27\ \dfrac{\text{ft}^3}{\text{yd}^3}} = \frac{\left(\frac{15}{1}\ \text{ft}\right)\left(\frac{13}{1}\ \text{ft}\right)\left(\frac{8}{1}\ \text{ft}\right)}{\dfrac{27\ \text{ft}^3}{1\ \text{yd}^3}} = \left(\frac{1560\ \text{ft}^3}{1}\right)\left(\frac{1\ \text{yd}^3}{27\ \text{ft}^3}\right) =$$

1.6.23. Therefore, the solution is: $57.77777777\ \text{yd}^3$.

1.6.24. As previously stated, a dimensional equation need not be used as long as the proper Conversion Factors are used. To see the use of conversion factors consider the following solution to the same problem.

1.6.25. Volume of the container is expressed as:

V = L * W * h

1.6.26. Where V = volume, L = length, W = width, and h = height. In the same example the equation would be written as:

V = 15 * 8 * 13
Therefore,

V = 1560 cubic feet

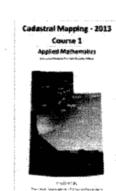
1.6.27. Since the problem was to find out the cubic yards in the container and we know that there are 27 cubic feet in a cubic yard the volume in cubic yards is:

$$V = \frac{1560}{27} = 57.77777777\ \text{yd}^3$$

1.6.28. One more example that is probably familiar to the cadastral mapper is converting chain measures to feet measures. A distance of 20 chains can be converted to the equivalent foot value by knowing the conversion factor is 66 feet in one chain. So the conversion would be made:

20 * 66 = 1320 feet

1.6.29. In the appendix there is a section on conversion factors that can be used to make dimensional conversion.

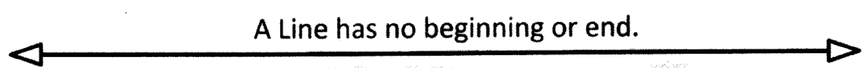


1.6.30. Geometric Shapes:

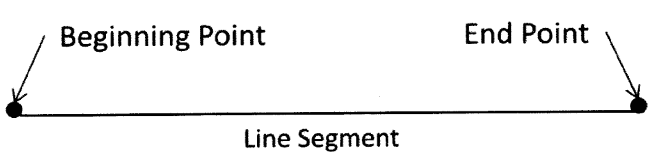
1.6.31. A basic definition of these geometric objects or geometric shapes should be understood by the cadastral mapper because the use of drafting tools is necessary to produce maps whether the work is done by hand (board) drafting or in modern CAD systems. To begin this subject a detailed definition of geometric objects and shapes is presented. The definitions will use terms associated with surveying, mapping and mathematics. These definitions should be memorized by the student.

1.6.32. Types of Lines:

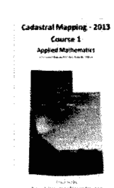
1.6.33. A **Line** in mathematics is defined as an object having no width, but having a length of infinite distance. Euclid stated this concept as "[a] line is breadthless length.", in other words, a line has no width. He also stated that "[t]he extremities of a line are points." (thus creating a Line Segment), this has a direct relationship to the definition of a Point. In other words a Line has no beginning or end and is not a surface, but can reside on a surface or two intersecting surfaces. A Line in surveying may be created by the "line of sight" that a surveying instrument defines by the telescope cross hair when the instrument is setup on a point and oriented to look at another object, generally another point. This creates a line between the occupied points. In mapping the line is that which is drawn with the pencil or pen to identify a single mapped location.



1.6.34. The **Point** in mathematics is defined as a line having no length, essentially it begins and ends at the same place. Euclid stated it as being "...that which has no part.", and the Point also resides at the beginning or ending of Line Segments. Euclid also stated that "[a] straight line is a line which lies evenly with the points on itself.", therefore, the straight line and Points reside in the same Plane and coincide with one another. The Point can be related to any object that is used by a surveyor to set an instrument on or by a mapper to identify a single location. Once a line is defined the surveyor or mapper can establish points along the line at any given position. In surveying and mapping the point can be defined by many different coordinate systems or datums.

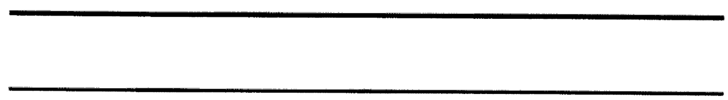


1.6.35. A **Line Segment** is a line that has a beginning at a specific point and an end at a specific point. Surveying measurements define or identify this type of line. Deed descriptions also use this type of line when each course of the parcel is being described. For instance, a part of a description may read:



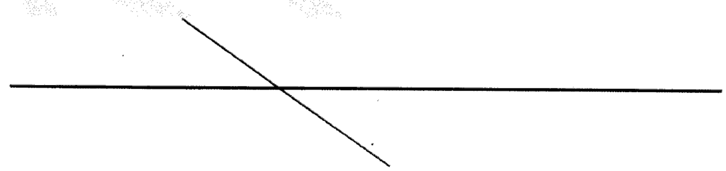
… Thence North 88°10' East 200.0 feet, thence South 87°45' East 153.8 feet, …

1.6.36. **Parallel lines** are two or more lines which reside in the same plane and do not intersect.



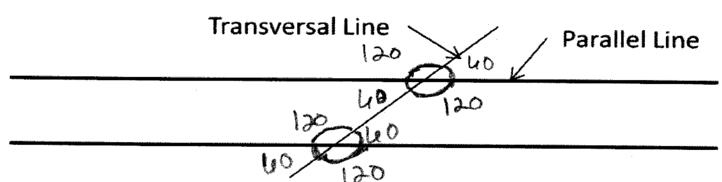
Parallel Lines

1.6.37. **Intersecting lines** are two or more lines which reside in the same plane and will cross or intersect at some point or location.



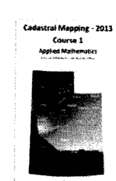
Intersecting Lines

1.6.38. A **Transversal** is a line that crosses parallel lines and resides in the same Plane as the parallel lines.

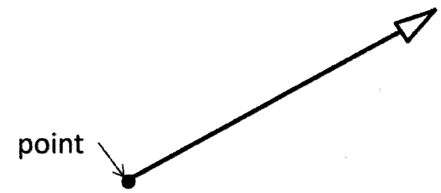


Transversal

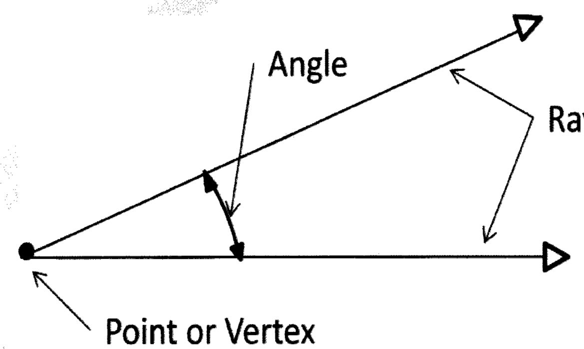
1.6.39. A **Ray** is a line that has a beginning at a specific point but no end. Ray's are found in many aspects of mapping. The radius of a circle is a form of a ray where the center of the circle originates at a point and the ray terminates at the edge of the circle. The length of the



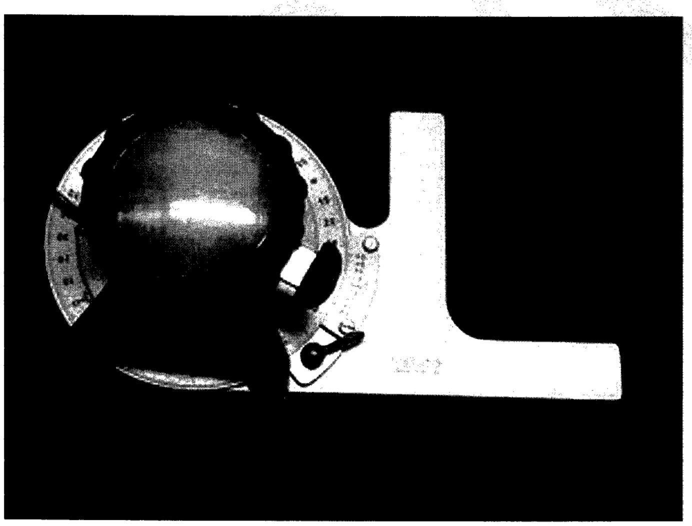
radius may be infinitely close to zero but not zero and infinitely long. Until some specific value is attached to the ray the ray will not have a defined end.



1.6.40. The **Vertex** of an Angle is the Point at which two Line Segments or Rays begin. The Vertex is the common point of two converging lines. In surveying and mapping the vertex may be referred to as a Corner or Point and is usually at the location where a monument, permanent or temporary, may occupy.

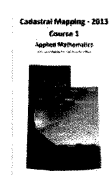


1.6.41. An **Angle** is defined as the difference in bearing or azimuth between two lines that are not parallel which converge upon a single point or vertex. The size or magnitude of an angle can be measured with many different tools. Prior to electronic drafting equipment (CAD systems) to produce a hard copy plat the mapper or drafter could use a protractor to measure an approximate angle. For more precise angular measurement the mapper would utilize a mechanical drafting arm with a drafting head. The drafting head would have a more precise protractor and usually incorporated some type of vernier system for being able to measure or layout angles to the nearest minute of arc. The surveyor uses precision tools to measure or layout angles. High precision surveying instruments can measure angles to precisions as accurate as tenths of a second of arc.

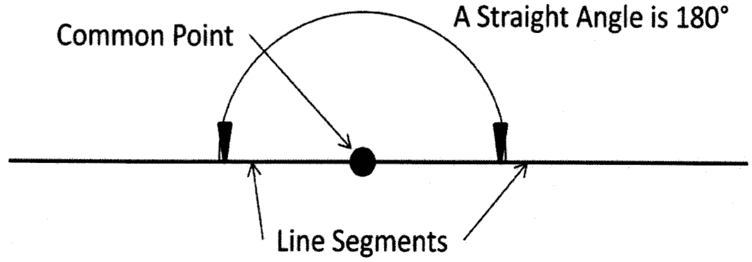


1.6.42. **Types of Angles:**

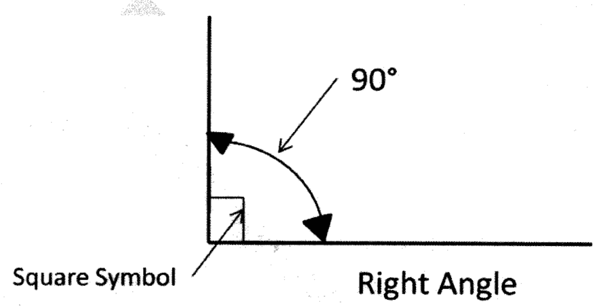
1.6.43. There are several different types of angles which are defined as follows;



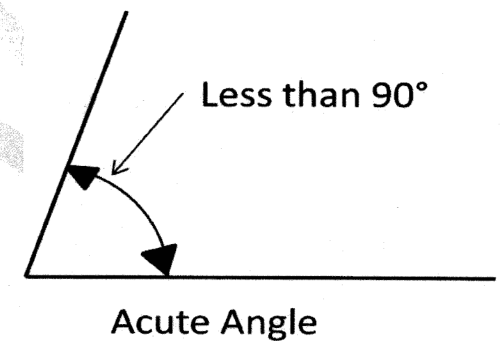
a. **Straight angle** which is formed by two Line Segments or Rays originating at a common point and extending in opposite directions. The angle between these two Line Segments is precisely 180°,



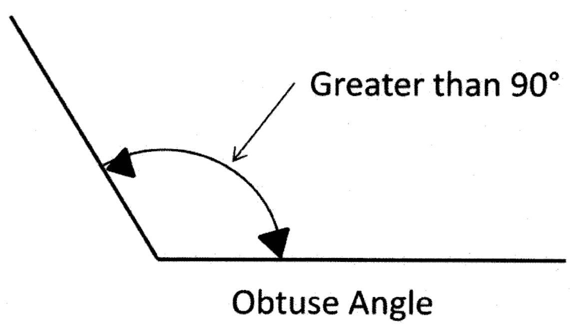
b. **Right angle** is formed by two Line Segments or Rays originating at a common point and extending in directions that are perpendicular to one another. Typically when a Right angle is noted a "square" symbol is used as shown. The angle between the Line Segments or Rays is precisely 90°.



c. An **Acute angle** is formed when two Line Segments or Rays, originating at a common point, are extended in a direction where the included angle is **less** than 90°.

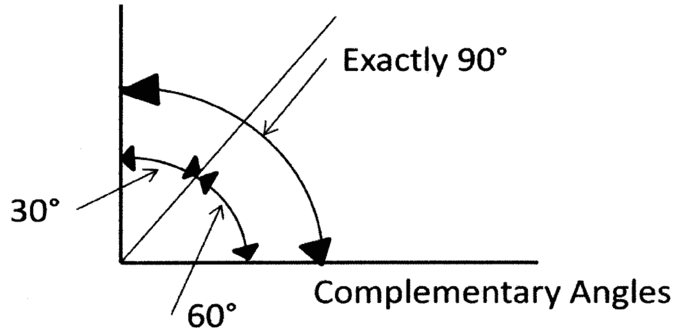


d. An **Obtuse angle** is formed when two Line Segments or Rays, originating at a common point, are extended in a direction where the included angle is **greater** than 90°.

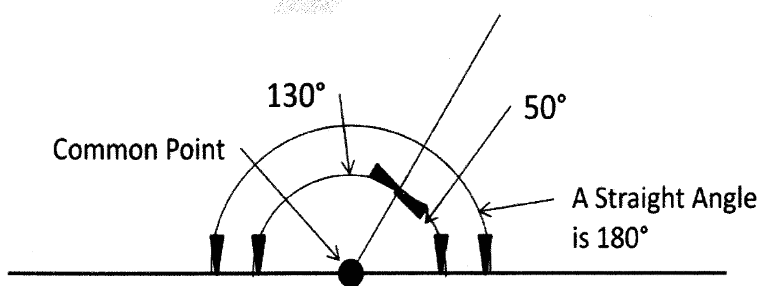


e. **Complementary angles** are formed when three Line Segments or Rays, originating at a common point, are extended in three different directions, two of which are at Right

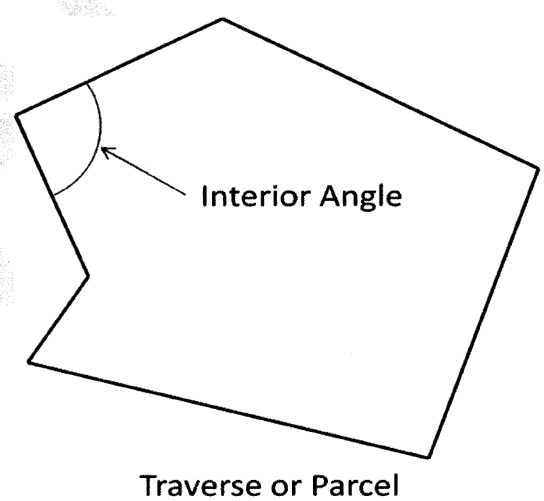
angles (perpendicular) to one another. The 30° and 60° angles are Complementary to one another because they summate to 90°.



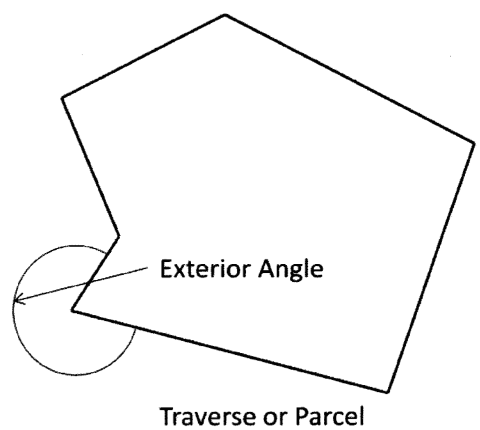
f. **Supplementary angles** are formed when three Line Segments or Rays, originating at a common point, are extended in three different directions, two of which for a Straight angle. The 130° and 50° angles are Supplementary to one another because they summate to 180°.

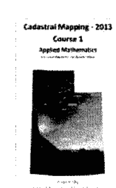


g. **Interior angles** can be either Right, Obtuse, or Acute angles formed by the Line Segments of a Polygon (Traverse or Parcel) and are on the interior or the polygon.

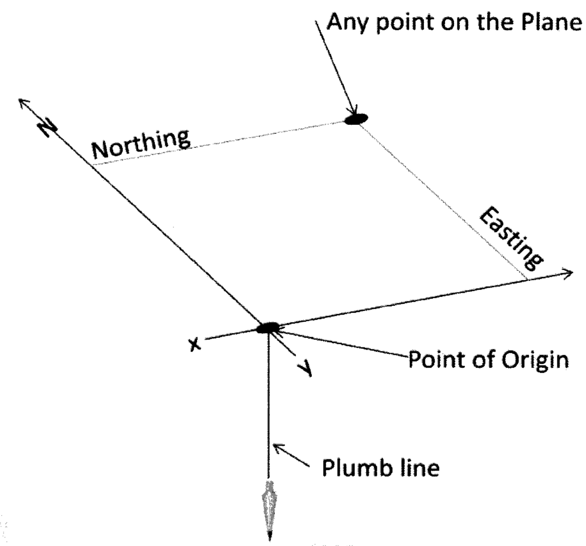


h. **Exterior angles** can be either Right, Obtuse, or Acute angles formed by the Line Segments of a Polygon (Traverse or Parcel) and are on the exterior of the polygon.



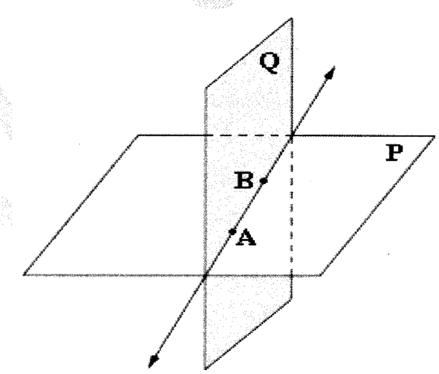


1.6.44. The **Plane** is a flat two-dimensional surface on which a coordinate system of 'x' and 'y' values or Northing and Easting values can be located. The plane is generally thought of as a level surface especially in terms of the cadastral map, however, a Plane can also be a non-level surface which may or may not be related to a coordinate system. For example, the flat surface of a table, a wall, or other surface.



1.6.45. The Plane is defined with a **point of origin**, an **orientation**, and a **reference to the level** (level being a horizontal plane or surface).

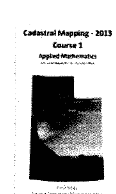
1.6.46. Planes may be parallel or intersecting surfaces. Parallel surfaces will not intersect but a non-parallel surface will always intersect somewhere. Intersecting Planes create a line which is common to and parallel to each Plane or surface. In the figure, line AB is the line created by the intersections of the surfaces Q and P.



1.6.47. The angle between two intersecting Planes is known as the dihedral angle. The dihedral angle in a two dimensional plane are the angles that we use in Plane Trigonometry. Three intersecting planes whose lines of intersections are parallel create a triangle in the two dimensional plane when the three intersecting planes are viewed from the edge.

1.6.48. The cadastral mapper will not need to be concerned with intersecting Planes or surfaces because a cadastral map is produced to reside in a single horizontal Plane. This plane is always being viewed from the edge of all the other intersecting planes so what we see is simply the lines of the edges of the planes.

1.6.49. The surveyor, however, must understand intersecting Planes or surfaces when constructing topographic maps that contain elevation contour data. Modern surveying equipment measures points in 3 dimensional space and calculates the 'x', 'y', and 'z' (easting, northing, and elevation) coordinate of points which can represent the corners of property.



1.6.50. **Geometric Construction.**

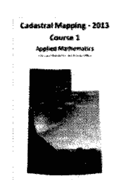
1.6.51. The principles of Constructs we briefly introduced in the preceding section. The next several sections will expand on the use of Constructs to develop geometric shapes and solve geometric problems.

1.6.52. Whether the mapper is using hand drafting techniques with pencil and pen or using electronic drafting systems (CAD systems), understanding how to construct basic geometric shapes must be understood to properly map property descriptions. Sometimes there is not enough information or data present in a written description to be able to accurately draft the parcel without doing some type of calculation. This calculation can be made utilizing electronic calculators or it can be made using geometric constructs.

1.6.53. A knowledge of the principles of geometric construction will give the mapper additional tools that can be used to solve problems of the type mentioned. The principles discussed herein will be assumed to be located in a single horizontal Plane in which the use of Plane Geometry will be made.

1.6.54. When this subject was developed by the Greek Mathematician Euclid, there was only two basic tools used to solve geometric problems, the Compass and the Straightedge. Because the Greek straightedge had no graduations, the principles of geometric construction used only *Points, Lines and Arcs* to solve problems.

1.6.55. What is interesting is that in geometric constructs there are no numbers used to solve many problems of mathematics which relate to geometry. In fact, the ancient Greek system of numbers did not have the value zero (0) nor did it allow for place holders which the modern use of the zero facilitates in many instances. The concept of *place holders* was discussed under the subject of significant digits. The challenges that the lack of the zero in the Greek system created was overcome by the use of *Constructs* in geometry and, the lack of the zero, made mathematics as we know it today almost impossible to solve except by the use of Constructs.



1.6.56. **Lines.**

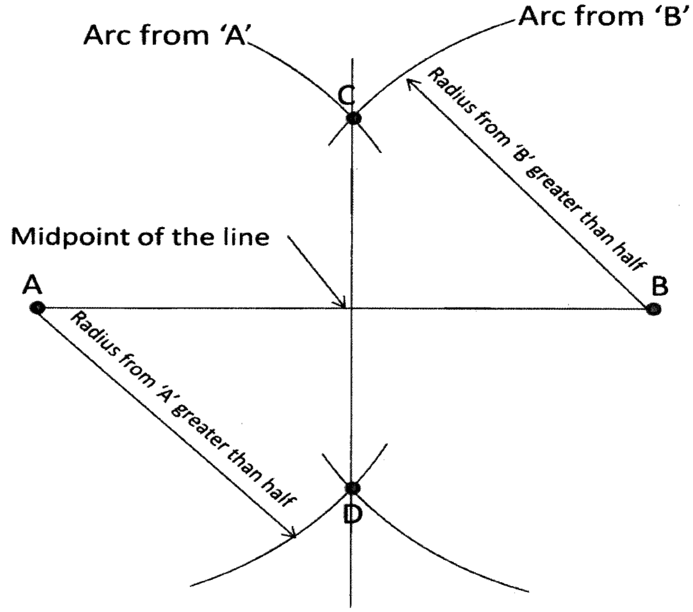
1.6.57. To **BISECT A LINE** identifying the midpoint of the line, the following Constructs are made.

a. Construct a line that is drawn from point A to point B.

b. Next construct two arc segments from point A which have a radius greater than half the length of the line. In the graphic this is shown as Arc from 'A' and is drawn above and below the line.

c. Two more arcs are drawn from point B in the same manner as were drawn from point A.

d. These 4 arcs will cross at points C and D from which a line is drawn between these points.

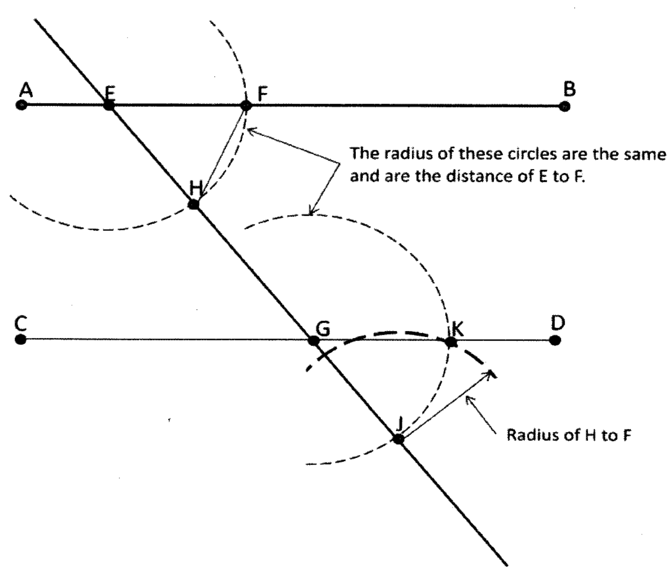


Construct Midpoint of a Line

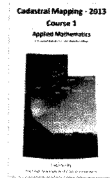
e. These steps will draw a perpendicular line from point C to point D and will Bisect the line AB at the Midpoint of the line. NOTE: The radius used from point A and point B must be precisely the same, they cannot be different.

1.6.58. To create **PARALLEL LINES** the following constructs are made.

a. Line AB is a line that we want to construct another line parallel to eventually drawing line CD. Point G is also a known point that has been previously



Construct of Parallel Lines



established at a specific distance from line AB.

b. Draw a line through point G crossing line AB at point E. It is not necessary to have point E at any specific location.

c. Construct a Circle or Arc with a radius of E to F identifying point H. The length of this radius does not need to be a specific length.

d. Construct a Circle or Arc from point G with precisely the same length radius as point E to F. This Construct identifies point J.

e. Set the radius equal to the distance of point H to F.

f. Using point J as the radius point construct an arc that will identify point K. Again, the distance of point J to K being equal to point H to F and intersecting two circles or arcs.

g. Finally, line CD is constructed through points G and K. NOTE: It may be easier and a more precise method of construction to draw line EG so that it crosses line AB at an approximately a 45° angle.

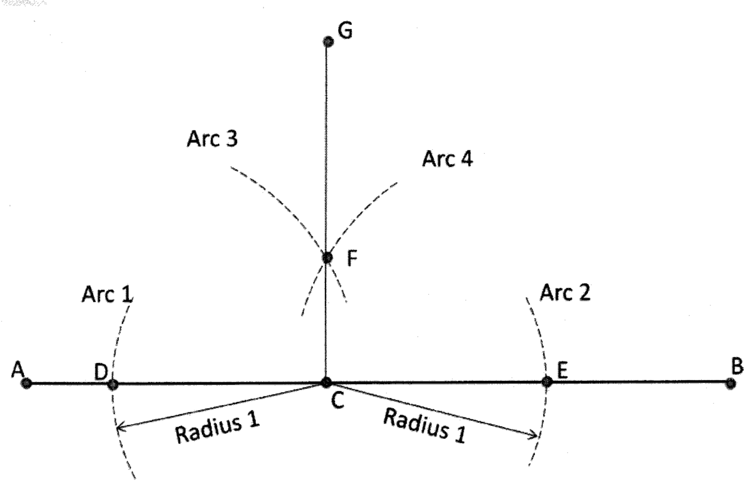
1.6.59. A **PERPENDICULAR LINE THRU A POINT ON A LINE** is made using the following geometric constructs.

a. In this example line AB is drawn with point C lying somewhere on the line.

b. A compass is set to a random radius and two arcs are scribed (arc 1 and arc 2) using the same radius length as radius 1 creating points D and E.

c. Using point D as the radius point and adjusting the compass so that the radius is a little longer than radius 1, scribe arc 3.

d. Using point E and holding the same radius that was used to scribe



Perpendicular Line
Thru a Point on a Line

arc 3 scribe another arc that being arc 4.

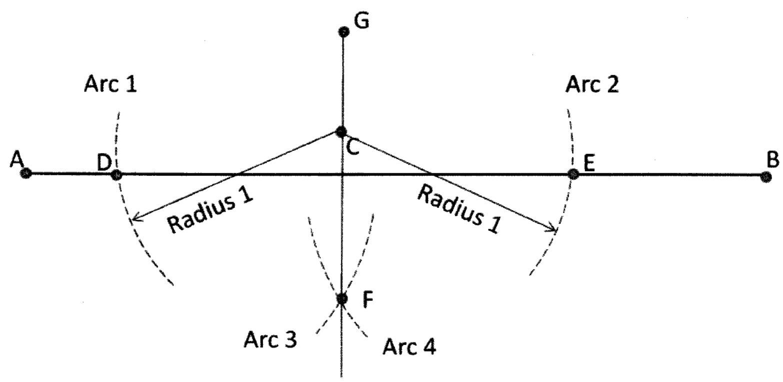
e. At the intersection of arc 3 and 4 point F is created.

f. A line is then drawn through points C and F creating line CG. Line CG is perpendicular to the line and passes through point C.

1.6.60. A **LINE PERPENDICULAR TO A LINE THRU A POINT NOT ON A LINE** is made using the following geometric constructs.

a. In this example line AB is drawn with point C being a point that is not on line AB. From point C a compass is set to a random radius and arcs 1 and 2 are drawn creating points D and E.

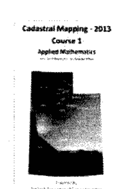
b. Using point D the compass is adjusted so that the radius is a little longer than the distance from point D to where a perpendicular line from point C would intersect the line.



A Line Perpendicular to a Line
Thru a Point NOT on a Line

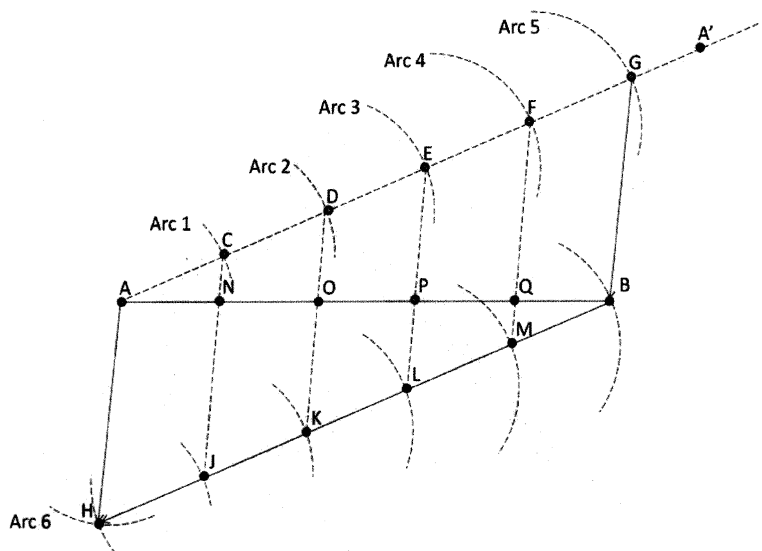
c. Arc 3 and arc 4 are then drafted with this radius using point D and Point E, respectively, as the radius point creating point F.

d. Line FG is then drawn so that the line passes through points F and C and creating a line perpendicular to line AB.



1.6.61. To **DIVIDE A LINE INTO EQUAL SEGMENTS** the following constructs are made. There are two methods for accomplishing this task the first will be demonstrated utilizing only geometric construction using the straight edge and compass, the second will be demonstrated using the drafting scale.

1.6.62. **Method One.**



Dividing a Line into Equal Segments

a. Line AB is the line that is going to be divided into 5 equal segments eventually establishing points N, O, P, and Q. The process is started by drawing a line of random direction from point A (this is line AA').

b. Using a compass mark line AA' with arcs of a radius which will draw 5 arcs that will end beyond point B. These arcs will create points C, D, E, F, and G by using point A as the radius point for the first arc and drawing point C. The point C is used as the radius point to draw the arc for point D (also using the same radius distance from A to C in each case). Then point D is used for the radius point to draw the arc for point E. This process is followed for each subsequent point until point G is drawn.

c. The radius of the compass is then adjusted for the distance from G to B.

d. Using this radius an arc is drawn using point A as the radius point from which arc 6 is drawn passing through point H.

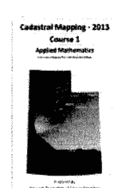
e. The radius of the compass is then adjusted for the distance from A to G.

f. Using this radius an arc is drawn using point B as the radius point from which arc 7 is drawn passing through point H.

g. A line is drawn from point H to point B.

h. The compass is then set to the radius of A to C.

i. Line HB is then divided in the same manner as line AG.

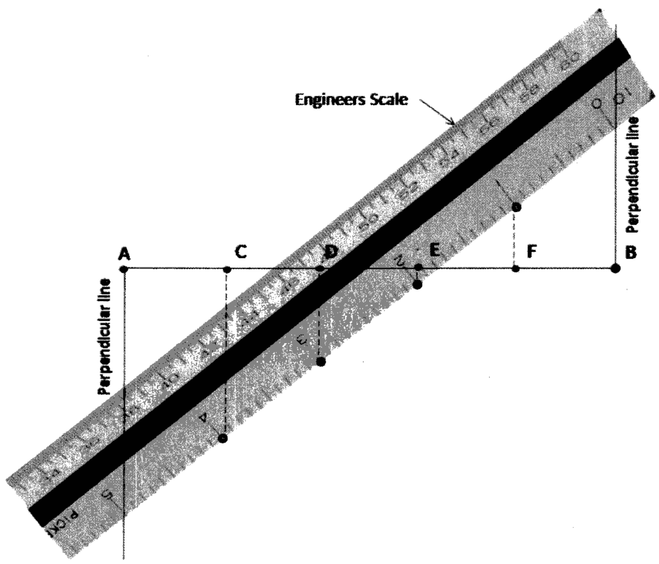


j. A line is then drawn between points C and J, points D and K, points E and L, and points F and M.

k. The intersection of the lines just drawn and line AB create points N, O, P, and Q which divide the line into 5 equal segments.

1.6.63. **Method Two.** While this method is a principle of modern geometric construction, it is not a method that was developed by the ancient Greeks utilizing only a compass and un-marked straight edge.

a. This method utilizes an engineering scale (drafting scale) to make the division marks instead of a compass. In this method line AB is drawn and a perpendicular line from point A is drawn and a perpendicular line from point B is also drawn.



Dividing a Line into Equal Segments using Engineers Scale

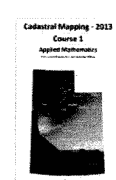
b. An appropriate scale is chosen to use, in this case the 10 scale will work.

c. The scale is lined up with the perpendiculars with the 0 (zero) on one perpendicular and the 5 (five) on the other perpendicular. The 5 is used to divide the line into 5 equal segments.

d. Marks are made at each numbered index (each inch).

e. From these index marks perpendicular lines are drafted to intersect line AB which creates points C, D, E, and F.

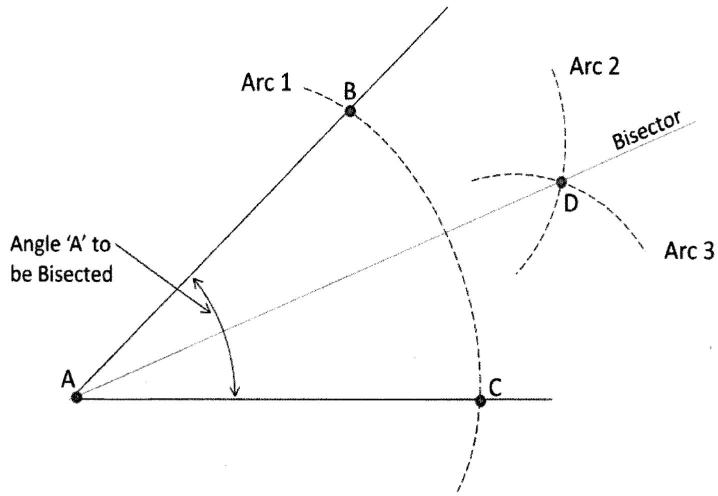
f. The line is now divided into 5 equal segments.



1.6.64. **Angles.**

1.6.65. **BISECTING AN ANGLE** into two equal angles is done by using the following constructs.

a. In this example Angle ∟BAC is to be divided into two equal parts (bisecting angle A). This is done by using a compass to draw an arc or random radius holding point A as the radius point. Creating arc 1, this arc creates points B and C.

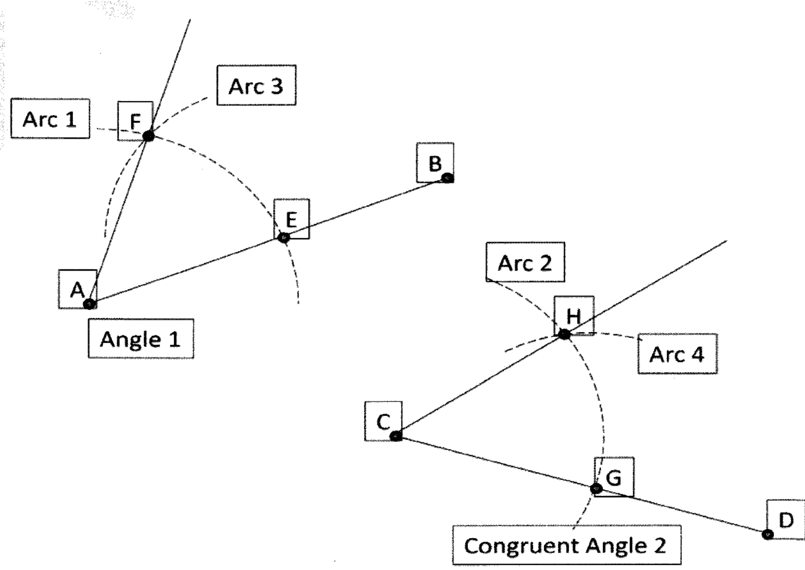


Bisecting an Angle
(Creating two angles of equal magnitude)

b. The compass is adjusted so that two arcs will be created that intersect at point D. These two arcs are drawn by using point B as the radius point and scribing arc 2 then using point C as the radius point scribing arc 3.

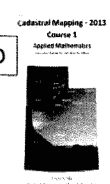
c. The bisecting line is then drawing from point A through point D creating two equal angles ∟BAD and ∟DAC.

1.6.66. **CONGRUENT ANGLES** are angles that are equal to one another such as ∟BAD and ∟DAC which resulted from bisecting an angle shown in section 2.6.1 above. Congruent angles can be constructed as follows.



Congruent Angles

a. Line AB is drafted and the line passing through point F from point A is also drafted. For purposes of this example line AF does



not need to be a specific angular value from line AB.

b. Line CD is then drafted and an angle that is congruent to angle ∟BAF is to be constructed using line CD as a terminal side of the angle.

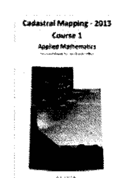
c. Scribe Arc 1 at a random radius. Our example scribes the arc passing through points E and F.

d. Arc 2 is of the same radius as the distance from points A to E and is scribed using point C as the radius point creating point G on line CD.

e. The compass is then set equal to the distance from points E and F and point G is used as the radius point to scribe Arc 4.

f. The intersection of Arc 2 and Arc 4 create point H.

g. A line is constructed beginning at point C and passing through point H. The angle ∟DCH is congruent to angle ∟BAF.



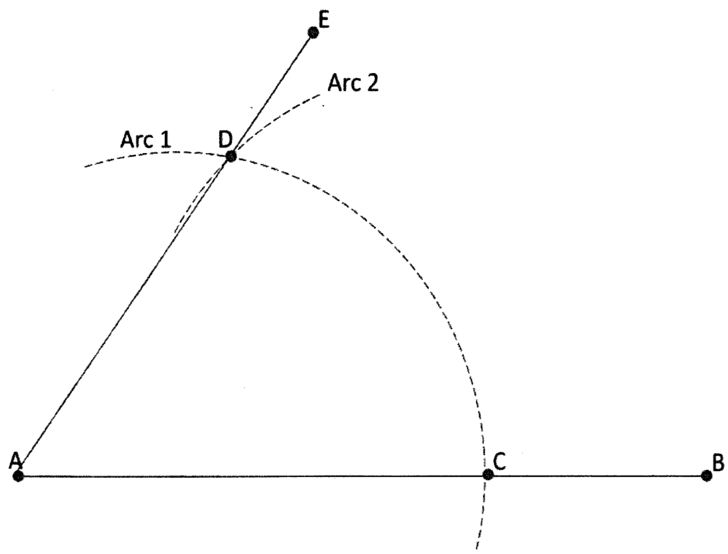
1.6.67. Creating **30°, 60°, and 45° ANGLES.** In geometry and drafting the 30°, 60° and 45° angles are very commonly used. The following steps will illustrate how to construct these angles.

1.6.68. A **60° ANGLE** is drafted using the following geometric constructs.

a. This is done by setting a compass to a random radius and scribing arc 1 by using point A as the radius point of arc 1 creating point C. Arc 1 should be drafted long enough that point D can be created on arc 1.

b. Using the same radius from points A to C - point C is used as the radius point and arc 2 is drawn to intersect arc 1 creating point D.

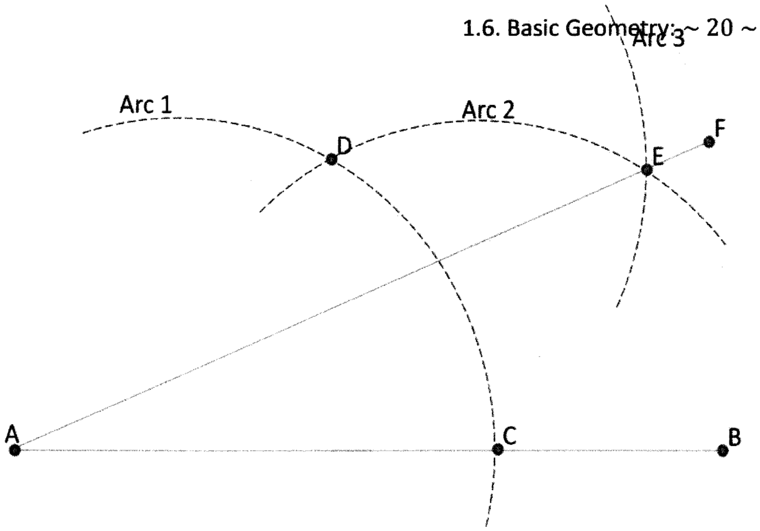
c. A line is then drawn through points A and D creating line AE and the angle ∟EAB is then drafted to be 60°.



Constructing a 60° Angle

1.6.69. A **30° ANGLE** is drafted using the following geometric constructs. The beginning is identical to that of constructing a 60° angle but an additional step is added to construct the 30° angle.

a. This is done by setting a compass to a random radius and scribing arc 1 by using point A as the radius point of arc 1 creating point C. Arc 1 should be drafted long enough that point D can be created on arc 1.



Constructing a 30° Angle

b. Using the same radius from points A to C point C is used as the radius point and arc 2 is drawn to intersect arc 1 creating point D.

c. Moving the radius point of the compass to point D and holding the same radius used for arc 1 and 2 - arc 3 is drawn intersecting arc 2 at point E.

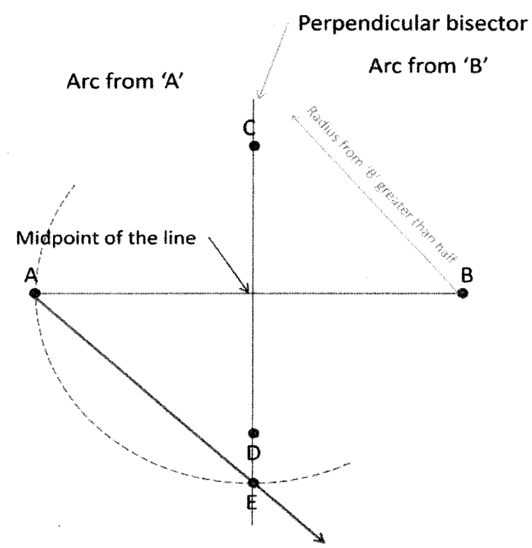
d. A line is then drawn passing through points A and E creating line AF. The angle ∟FAB is now drafted to be a 30° angle.

1.6.70. A **45° ANGLE** is drafted using the following geometric constructs. The process to construct a 45° Angle begins by drawing a perpendicular line at the midpoint of a line segment as shown in section 1.6.23. After constructing a perpendicular bisector the final steps are as follows.

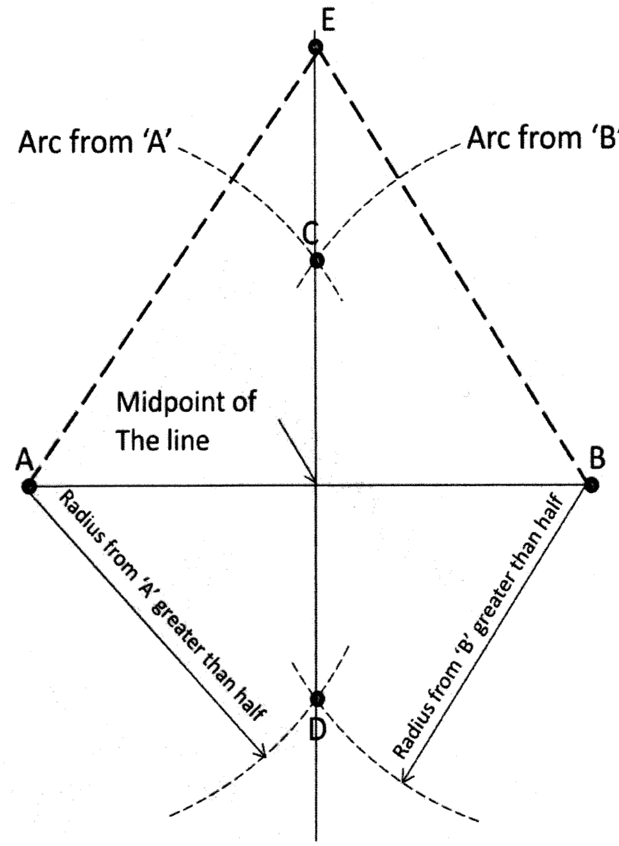
a. Set the compass radius equal to the distance from the midpoint of the line to point A.

b. Construct an arc using the midpoint of the line as the radius point so that the arc passes through point A and intersects the extensions of line CD creating point E.

c. A line is then constructed which passes through points A and E. Creating angle ∟EAB to be 45°.



1.6.71. An **ISOSCELES TRIANGLE** is used in algebra to help identify when a quadratic equation is to be used in solving a problem known in surveying as the Bearing-Distance Intersection. Such triangle has the characteristics of having two angles of the triangle equal with the same value. In the example shown, angles at points A and B have the same magnitude (size). In other words, ∟EAB and ∟EBA are equal.



Construct an Isosceles Triangle

a. To construct an Isosceles Triangle the geometric constructs shown in section 1.6.23 to bisect a line is followed using the next steps (The perpendicular bisect line is not necessary to draft, however, it may be helpful to complete the process.).

b. Having line CD drafted, establish a point on line CD (which may extend beyond points C and D for any distance) at any location.

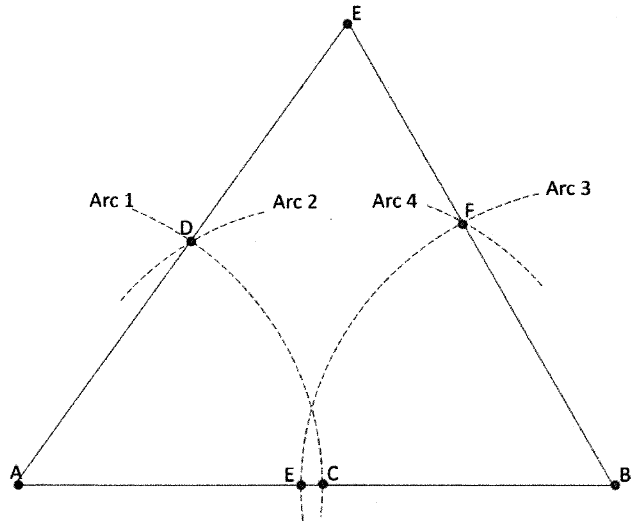
c. We can use point E for this random position.

d. Draw lines between point A and E then between points B and E.

e. The triangle ∟ABEA is and isosceles triangle.

f. The angular relationship between these angles is such that if point E was moved to any location on line CD the angles at points A and B would remain the same.

1.6.72. The **EQUILATERAL TRIANGLE** is a triangle in which all three angles equal the same value, which is 60°. The equilateral triangle is constructed in the same manner as that of developing a 60° angle as shown in section 1.6.34. where the steps are detailed using point A to develop the 60° angle at point A.



Constructing an Equilateral Triangle

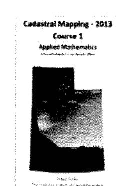
a. The same steps as noted are repeated using point B to construct another 60° angle at point B. Once this is completed the location at which the two newly constructed lines intersect will form the final 60°angle and complete the construction of the equilateral triangle.

1.6.73. An easier or alternative method of constructing an equilateral triangle is to use the following steps.

a. Using line AB, set a compass radius equal to the distance between points A and B.

b. Scribe two arcs the first holding point A as the radius point and then holding point B as the radius point.

c. Where these two arcs cross or intersect will create point E from which the lines AE and BE can be drawn. The resulting triangle is equilateral.

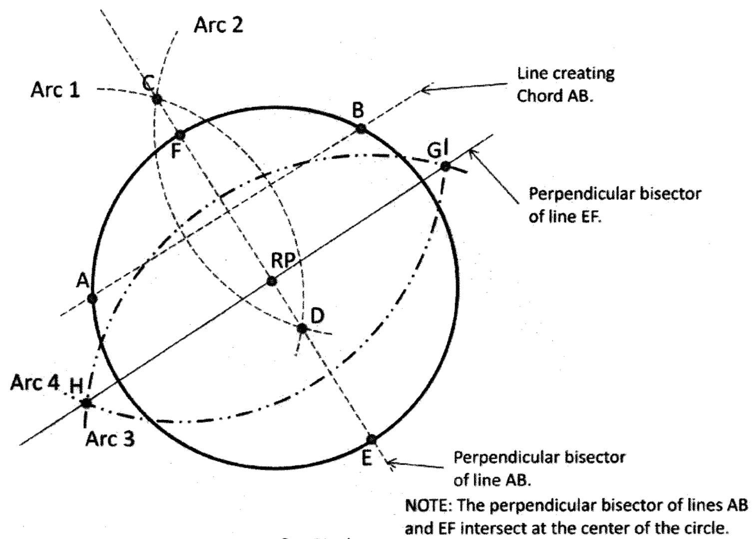


1.6.74. **Circles.**

1.6.75. The **CENTER OF A CIRCLE** is found by following the steps detailed herein. It may look confusing but if the steps are followed, which utilize some of the principles discussed previously, the identification of the radius point (center of the circle) is straight forward. Essentially, the process includes bisecting a line twice.

a. The first step is to construct a line (line AB) that cuts the circle in a manner which creates a chord.

b. The radius of a compass is then set so that it is greater than the mid-point of line AB so that Arc 1 and Arc 2 can be constructed using point A and point B as the radius point of each arc, respectively.



Center of a Circle

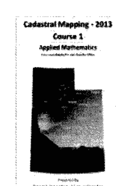
c. Point C and point D are created at the intersections of both Arc's 1 and 2.

d. Drafting line CD then bisects chord AB. Line CD is extended so that it passes through the entire circle intersecting the circle at points E and F. This line is the perpendicular bisector of chord AB.

e. Next is to construct a perpendicular bisector of line EF by setting the compass radius to be greater than the mid-point of line EF so that Arc 3 and Arc 4 can be constructed using point E and point F as the radius point of each arc, respectively.

f. Point G and point H are the created at the intersections of both Arc's 3 and 4.

g. A line is drawn between points G and H that intersects the Perpendicular bisector of line AB at point RP (the center of the circle).

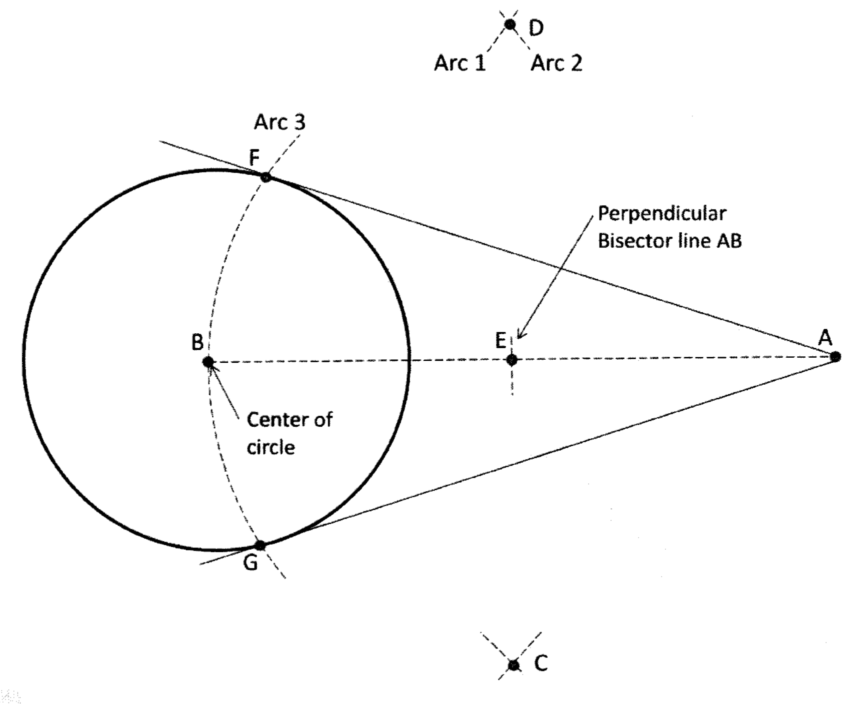


1.6.76. **TANGENTS TO A CIRCLE** (the center which is point B) from a point (point A) are found as follows.

a. Start by drawing a line from Point A to Point B.

b. Next is to find the midpoint of the line AB by constructing Arc 1 and Arc 2 which will identify Point C and Point D. The midpoint is found in the same manner as previously described.

c. The midpoint of the line is then completed by intersecting Line AB and a line between Points C and D, creating point E.



Tangents To A Circle

d. Next adjust a compass to the distance between Point E and Point B and draw Arc 3 which will pass through point B creating points F and G.

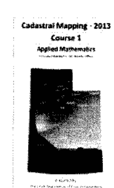
e. To finish construct straight lines from point A through points F and G as shown. The lines AF and AG are tangent to the circle.

1.6.77. **THREE POINT ARC** or a circle which touchs three points is constructed in the following manner.

a. Start by identifying 3 points (points 1, 2, and 3) in a random location. In surveying these points could be points on a curb or edge of pavement or even on the centerline of a highway curve.

b. Construct a straight line between points 1 and 2.

c. Construct another straight line between points 2 and 3.



d. Next find the midpoint of each line, line 1-2 and line 2-3 in the same manner as previously described, by

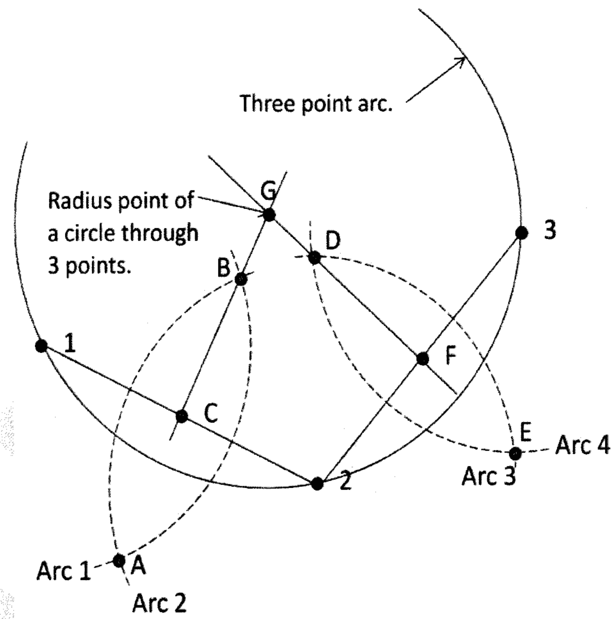
e. Setting a compass to construct Arc 1 and Arc 2 using a radius which is greater than the midpoint of the line using point 1 and point 2 as the radius point for Arc 1 and Arc 2.

f. This will create points A and B.

g. Construct a line which passes through points A and B and creates point C on line 1-2.

h. Then the midpoint of line 2-3 is found in the same manner creating Arc 3 and Arc 4.

i. The intersect of Arc 3 and Arc 4 create point D and point E.



Three Point Arc

j. Then a straight line is constructed between point D and point E which intersects line 2-3 at point F.

k. Line AB and line DE are extended so that they intersect at point G creating line CG and line FG.

l. Point G is the center of a circle that will pass through points 1, 2 and 3.

1.6.78. The Tie to Algebra.

1.6.79. "In ancient Greece the Pythagoreans[2] considered the role of numbers in geometry. However, the discovery of incommensurable lengths, which contradicted their philosophical views, made them abandon abstract numbers in favor of concrete geometric quantities, such as length and area of figures.

1.6.80. "Numbers were reintroduced into geometry in the form of coordinates by Descartes, who realized that the study of geometric shapes can be facilitated by their algebraic representation, and whom the Cartesian plane is named after.

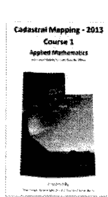
[2]. **Pythagoreanism** was the system of esoteric and metaphysical beliefs held by Pythagoras and his followers, the Pythagorean cult, who were considerably influenced by mathematics, music and astronomy. Pythagoreanism originated in the 5th century BCE and greatly influenced Platonism.

1.6.81. “Analytic geometry applies methods of algebra to geometric questions, typically by relating geometric curves and algebraic equations.

1.6.82. “These ideas played a key role in the development of calculus in the 17th century and led to discovery of many new properties of plane curves. Modern algebraic geometry considers similar questions on a vastly more abstract level.”[3]

1.6.83.

[3]. The information in the last 4 paragraphs is from Wikipedia, The Free Online Encyclopedia from the search term “Geometry”.



APPLIED MATHEMATICS:
Course 1. Chapter 7.

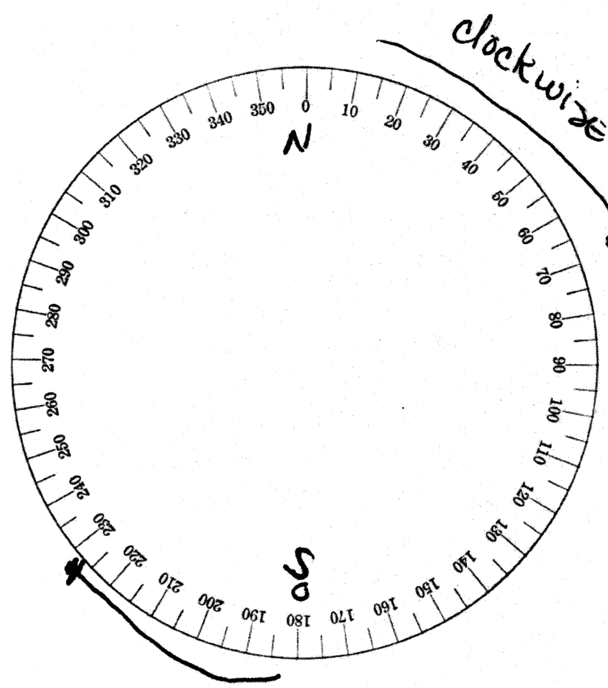
1.7 Bearing and Azimuth Calculations:

1.7.1. This may seem like a basic calculation process. However, experience has revealed that this subject is not necessarily the easiest subject to understand or master for many of the students that I have taught over the last decade. The calculation process is somewhat basic, however, the ability to visualize the geometry necessary to solve a variety of types of problems may require practice.

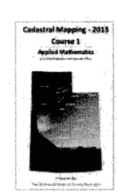
1.7.2. The cadastral mapper may be required to perform these types of calculations with regularity. With that in mind there will be a number of problems in this section to solve. The ability to add and subtract angles will be necessary. If the student does not understand the concept of adding and subtracting angles is will be necessary to review the material in Chapter 5 - HMS Conversions. Additionally, Chapter 5 discussed the concept of the angle. Trigonometry will be necessary to understand to gain a complete understanding of this subject.

1.7.3. The concepts of Azimuths, Back Azimuths, Bearings, Back Bearings, and interior angles are also necessary for the student to fully understand calculations of Bearings and Azimuths. This is where this subject material will begin.

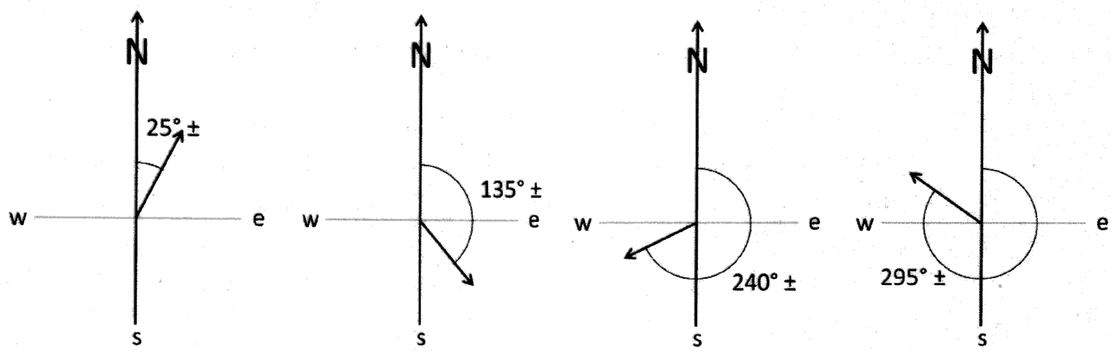
1.7.4. **An Azimuth** is generally a clockwise angle being reckoned from North. Sometimes the Azimuth can be a clockwise angle being reckoned from South. This South reckoning was usually used by the USC&GS (United States Coast and Geodetic Survey), the predecessor to the NGS (National Geodetic Survey) for accomplishing the national triangulation survey network. In this text it will be assumed that Azimuths are reckoned from North, however, the concept of North and what North means will be discussed in detail in a subsequent chapter so that cadastral mapper will understand what it means to have a "common basis of bearing" in the mapping process.



1.7.5. The magnitude of an Azimuth or the angle of the azimuth begins at zero degrees (0°) at the North point. As the angle rotates in the clockwise direction the value of the angle



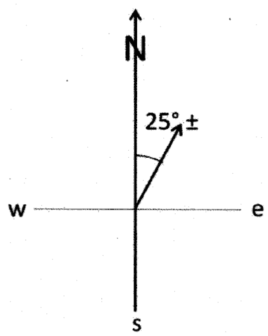
increases. The quarter circle being equal to 90°, the half circle being equal to 180°, the 3/4 circle being equal to 270°, and the full circle being equal to 360°. Some examples of azimuths in each quadrant is shown below.



Azimuth in 1st quadrant | Azimuth in 2nd quadrant | Azimuth in 3rd quadrant | Azimuth in 4th quadrant

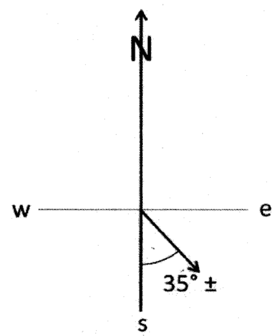
1.7.6. Azimuths are written without quadrant directors but bearings must have a quadrant director to know in what quadrant the angle resides. An azimuth is noted in some manner so that the user knows that it is not just a random angle. For instance, 25°10'30" AZ, 154°49'30" AZ, 205°10'30" AZ, and 334°49'30" AZ are azimuth angles in each of the four quadrants. The "AZ" is the notation that will be used in this text to indicate the angle is an azimuth.

1.7.7. **The Bearing Angle** is another a means of defining direction that is related to the cardinal points of the compass, North, South, East and West. This is probably the most common means of defining direction. Each bearing angle is written beginning with either a North or South director followed by the magnitude of the angle and ending with either an East or West director, in the following pattern. N 25°10'30" E, S 25°10'30" E, S 25°10'30" W, and N 25°10'30" W. The 4 examples describe an angle in each of the four compass quadrants. Other examples are shown below, the plus/minus sign is included with these examples to indicate that any angle or size could be shown.



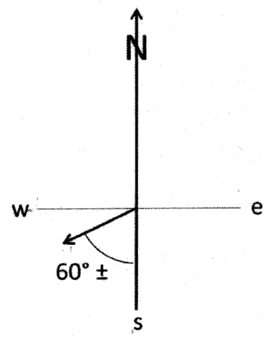
Bearing in 1st quadrant

N 25° E



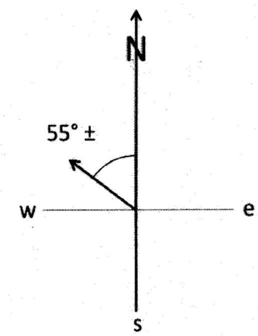
Bearing in 2nd quadrant

S 35° E



Bearing in 3rd quadrant

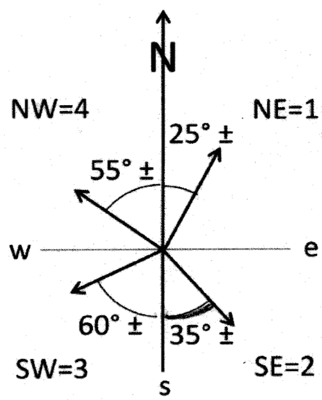
S 60° W



Bearing in 4th quadrant

N 55° W

1.7.8. Notice that the bearing angles are reckoned from either North or South (the north or south line of the compass being the initial side of the angle) increasing in magnitude toward the East or West resulting in the terminal side of the angle.



Bearing quadrants

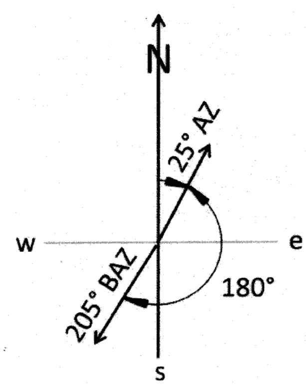
1.7.9. From the diagram the quadrants of the compass are also defined. When utilizing most computer software the numbers that correspond to the quadrants are used to identify bearings that reside in the Northeast (equaling quadrant 1), Southeast (equaling quadrant 2), Southwest (equaling quadrant 3), and Northwest (equaling quadrant 4).

1.7.10. With the Azimuth and Bearing defined the meaning of Back Azimuth and Back Bearing will now be discussed.

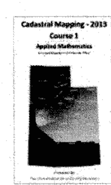
1.7.11. To determine a **Back Azimuth (BAZ)** or Back Bearing a Straight Angle is applied to the value, for instance, an Azimuth of 25° AZ, the back azimuth would be 205° BAZ. The difference being precisely 180°.

1.7.12. Examining the other three quadrant azimuths and their associated back azimuths we find; 135° AZ = 315° BAZ, 240° AZ = 60° BAZ, and finally 295° AZ = 115° BAZ.

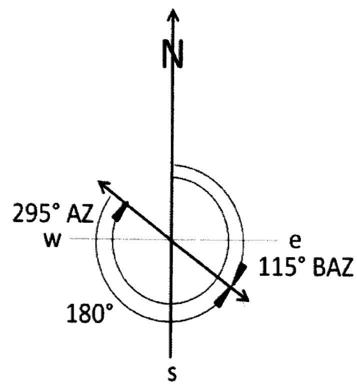
1.7.13. Graphically we can see how this works. Where the AZ is in the NE quadrant with a magnitude of 25° the BAZ is calculated by adding 180° to the AZ equaling 205°.



Azimuth in 1st quadrant



1.7.14. **RULE:** To obtain the BAZ for any AZ in either the NE or SE quadrants the addition of 180° is performed. For an AZ in the SW or NW quadrants 180° is subtracted from the AZ to obtain the BAZ.



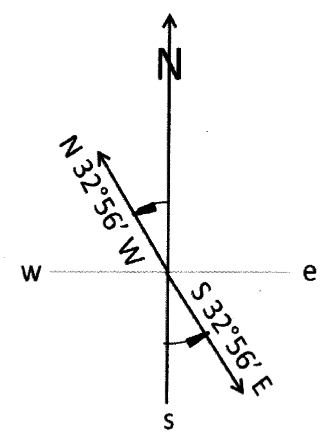
Azimuth in 4th quadrant

1.7.15. In the example to the left the AZ equals 295°, by subtracting 180° from the AZ the remainder is equal to 115° and is the BAZ.

1.7.16. In cadastral mapping azimuths and bearings are used in written descriptions which are interpreted into a graphical representation or map. Sometimes descriptions only contain the metes (bearings and distances) of a description. In the examples used in this section of the text only descriptions containing the metes will be used.

1.7.17. The **Back Bearing (BB)** is actually easier to obtain than that of the back azimuth because there is no calculation to perform. To obtain the Back Bearing it is as simple as using the opposite North-South or East-West director. For instance, a bearing of N 25° E the back bearing is obtained by changing the North to South and changing the East to West as in S 25° W BB[1]. The angular value of the bearing always remains the same.

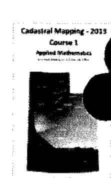
1.7.18. To describe this in reference to the graphic example to the right we see that a line from point A to point B has been drawn to represent a bearing of S 32°56' E. The back bearing (from point B to point A) is also represented with the same angle but now the opposite quadrant is being used and equals N 32°56' W.



Bearing and Back Bearing

1.7.19. Therefore, if we examine the compass quadrants we see that; for any bearing in the Northeast (NE) quadrant the back bearing is in the Southwest (SW), for any bearing in the Southeast (SE) quadrant the back bearing is in the Northwest (NW) quadrant, for any bearing in the Southwest (SW) quadrant the back bearing is in the Northeast (NE) quadrant, and finally for any bearing in the Northwest (NW) quadrant the back bearing is in the Southeast (SE) quadrant.

1 . The BB shown here is not usually used in surveying or mapping. It may be used only in this material to help indicate when the bearing is being used as a Back Bearing (BB).



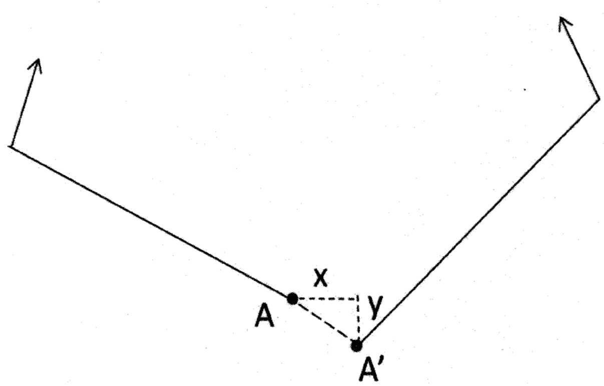
1.7.20. **AZIMUTH and BEARING CALCULATIONS:** The following exercise will show how to make azimuth and bearing calculations of a traverse using only interior angles. Prior to working through the calculations for this traverse the concept of "error of closure" needs to be developed.

1.7.21. **Error of Closure** is the error associated with any traverse (parcel) and the surveyed angles and distances of that parcel. When a parcel is being Traversed measurements are being made of the angles and distances of each leg or side of the parcel. The cadastral mapper must understand that inherant with every angle and distance measurement made by surveying there are errors contained in the data. This concept was introduced in Chapter 3 – Siginificant Digits.

1.7.22. Errors in surveying measurements are always present in any data and can not be eliminated. Mistakes on the other hand, also called blunders, should not exist in surveying measurements. Mistakes should be found or identified and corrected or eliminated. Errors can not be eliminated and in most cases they are not able to be detected. It is because of this fact that surveyors have developed tools to help compensate for errors. These tools are based in statistics and is beyond the scope of this discussion.

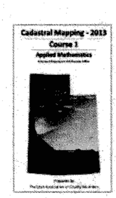
1.7.23. Some of the methods used by surveyors are Least Squares Adjustments, Transite Rule Adjustment, even the Compass Rule Adjustment. These adjustments are intended to distribute systematic errors in a manner that will best represent a survey of a parcel in a mathematical sense. Error of Closure is a term used in the Compass Rule Adjustment, as well as other adjustment types, to identify the survey errors in a given data set and let the surveyor know how accurate his work was to help them determine if the work needs to be redone or if it meets the required specifications or standards.

1.7.24. Error of Closure is illustrated at the right with the diagram showing two points A, one being point A and the other being A'. In any traverse/parcel the intent is that there should only be one coordinate value for each point of a traverse/parcel. Due to the errors in surveying measurements there are generally two points for the beginning point. The mathematical difference in both y or Northing and x or Easting produce two sides of a right triangle.



Error of Closure

1.7.25. Using pythagoreans theorem the length of the hypotnuse can be calculated which is the error of closure. Let's say that $x = 0.12$ and that $y = 0.07$ units. Calculating the length of AA';



$$AA'^2 = x^2 + y^2$$

a. filling in the variables,

$$AA'^2 = 0.12^2 + 0.07^2$$

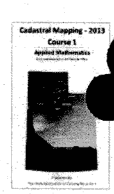
b. solving for AA' we get,

$$AA' = \sqrt{0.12^2 + 0.07^2}$$

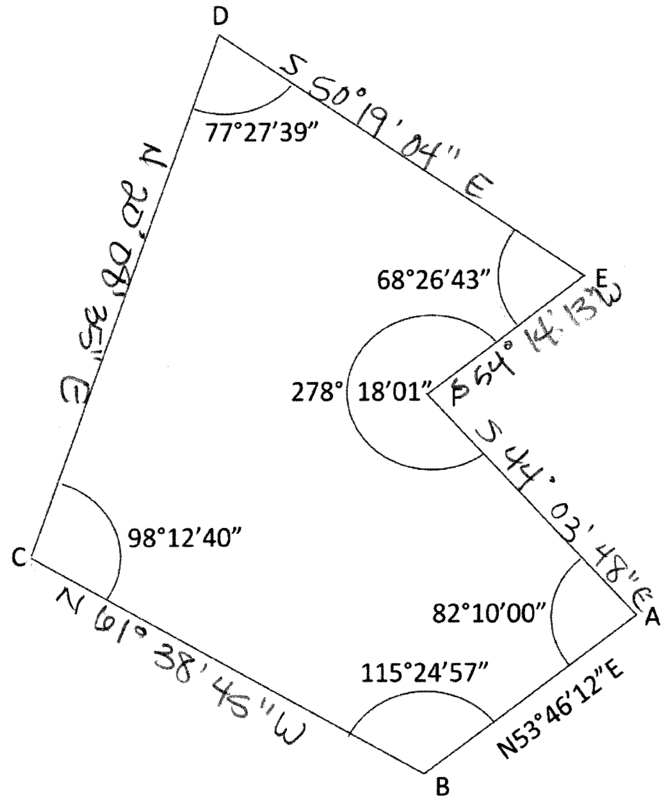
$$AA' = 0.1389$$

1.7.26. Therefore, the statement is that the Error of Closure for the traverse is equal to 0.1389 units with respect to the total distance of each leg of the traverse/parcel. When relating this concept to surveying measurements it can be said that there was approximately 0.14 feet of accumulated error in the traverse.

1.7.27. The purpose of discussing this concept is so that it is understood that not all traverses or property descriptions will close and that there are methods available to surveyors to distribute measurement errors. That the calculations of bearing and distances around a parcel may contain these errors and that for a parcel or description to properly close some survey adjustments to the data may be necessary. A discussion on surveying measurements and associated errors in legal descriptions and the analysis which the mapper may be required to perform will be discussed in a subsequent chapter. For this discussion we will check for mathematical closure but not employ any error analysis.



1.7.28. **BEARING CALCULATION**. To begin with, the **line AB** is identified with a bearing of N 53°46'12" E and is considered the basis of bearing for this traverse. With the direction of the line being noted as a Northeast bearing (line BA) and the line being labeled from point A running to the Southwest to point B the bearing is also the Back Bearing of the line BA.



Bearing Calculation from Interior Angles

1.7.29. With this information we can check to see if the traverse "closes". What this means is that the interior angles of any traverse "should" close using the following formula.

Sum of the Angles = 180 (N-2)

1.7.30. We have used this equation in previous chapters so you should be familiar with the form of the equation and what it does. The variable "N" equals the number of interior angles in the traverse. To evaluate this problem identify the number of interior angles. In this case there are 6 interior angles. Six is then put into the equation for the variable "N" and evaluated as follows.

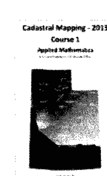
SA = 180 (6-2)

SA = 180 (4)

SA = 720°

1.7.31. Therefore, the sum of the interior angles should equal precisely 720°. To check the traverse add the interior angles:

82°10'00"
115°24'57"
98°12'40"
77°27'39"



68°26'43"
278°18'01"
Σ 720°00'00"

1.7.32. From this calculation we know that the traverse closes in angle it may still not close in distance, however, that is for another discussion. We can now calculate the bearings of each leg of the traverse.

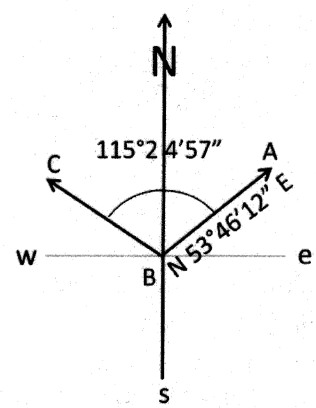
1.7.33. Line BC.

a. To begin the calculation for the forward bearing of **line BC** draw a compass with the cardinal points. Then add the back bearing line of line BA and the forward bearing of line BC as shown.

b. The next step is to identify the bearing angle for line BC. Which as can be seen from the graphic line BC falls within the Northwest quarter so the direction will be North and West.

c. To calculate the angle in this case the bearing of line BA will be subtracted from the interior angle ABC.

115°24'57" interior angle ABC
- 53°46'12" back bearing angle line BA
61°38'45" forward bearing angle line BC



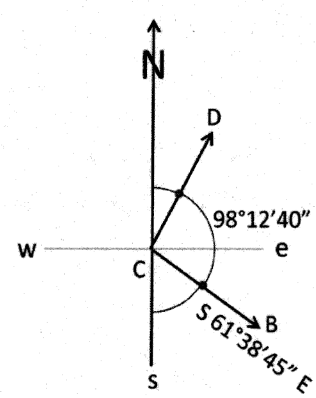
Line BC

Combining the calculated angle and the quadrant of the forward bearing of the line BC the bearing equals **N 61°38'45" W.**

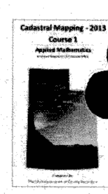
1.7.34. Line CD.

a. Calculating the forward bearing of **line CD** begins by drawing the compass with the cardinal points as shown. Then identify the back bearing of line CB. Draw and label line CB on the compass.

b. Identify the quadrant of the forward line by estimating using the back bearing angle and the interior angle of the traverse at point C. This estimation shows that the forward bearing



Line CD



of line CD will be in quadrant 1 or the Northeast quadrant.

c. Next identify the bearing angle for line CD. Label the interior angle C (the angle between lines CB and CD). From inspection the bearing angle of line CD is between the North compass point and line CD. To calculate the angle in this case the bearing of line CB and the interior angle C will be subtracted from 180°.

180°00'00" straight angle from north to south
<u>- 61°38'45"</u> back bearing angle line CB
118°21'15"

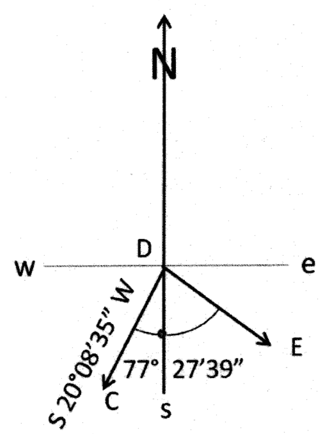
118°21'15"
<u>- 98°12'40"</u> interior angle C
20°08'35" forward bearing angle line CD

Combining the calculated angle and the quadrant of the forward bearing of the line CD equals **N 20°08'35" E.**

1.7.35. Line DE.

a. Calculating the forward bearing of **line DE** begins by drawing the compass with the cardinal points as shown. Then identifying the back bearing of line DC. Draw and label line DC on the compass.

b. Identify the quadrant of the forward line by estimating using the back bearing angle and the interior angle of the traverse at point D. This estimation shows that the forward bearing of line DE will be in quadrant 2 or the Southeast quadrant.

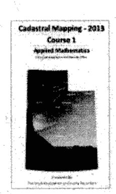


Line DE

c. Next identify the bearing angle for line DE. Label the interior angle D (the angle between lines DC and DE). From inspection the bearing angle of line DE is between the South compass point and line DE.

d. To calculate the angle in this case the bearing of line DC is subtracted from the interior angle D.

77°27'39" interior angle D
<u>- 20°08'35"</u> back bearing angle line DC
57°19'04" forward bearing line DE



Combining the calculated angle and the quadrant of the forward bearing of the line DE equals **S 57°19'04" E.**

1.7.36. Line EF.

a. Calculating the forward bearing of **line EF** begins by drawing the compass with the cardinal points as shown. Then identifying the back bearing of line ED. Draw and label line ED on the compass.

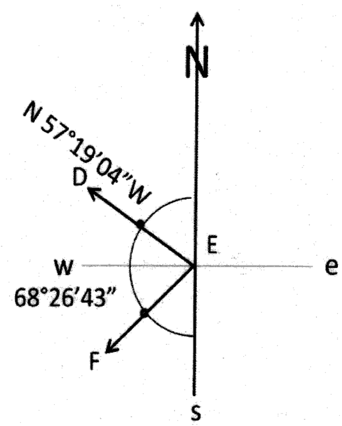
b. Identify the quadrant of the forward bearing line by estimating using the back bearing angle and the interior angle of the traverse at point E. This estimation shows that the forward bearing of line EF will be in quadrant 3 or the Southwest quadrant.

c. Next identify the bearing angle for line ED. Label the interior angle E (the angle between lines ED and EF). From inspection the bearing angle of line EF is between the South compass point and line EF.

d. To calculate the angle in this case the bearing of line ED and the interior angle E are subtracted from a straight angle of 180°.

180°00'00" straight angle from north to south
- 57°19'04" back bearing angle line ED
122°40'56"

122°40'56"
- 68°26'43" interior angle E
54°14'13" forward bearing angle line EF

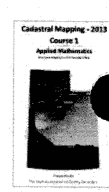


Line EF

Combining the calculated angle and the quadrant of the forward bearing of the line EF equals **S 54°14'13" W.**

1.7.37. Line FA.

a. Calculating the forward bearing of **line FA** begins by drawing the compass with the cardinal points as shown. Then identifying the back bearing of line FE. Draw and label line FE on the compass.



b. Identify the quadrant of the forward bearing line by estimating using the back bearing angle and the interior angle of the traverse at point F. This estimation shows that the forward bearing of line FA will be in quadrant 2 or the Southeast quadrant.

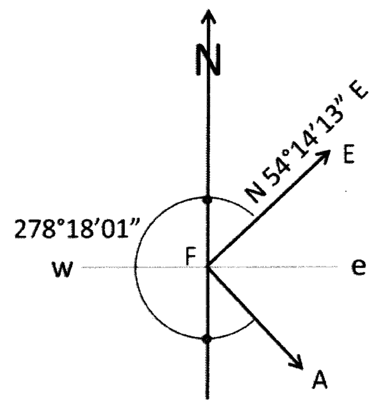
c. Next identify the bearing angle for line FE. Label the interior angle F (the angle between lines FE and FA rotating in a counter-clockwise direction). From inspection the bearing angle of line FA is between the South compass point and line FA.

d. To calculate the angle in this case the bearing of line FE and a straight angle of 180° is subtracted from the interior angle F.

278°18'01" interior angle F
-54°14'13" back bearing angle line FE
224°03'48"

224°03'48"
- 180°00'00" straight angle from north to south
44°03'48" forward bearing angle line FA

Combining the calculated angle and the quadrant of the forward bearing of the line FA equals **S 44°03'48" E.**



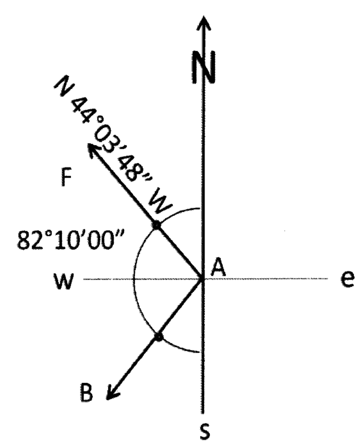
Line FA

1.7.38. This completes the calculations for the bearings of each of the lines of the traverse/parcel. There is, however, one additional step that needs to be done to ensure that there was not a calculation error in the individual legs of the traverse. That is to calculate the bearing of line AB from point A. To **check the calculations** begin the process the same as before.

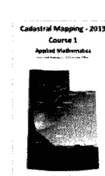
1.7.39. Line AB.

a. Calculating the forward bearing of **line AB** begins by drawing the compass with the cardinal points as shown. Set the center of the compass as point A, then identify the back bearing of line AF. Draw and label line AF on the compass.

b. Identify the quadrant of the forward bearing line by estimating using the back bearing angle and the interior angle of the traverse at point A. This estimation shows



Line AB to check



that the forward bearing of line AB will be in quadrant 3 or the Southwest quadrant.

c. Next identify the bearing angle for line AF. Label the interior angle A (the angle between lines AF and AB rotating in a counter-clockwise direction). From inspection the bearing angle of line AB is between the South compass point and line AB.

d. To calculate the angle in this case subtract the bearing of line AF and the interior angle A from 180°.

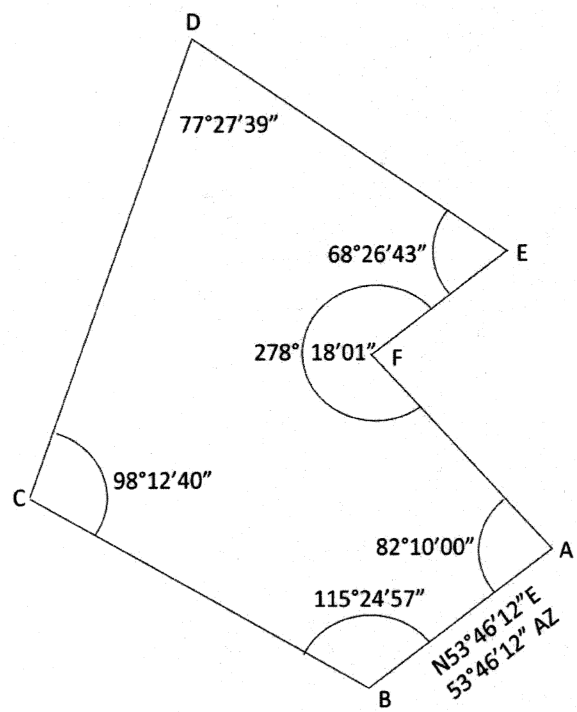
180°00'00" straight angle from north to south
- 44°03'48" back bearing angle AF
135°56'12"

135°56'12"
- 82°10'00" interior angle A
53°46'12" forward bearing line AB

Combining the calculated angle and the quadrant of the forward bearing of the line AB equals **S 53°46'12" W.** This calculation checks the initial bearing of line AB.

1.7.40. **AZIMUTH CALCULATIONS.**

1.7.41. Examining the same traverse/parcel used in the above sample calculate the forward azimuths of each leg of the traverse/parcel. We can begin the process by converting the Back Bearing of line BA to the Back Azimuth. To make the conversion in this case simply drop the quadrant directors and show the bearing as the AZ. This can be done because the back bearing of the line is in the Northeast quadrant. In any other quadrant this is not possible.



Azimuth Calculation from Interior Angles

1.7.42. The line AB is identified with an azimuth of 53°46'12" AZ and is considered the basis of bearing for this traverse. Quadrants with an Azimuth is not necessary to know for purposes of direction identification, however, knowing the quadrant which an azimuth resides in will help in the calculation process.

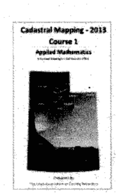
1.7.43. Because the AZ is less than 90° we know that the direction of the line being noted is in the Northeast quadrant which further indicates that the AZ as noted is the Back Azimuth of the line BA.

1.7.44. Line BC.

1.7.45. To begin the calculation for the forward azimuth of **line BC** draw a compass with the cardinal points. Then draw the back azimuth line of line BA and using the interior angle estimate the location of the forward azimuth of line BC as shown.

a. The next step is to identify the azimuth angle for line BC which as can be seen from the graphic that line BC falls within the Northwest quadrant. This will affect how the azimuth angle will be calculated. The azimuth angle is also shown to be a clockwise angle beginning at the North compass point and terminating on the line BC.

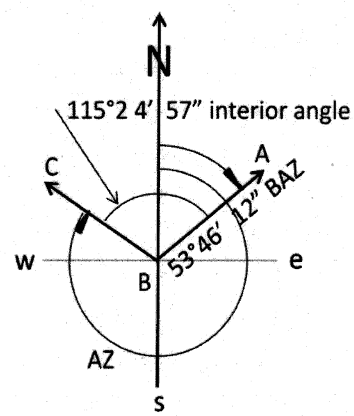
b. To calculate the azimuth angle the azimuth of line BA will be subtracted from the interior angle B resulting in;



115°24'57" interior angle B
- 53°46'12" BAZ - back azimuth angle line BA
61°38'45" forward **bearing** angle line BC

Note: the resulting angle in this case is the Bearing of the line BC. To obtain the Azimuth of line BC this forward bearing must be subtracted from 360°.

360°00'00"
- 61°38'45" forward bearing angle line BC
298°21'15" AZ - azimuth of line BC



Azimuth Line BC

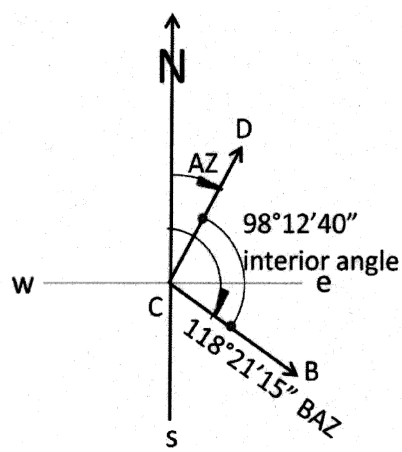
Again, remember that in this text an azimuth is reckoned from North and is a clockwise angle (angle to the right).

1.7.46. Line CD.

a. Start the calculation for the forward azimuth of **line CD** by drawing a compass with the cardinal points. The back azimuth of line BC can be calculated by subtracting 180° from the AZ of line BC.

   298°21'15" AZ - line BC
   - 180°00'00"
   118°21'15" BAZ - line CB

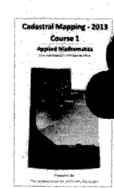
b. Then draw the back azimuth line of line CB. The forward direction of line CD can be estimated using the interior angle at C as shown which indicates the line to be in the Northeast quadrant.



Azimuth Line CD

c. The next step is to identify the azimuth angle for line CB and draw it on the compass. With this information it can be seen that to calculate the AZ of line CD the interior angle C is subtracted from the Back Azimuth of line CB.

   118°21'15" BAZ - line CB
   - 98°12'40" interior angle C
   20°08'35" AZ - forward azimuth angle line CD

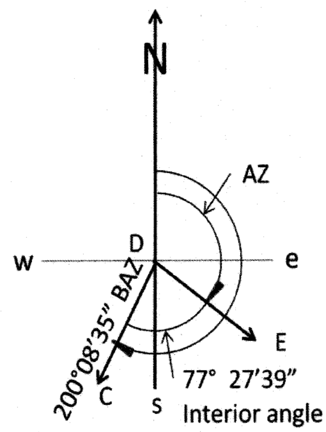


Note: the resulting angle in this case is also the Bearing of the line CD because it is in the Northeast quadrant.

1.7.47. Line DE.

a. Start the calculation for the forward azimuth of **line DE** by drawing a compass with the cardinal points. The back azimuth of line CD can be calculated by adding 180° to the AZ of line CD.

  20°08'35" AZ - line CD
<u>+ 180°00'00"</u>
 200°08'35" BAZ - line DC



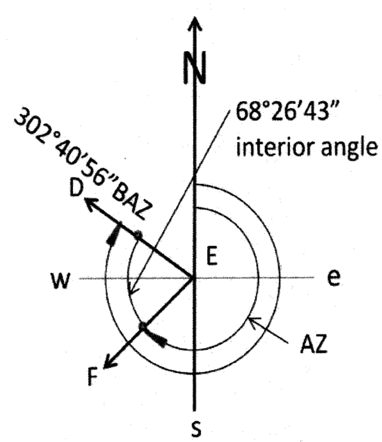
Azimuth Line DE

b. Then draw the back azimuth line of line DC. The forward direction of line DE can be estimated using the interior angle at D as shown which indicates the line to be in the Southeast quadrant.

c. The next step is to identify the azimuth angle for line DE and draw it on the compass. With this information it can be seen that to calculate the AZ of line DE the interior angle D is subtracted from the Back Azimuth of line DC.

200°08'35" BAZ - line DC
<u>- 77°27'39"</u> interior angle C
122°40'56" AZ - forward azimuth angle line DE

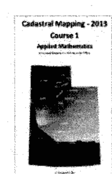
1.7.48. Line EF.



Azimuth Line EF

a. Start the calculation for the forward azimuth of **line EF** by drawing a compass with the cardinal points. The back azimuth of line ED can be calculated by adding 180° to the AZ of line DE.

  122°40'56" AZ - line DE
<u>+ 180°00'00"</u>
 302°40'56" BAZ - line ED



b. Then draw the back azimuth line of line ED. The forward direction of line EF can be estimated using the interior angle at E as shown which indicates the line to be in the Southwest quadrant.

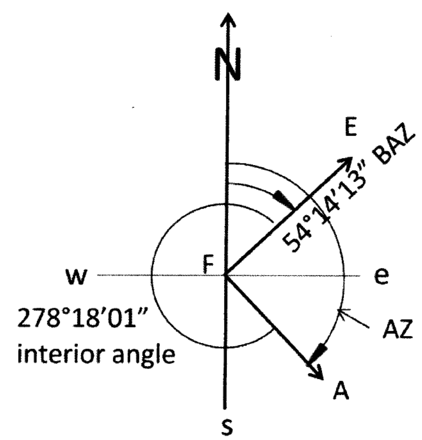
c. The next step is to identify the azimuth angle for line EF and draw it on the compass. With this information it can be seen that to calculate the AZ of line EF the interior angle E is subtracted from the Back Azimuth of line ED.

302°40'56" BAZ - line ED
<u>- 68°26'43"</u> interior angle E
234°14'13" AZ - forward azimuth angle line EF

1.7.49. Line FA.

a. Start the calculation for the forward azimuth of **line FA** by drawing a compass with the cardinal points. The back azimuth of line FE can be calculated by subtracting 180° from the AZ of line EF.

234°14'13" AZ - line EF
<u>- 180°00'00"</u>
54°14'13" BAZ - line FE



Azimuth Line FA

b. Then draw the back azimuth line of line FE. The forward direction of line FA can be estimated using the interior angle at F as shown which indicates the line to be in the Southeast quadrant.

c. The next step is to identify the azimuth angle for line FA and draw it on the compass. With this information it can be seen that to calculate the AZ of line FA there will need to be a two step process beginning with subtracting the BAZ of line FE from the interior angle F.

278°18'01" interior angle F
<u>- 54°14'13"</u> BAZ - line FE
224°03'48"

Next the result is subtracted from 360° which will give the AZ for line FA.

360°00'00"
<u>- 224°03'48"</u>
135°56'12" AZ - line FA

1.7.50. Again, this completes the calculations for the azimuths of each of the lines of the traverse/parcel. Which means that one additional step needs to be done to ensure that there was not a calculation error. This is to calculate the azimuth of line AB from point A.

1.7.51. To **check the calculations** begin the process the same as before.

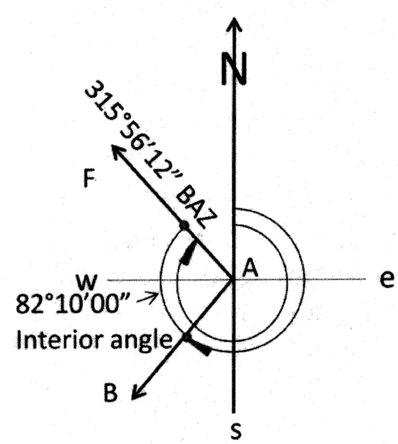
1.7.52. Line AB.

a. Start the calculation for the forward azimuth of **line AB** by drawing a compass with the cardinal points. The back azimuth of line AF can be calculated by adding 180° to the AZ of line FA.

135°56'12" AZ - line FA
<u>+ 180°00'00"</u>
315°56'12" BAZ - line FA

b. Then draw the back azimuth line of line AF. The forward direction of line AB can be estimated using the interior angle at A as shown which indicates the line to be in the Southwest quadrant.

c. The next step is to identify the azimuth angle for line AB and draw it on the compass. Calculating the AZ of line AB is made by subtracting the interior angle A from the BAZ of line AF.

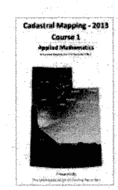


Azimuth Line AB to check

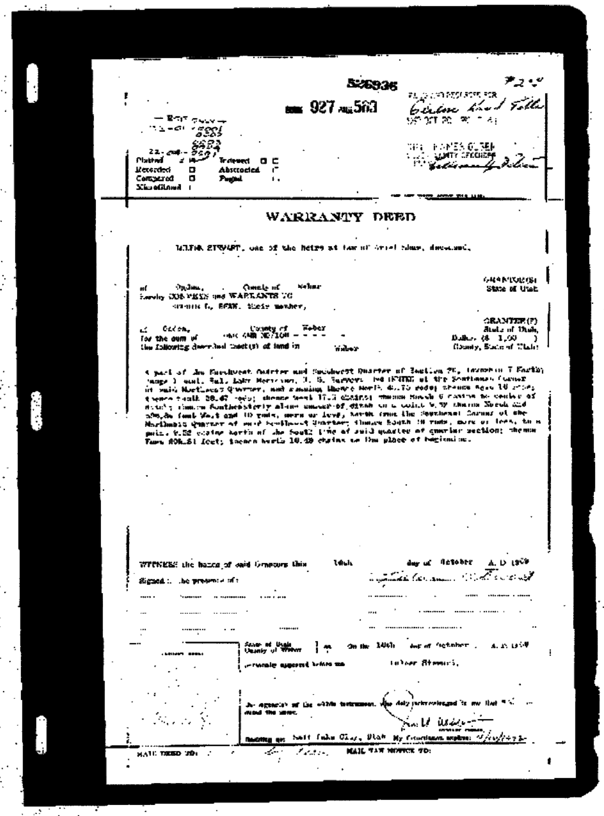
315°56'12" BAZ - line AF
<u>- 82°10'00"</u> interior angle A
233°46'12" AZ – line AB

233°46'12" AZ – line AB
<u>-180°00'00"</u>
53°46'12" BAZ - line AB

1.7.53. From this check is found that our calculations were correct.



1.7.54. **Reversed directions.** There are times when deeds require using the bearings in reverse direction to be able to draft them. This occurs most often when a deed description at some point in the courses simply uses a general direction and no distance information for the course and then continues with bearings and distances after. An example of how to resolve this type of description follows.


526936

927 563

Plotted
Recorded
Compared
Indexed
Abstracted
Paged

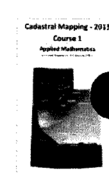
WARRANTY DEED

1.7.55. The description shown in the document at the right reads as follows:

1.7.56. "A part of the Northwest Quarter and Southwest Quarter of Section 20, Township 7 North, Range 1 East, Salt Lake Meridian, U. S. Survey: BEGINNING at the Southeast Corner of said Northwest Quarter, and running thence North 45.75 rods; thence West 10 rods; thence South 28.47 rods; thence West 17.5 chains; thence South 6 chains to center of ditch; **thence Southeasterly along center of ditch to a point 9.52 chains North and 696.84 feet West and 10 rods, more or less, North from the Southeast Corner of the Northeast**
B BEGIN **Quarter of said Southwest Quarter;** *thence South 10 rods, more or less, to a point 9.52 chains North of the South line of said quarter of quarter section; thence East 696.84 feet; thence North 10.48 chains* to the place of beginning."

1.7.57. As can be seen from a reading of the description there is a call which has no bearing or distance, "thence Southeasterly along center of ditch to a point 9.52 chains North and 696.84 feet West and 10 rods, more or less, North from the Southeast Corner of the Northeast Quarter of said Southwest Quarter;".

1.7.58. This description is actually written in a manner that would reverse the calls of the description which is the point of this discussion. To accomplish the task of mapping this parcel there are three approaches that could be made. One, using the calls of the 9.52 chains north etc. to establish the end of the call along the ditch (**this is the bold text in the description**), two, using the last calls of the description by using the back bearings (*this is the italicized text in the description), and* three, with aerial photography to help



identify the ditch. Lets illustrate each.

1.7.59. **First Method.** Using the calls of the 9.52 chains north etc. to establish the end of the call along the ditch.

1.7.60. Starting each method we will plot the description to the location of the beginning of the call along the ditch. The red lines are the courses up to the point of the ditch call.

1.7.61. When examining the description it can be seen that the scrivener intended to use the last calls in the reverse. This is evident from the bold text which is essentially a reversal of the final closing calls with the exception that the bold calls begin from the C-S 1/16th corner[2] instead of from the beginning location.

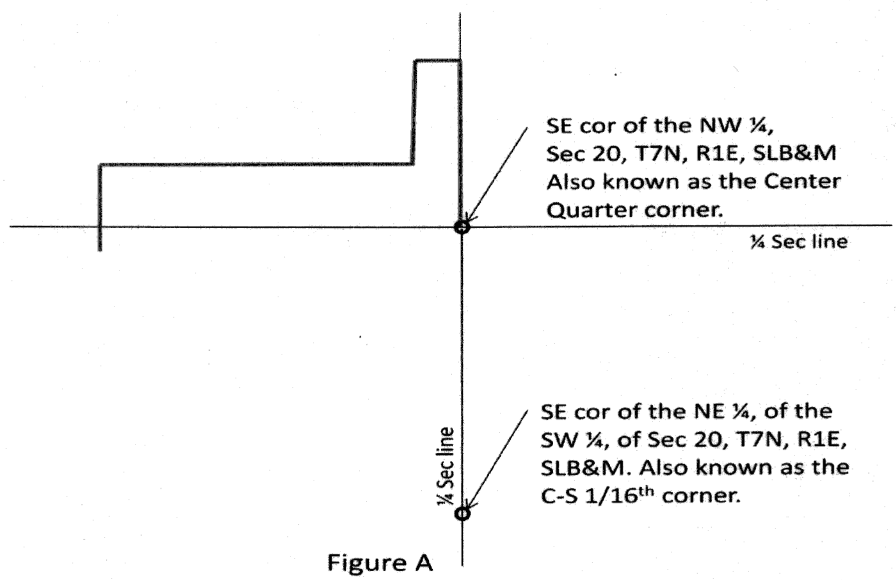
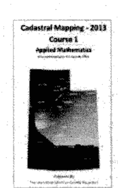


Figure A

1.7.62. If we then plot these calls from the C-S 1/16th corner the description now looks like Figure B. Examining Figure B it can be seen that the only calls of the description that is no longer plotted are the "Southwesterly along the center of ditch" and the last call which returns to the point of beginning.

2 . The abbreviation of "C-S 1/16th corner" is the way that the Bureau of Land Management makes reference to this corner of the Section and marks the physical monument for this corner. The abbreviation is read as the Center-South One Sixteenth Corner.



1.7.63. **Second Method.** With method two we would be assuming that the bold text was not included in the description. Plotting the description up to the point of Figure A we would

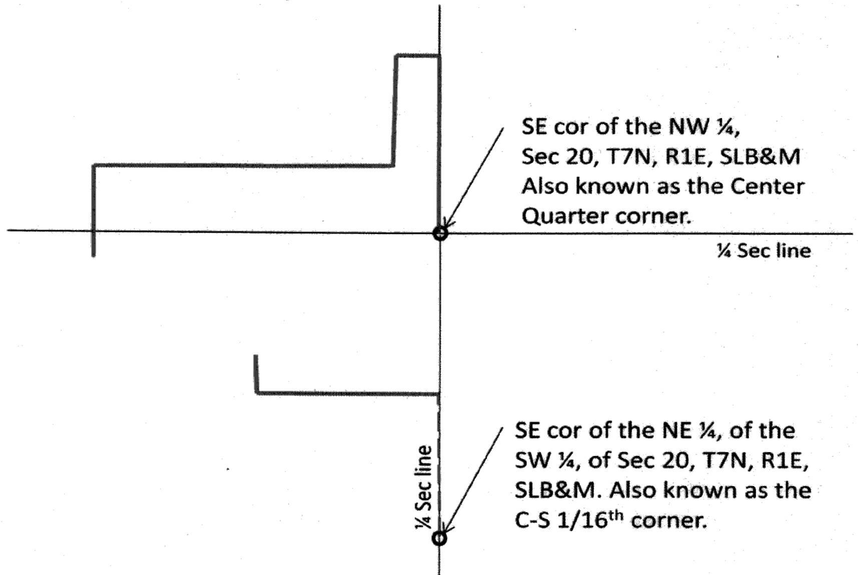


Figure B

then reverse the final calls of the description. However, instead of plotting the description from the C-S 1/16th corner the description calls in reverse would begin at the POB of the parcel (the C ¼ corner) and look like Figure C. The second method will result in the same configuration just using different tie points.

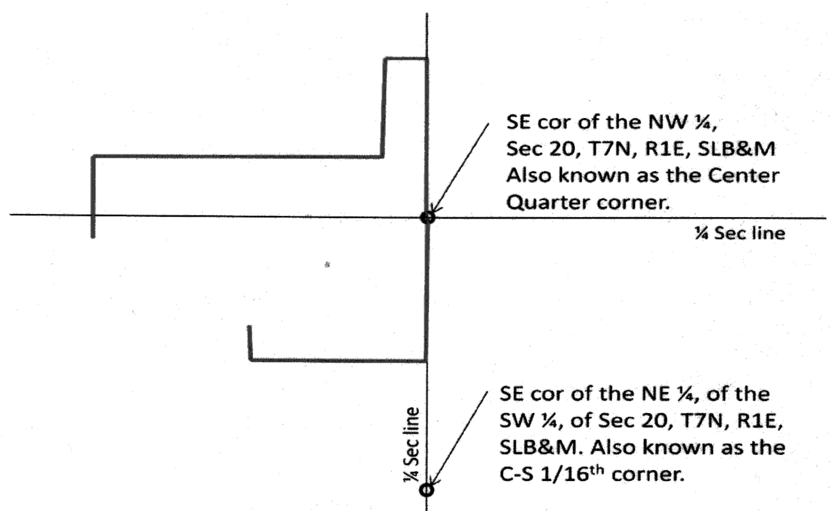


Figure C

1.7.64. The two methods may result in a line along the ditch that is not the same bearing or distance because the location of the C-S 1/16th corner may be closer or farther from the POB than the scrivener may have thought or known. The line that is used for the ditch is now assumed by the drafter to be a straight line between the two ending points. This is

because there is no information to tell the drafter otherwise in the written language of the description.

1.7.65. **Third Method**. Let's now examine what the use of Aerial photography would do to benefit the drafter in being able to correctly plat the description as it is written. The first

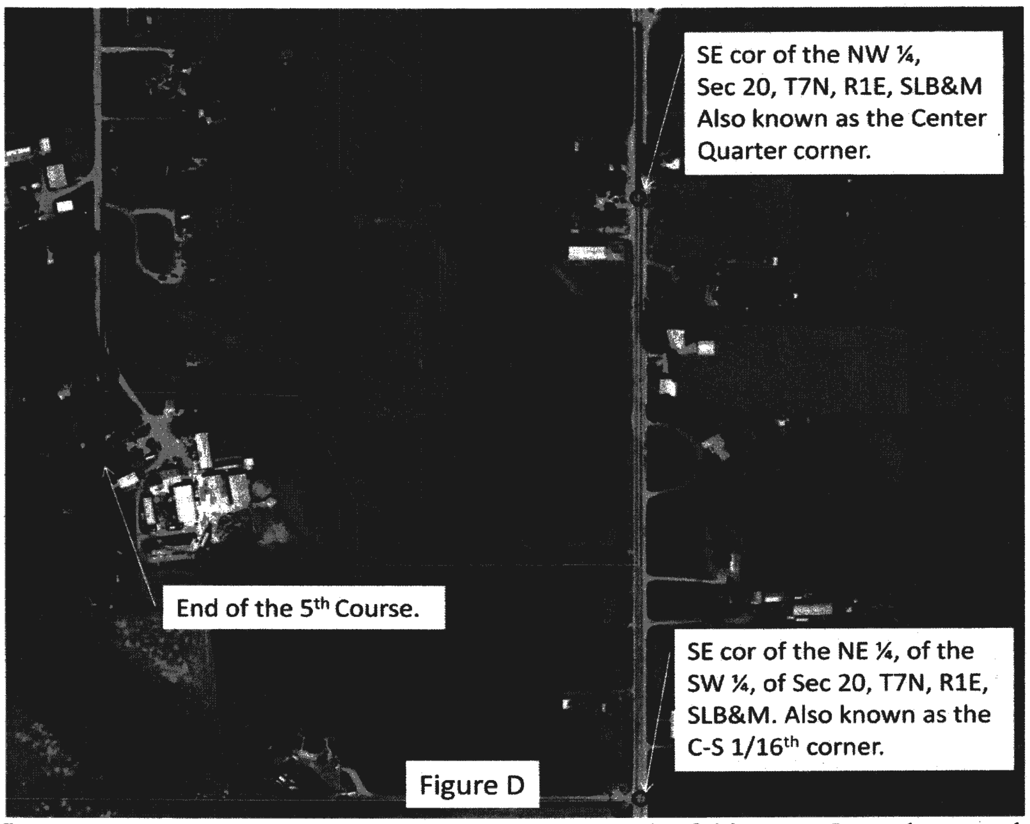
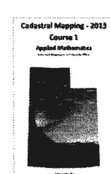


Figure D

five courses have been plotted on an aerial photograph of this area. It can be seen that they match fairly well with the lines of occupation (fences and roads).

1.7.66. The 5th course can be seen to extend beyond where the ditch would have been which shows that there is an error in the distance of this leg and that the distance is not 6 chains long as noted in the description. Without getting into the legalities of description interpretation, the description states, "thence South 6 chains to center of ditch;". The bounds of the description will stop the course at the ditch.

1.7.67. The next course of the description is the one that states, "thence Southeasterly along center of ditch …". This means that the property line is the ditch wherever it runs. From the aerial photo we can see that the ditch runs generally in an easterly direction then turns nearly 30 degrees southeast. This description then contains reverse calls stating that the property line ends in the ditch, "to a point 9.52 chains North and 696.84 feet West and 10



rods, more or less, North from the Southeast Corner of the Northeast Quarter of said Southwest Quarter;".

1.7.68. Figure E shows the description properly mapped using the deed description and aerial photography to help clarify the language of the description.

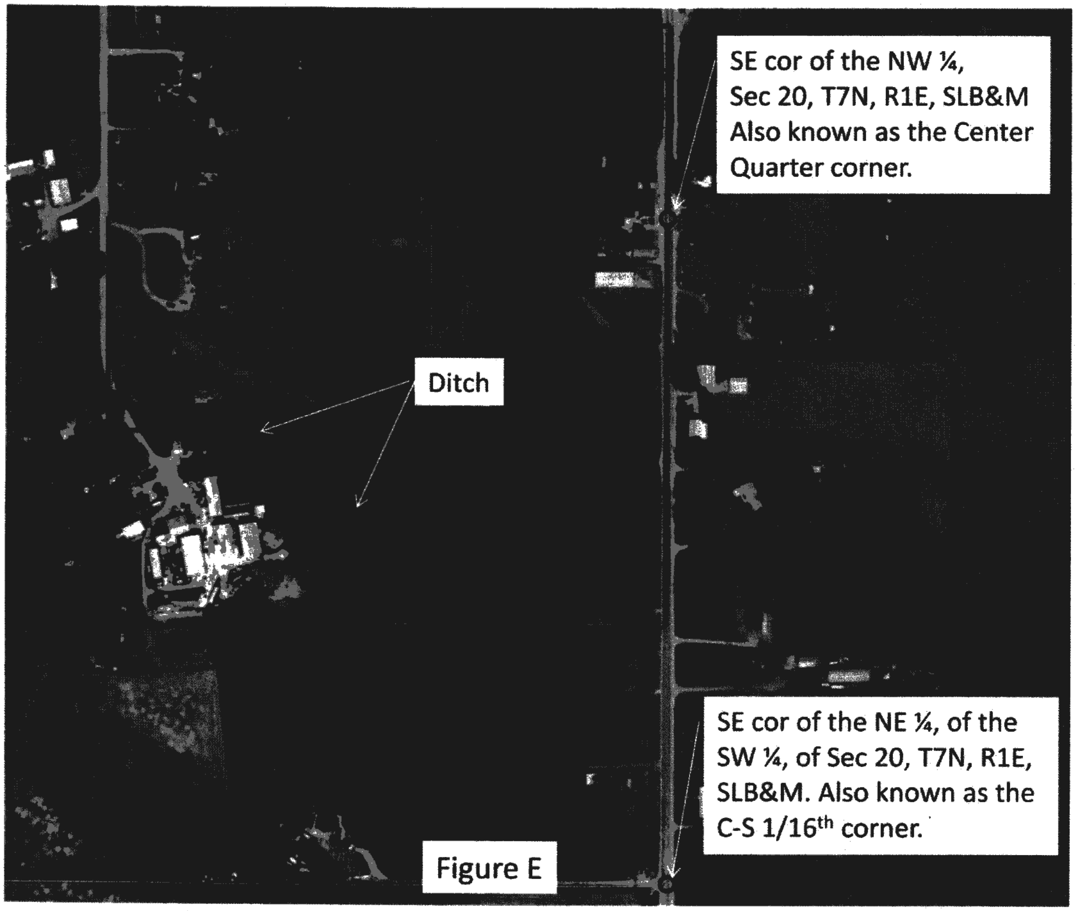
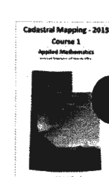


Figure E

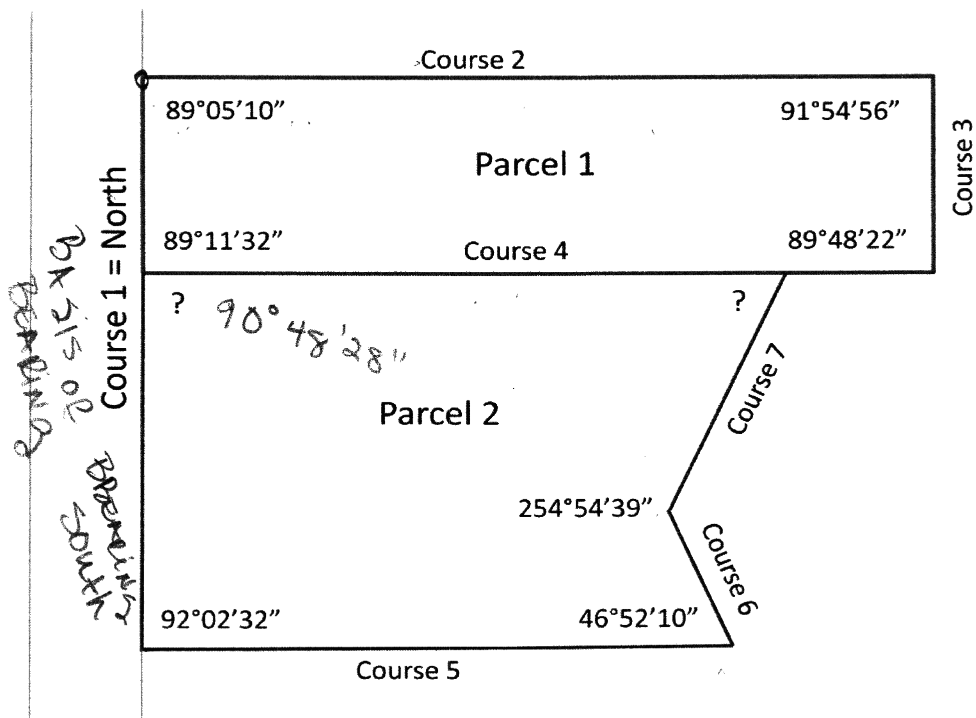
1.7.69. This is an example of how the use of reverse calls can help to resolve a description call that may be missing the bearing and distance. It also illustrates how the use of aerial photography can help in correctly platting descriptions.

A&RC
Aerial Photography



1.7.70. BEARING – AZIMUTH CALCULATION EXCERSIZES.

1.7.71. **Problem 1.** Calculate the Bearing of each line of the Parcel 1 and Parcel 2 with the given information.



Note : Each angle is at the interior angle of the parcel.

a. Course 1 Bearing = North

b. Course 2 Bearing = S 89° 05' 10" E

c. Course 3 Bearing = S01° 00' 06" E

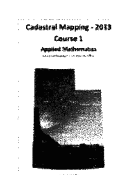
d. Course 4 Bearing = S 89° 11' 32 W

e. Course 5 Bearing = S87° 57' 28" E

f. Course 6 Bearing = N 41° 05' 18" W

g. Course 7 Bearing = N 33° 49' 21" E

1.7.72. **Problem 2.** Calculate the Azimuth of each line of Parcel 1 and Parcel 2 with the given information.



a. Course 1 Azimuth = North

b. Course 2 Azimuth =

c. Course 3 Azimuth =

d. Course 4 Azimuth =

e. Course 5 Azimuth =

f. Course 6 Azimuth =

g. Course 7 Azimuth =

1.7.73. **Problem 3.** Calculate the Back Bearing and Back Azimuth of each line of Parcel 1 and Parcel 2 with the given information.

a. Course 1 Back Bearing = North

Back Azimuth = North

b. Course 2 Back Bearing = N 89° 05' 10" W

Back Azimuth =

c. Course 3 Back Bearing = N 01° 00' 06" W

Back Azimuth =

d. Course 4 Back Bearing = N 89° 11' 32 E

Back Azimuth =

e. Course 5 Back Bearing = N 87° 57' 28" W

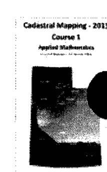
Back Azimuth =

f. Course 6 Back Bearing = S 41° 05' 18" E

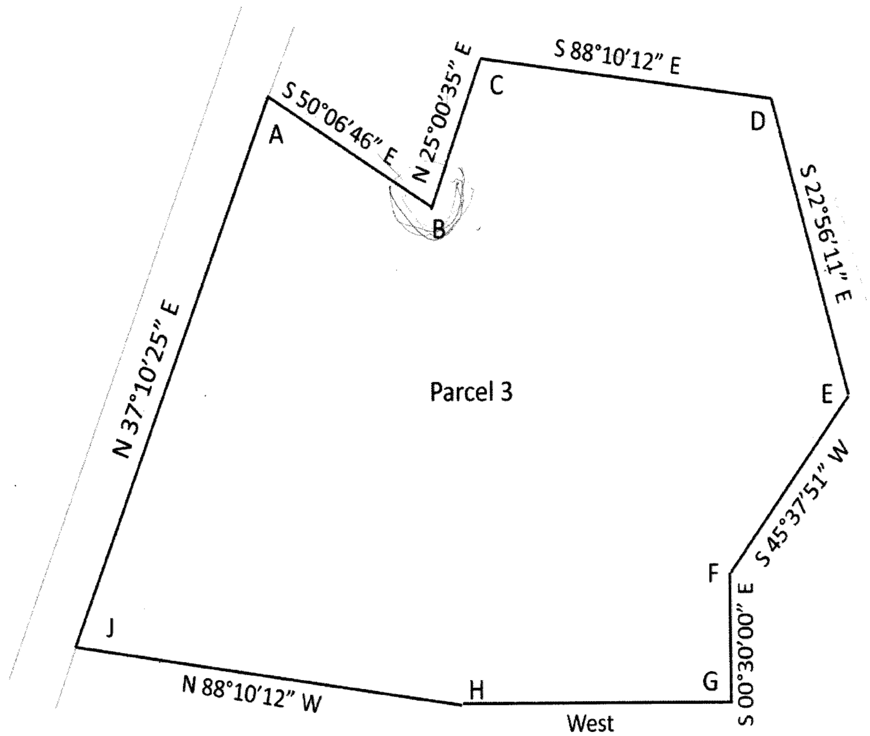
Back Azimuth =

g. Course 7 Back Bearing = S 33° 49' 24 W

Back Azimuth =



1.7.74. **Problem 4.** Using Parcel 3 below, calculate the interior angles for each angle point.



a. Angle A = 87° 17' 11"

b. Angle B = 284° 52' 39"

c. Angle C = 113° 10' 47"

d. Angle D = 111° 06' 23"

e. Angle E = 111° 25' 58"

f. Angle F = 226° 07' 51"

g. Angle G = 90° 30' 00"

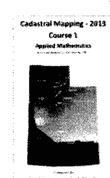
h. Angle H = 178° 10' 12"

i. Angle J = 54° 39' 23"

(n-2) 180° =

1,260

1.7.75.



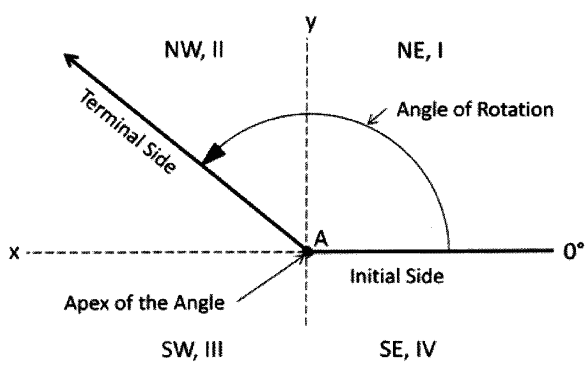
APPLIED MATHEMATICS:
Course 1.Chapter 8.

1.8 Trigonometry:

1.8.1. This subject is vital for the cadastral mapper to have a working knowledge of. Here again, we will only cover enough of the subject so that the cadastral mapper can effectively make the necessary calculations relating to the solution of geometric shapes as they relate to the mapping of land descriptions, parcels, roads, highways, railroads, subdivisions, easements, and other boundaries as may be required by the job or project at hand.

1.8.2. We will begin with the definition of some of the basic principles relating to the use of angles and distances in trigonometry.

1.8.3. The **ANGLE** has been defined previously and can be found in Appendix 5 - Definitions. From the Appendix you will learn that there are many types of angles which we will be using throughout this discussion. To begin we can define the individual parts of an Angle which are, The Apex, The Initial Side, The Terminal Side, and in some cases, the x axis and the y axis (when in Standard Position). An Angle is the rotational value (noted in this text as Hours, being the same as Degrees, Minutes and Seconds format) beginning on the Initial Side and proceeding counter-clockwise ending on The Terminal Side.

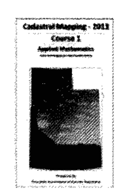


Definition of an Angle
In Mathematics

1.8.4. The counter-clockwise direction is the standard method of angular reckoning in mathematics. This is reverse from how an Azimuth as used in surveying is reckoned.

1.8.5. In the illustration what is known as The Standard Position of an Angle is shown. The Initial Side is coincident with the x axis but in mapping the Initial Side can be at any position with respect to the Axis of a coordinate system. When noted in Standard Position the direction of the Initial Side is running East-West (which is the direction of the x axis).

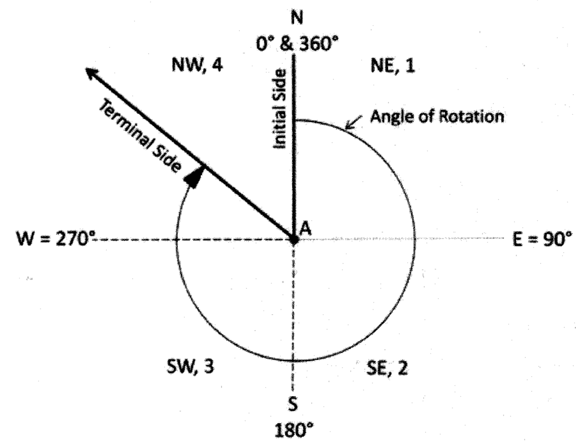
1.8.6. The x and y axes define the points of the compass where the x axis is east-west and the y axis is north-south. In the illustration the Apex of the Angle is located at the intersection of the x and y axis. With this configuration of the compass points, the four quadrants can be identified and labeled. Roman numerals (I, II, III, IV) are generally used to



identify the four quadrants in mathematics. In surveying Modern Arabic numerals (1, 2, 3, 4) are used to identify the four quadrants.

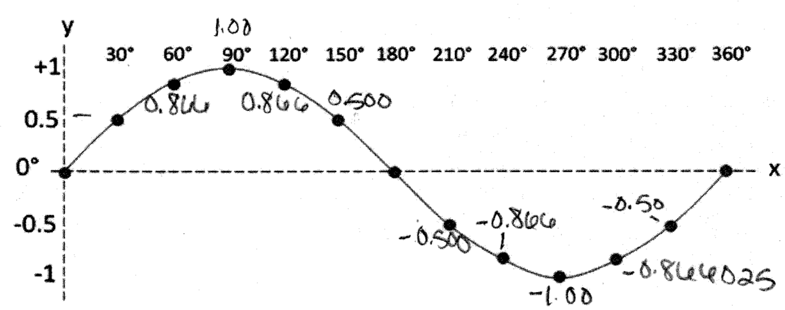
1.8.7. Another difference that can be seen from the illustration is that in mathematics the roman numerals begin in the Northeast quadrant with I and proceed in the counter-clockwise direction with II being the Northwest quadrant, III being the Southwest quadrant and IV being the Southeast quadrant. In surveying the Northeast quadrant is still the number 1 quadrant. The numbering of the quadrants proceed in a clockwise direction with 2 being the Southeast quadrant, 3 being the Southwest quadrant, and 4 being the Northwest quadrant as shown. When an Angle is said to be in the Northwest quadrant it is because the terminal side resides in that quadrant.

1.8.8. This illustration shows the definition of an Azimuth. The Azimuth will be defined in greater detail later but is shown here to point out the difference between how mathematics and surveying use angles. It should be noted that when you are using CAD systems and have not set the Units to use surveyor units the CAD system considers angles to be reckoned in the same manner as in mathematics.

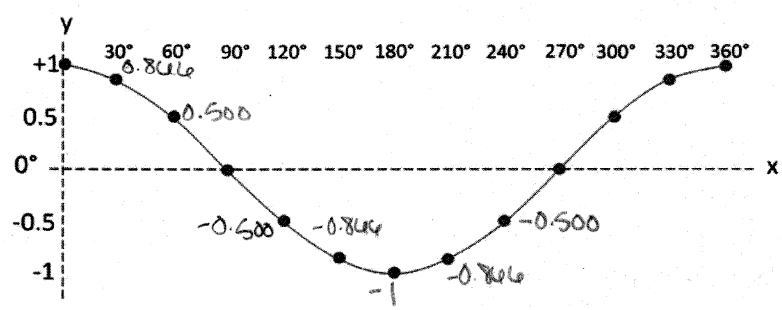


Definition of an Azimuth
In Surveying

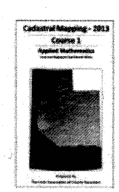
1.8.9. Using **TRIGONOMETRIC FUNCTIONS** (in the context of this work) is the means of solving missing parts of a triangle. There are basically 5 equations that we will be studying. These 5 equations should give you the tools necessary to solve virtually all triangle problems that may be required to be solved. Three of the 5 equations are for use in solving the Right Triangle. The other 2



The Sine wave plotted on 30° increments.



The Cosine wave plotted on 30° increments.



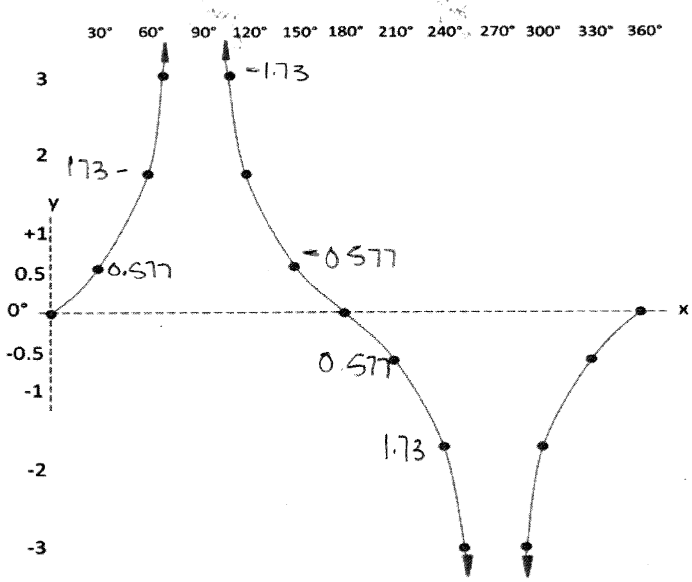
equations are used for solving Oblique Triangles.

1.8.10. **Sine, Cosine, and Tangent Functions.**A basic definition of the **Functions** Sine, Cosine, and Tangent needs to be developed. Generally, the Functions are abbreviated as Sine = sin, Cosine = cos, and Tangent = tan. A Function is an expression of a ratio of one side of a triangle divided by another side of a triangle. The ratio can be converted into an angular value using an Inverse Function (sometimes known as the reciprocal function).

1.8.11. The Function of an angle, as in the expression sin θ, is not a multiplication problem. The phrase "sin θ" means that the ratio of an angular value (θ) was derived from a fraction where the y axis was divided by the terminal side (or r) of a triangle. It may be easier to illustrate this principle with some examples of how to solve missing angles or distances of a triangle.

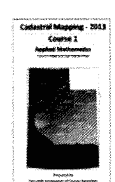
1.8.12. When performing a sin or cos function the resulting ratio must be less than 1 (positive one) or greater than -1 (negative one). The tan function will result in values that exceed these limits. The sin and cos functions when evaluated with respect to one rotation of a circle (360°) will create a wave and can be plotted on a graph. Examining the graphs of the sin and cos waves it can be seen that each are itentical to one another with the exception that they are offset by 90°. Meaning that the Sine of 0° = 0 and the Cosine of 0° = 1, etc.

1.8.13. The Tangent function does not create a wave. The values of 30° increments can also be plotted for the tangent functions and the graph will look similar to the one shown. What is not fully graphed is that for a 90° angle the result is ∞ (infinity). Therefore, there is NO TANGENT VALUE for a 90° angle and the tangent for an angle of 89°59'59" = 141.783789873, so the last second of arc contains all the values between 141.78 and ∞.



The Tangent function plotted on 30° increments.

1.8.14. **Precision of Trigonometric Functions**. Solving problems using trigonometric functions should be used with an understanding of the precision of the functions. Normally, when we think of mathematics we think of precise results and while this is true (depending on how many digits to the right of the decimal point are used) the precision that we see in our calculations will be dependent on how rounding is used.



1.8.15. If we perform the following analysis we can get a better understanding of the precision of the Sine, Cosine, and Tangent functions and how that relates to the number of digits used in an equation. If we evaluate the sine of several angles we find that for the 5th decimal place to experience a change in value it is necessary to reach an angle of at least 3 seconds (03") beginning at zero degrees.

sin 0° = 0.00 000 000 0 → infinite zeros
sin 0°00'02" = 0.00 000 969 6
sin 0°00'03" = 0.00 001 454 4

1.8.16. To effect a change in the 4th decimal place it is necessary to reach an angle of at least 21 seconds (21") again beginning at zero degrees.

sin 0°00'20" = 0.00 009 696 3
sin 0°00'21" = 0.00 010 181 1

1.8.17. We see from this analysis that angles between zero (0°) and 20 seconds (20") the 4th decimal place does not change. What this means is that if you use values of the sine rounded to 4 decimal places, results for angles between 0° and 20" there is a potential that a rounding error may occur. For instance, if the sine used is 0.0000 then the resulting angle is,

$\sin^{-1}$ 0.0000 = 0°

1.8.18. This would be the result for the first 20 seconds (20") of angle and could produce a potential angular error of 20". If we use 5 decimal places in the value of a sine from the above evaluation we see that a potential error of 2" would result. To further evaluate the decimal precision needed to maintain precision to the one second (1") of arc consider the following.

sin 0°00'01" = 0.00 000 484 8

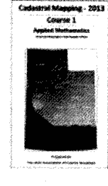
truncating to the 6th place,
$\sin^{-1}$ 0.00 000 4 = 0°00'00.8"

using the 7th place to round to the 6th place,
$\sin^{-1}$ 0.00 000 5 = 0°00'01.0"

Arc Sin
ASIN ($\sin^{-1}$)
Arc Cos
ACOS ($\cos^{-1}$)
ARC TAN
ATAN ($\tan^{-1}$)

1.8.19. What this illustrates is that when working with the Sine of angular values and to maintain precision to the nearest one second (01") of arc a minimum of 6 decimal places is required. If this is carried further and include the One Digit Rule (discussed in the section on Significant Digits) to prevent the rounding error, then it is required that a minimum of 7 decimal places is used to maintain the same precision. Personally, I go back to my introduction to the subject on Significant Digits and restate, "Don't Round", use all the digits that you have

* Truncate - to shorten a number by dropping one or more digits after decimal point



available.

1.8.20. The same evaluation will now be made using the Cosine function for the same angles previously evaluated.

cos 0° = 1.00 000 000 0
cos 0°00'06" = 1.00 000 000 0
cos 0°00'07" = 0.99 999 999 9

1.8.21. Indicating that there is no change in the cos of an angle for the first 6" to at least the 9th decimal place. Furthermore, before the 5th decimal place is affected it is necessary to reach an angle of 16 minutes (16').

cos 0°15'00" = 0.99 999 048 1
cos 0°16'00" = 0.99 998 916 9

1.8.22. If we continue the evaluation it is found that before the 4th decimal place is affected it is necessary to reach an angle of 47 minutes (47').
cos 0°46'00" = 0.99 991 047 8
cos 0°47'00" = 0.99 979 269 8

Indicating that for angles between zero (0°) and 47 minutes (47') there is a potential that a rounding error may occur.

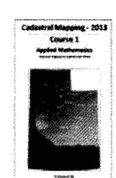
1.8.23. Therefore, when working with the cosine of an angle if the cosine is truncated to 4 decimal places the value returned will be zero (0°) for all angles between zero (0°) and six seconds (06") with the possibility of introducing a 6 second error in the calculations.

1.8.24. **Right Triangle Equations.**The 3 equations which will be studied here and used in solving the Right Triangle are:

$$\sin \theta = \frac{y}{r} \qquad \textbf{Equation 1.8.1}$$

$$\cos \theta = \frac{x}{r} \qquad \textbf{Equation 1.8.2}$$

$$\tan \theta = \frac{y}{x} \qquad \textbf{Equation 1.8.3}$$



$$\sec\theta = \frac{r}{x}$$ **Equation 1.8.4**

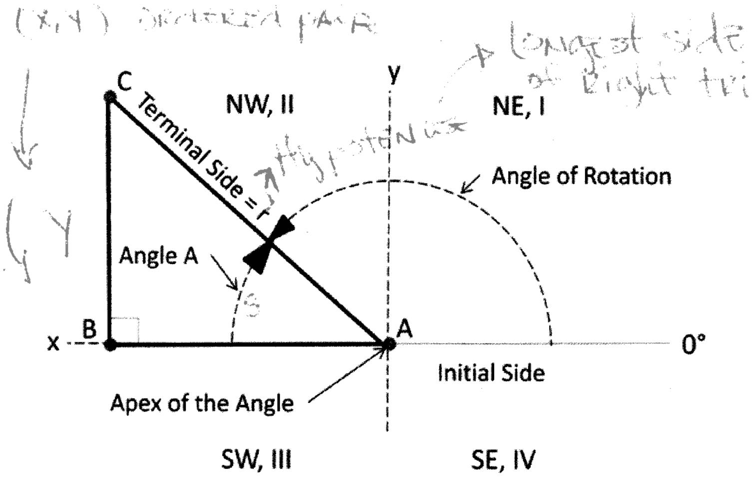
and

$$\sec\theta = \frac{1}{\cos\theta}$$ **Equation 1.8.5**

$$\text{vers}\,\theta = 1 - \cos\theta$$ **Equation 1.8.6**

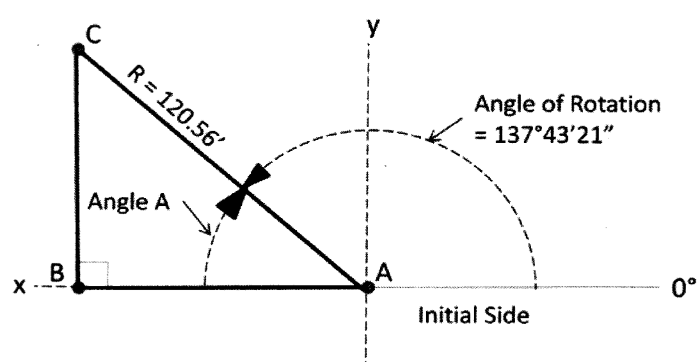
$$\text{exsec}\,\theta = \sec\theta - 1$$ **Equation 1.8.7**

1.8.25. These equations are labeled in accordance with an Angle in Standard Position as noted in the illustration "Definition of (x, y) ordered pair an Angle in Mathematics". To further explain the variables of x, y, and r we need to add some information to the illustration"Definition of a Triangle". In this example and the Equations noted above, θ will be used to represent the angle at point A being angle CAB.



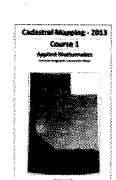
Definition of a Triangle

1.8.26. Drawing a line (line BC) perpendicular to the x axis (the line BC is also perpendicular to line AB which is coincident with the x axis). By doing so a triangle is created in which line AB is also on the x axis, line BC is parallel to the y axis, and line AC is the Terminal Side which is also designated as side r and known as the Hypotenuse of the triangle. This creates the right triangle known as Triangle ABCA. Every Terminal Side having any point C creates a right triangle which is the same as defined.



Solve Triangle ABCA

1.8.27. Also shown is Angle A which is an interior angle of the triangle and in this case it is the



supplementary angle to the "Angle of Rotation". To solve any right triangle one distance and one angle (in addition to the right angle) is required. Let's say that the Angle of Rotation is = 137°43'21" and that side r = 120.56 feet. Given this information the solution for each angle and each side of the triangle is accomplished as follows:

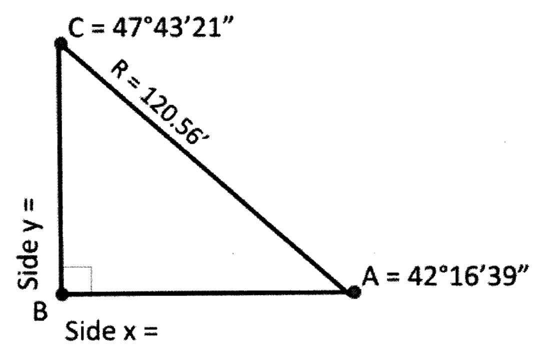
1.8.28. Solving Right Triangles.

1.8.29. **Angle A.** The first step in this process is to obtain Angle A by subtracting the Angle of Rotation from a Straight Angle (180°),

180° - 137°43'21" = 42°16'39"

Angle A = 42°16'39"

1.8.30. **Angle C.** Because we know that the sum of the interior angles of a triangle is 180° precisely, knowing **angle B** is precisely 90° and angle A equals 42°16'39", we can calculate angle C by adding angle A and angle B then subtracting the sum from 180°;



Example 1.03.01

90° + 42°16'39" = 132°16'39"

and,

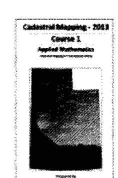
180° - 132°16'39" = 47°43'21"

Angle C = 47°43'21"

Therefore, all 3 of the interior angles of the triangle have been calculated.

1.8.31. **Side x.** Because we know all of the interior angles of the triangle to solve for variable x either Equation 1.03.01 or Equation 1.03.02 can be used. To use Equation 1.03.01 angle C must be used for angle θ, however, to use Equation 1.03.02 angle A must be used for angle θ.Solve for variable x (being side AB) using Equation 1.03.02 and angle A,

$$\cos\theta = \frac{x}{r}$$ Equation 1.03.02



1.8.32. Angle A equals 42°16'39" and is the value for θ in the equation 1.03.02 and distance r can also be filled in.

$$\cos 42°16'39" = \frac{x}{120.56}$$

Using algebraic rearrangement and solving for the variable x the equation becomes

$$\cos 42°16'39" * 120.56 = x$$

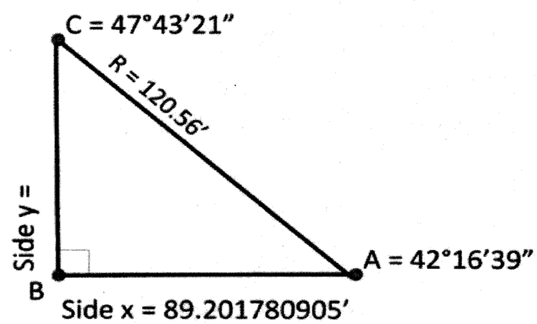
or,

$$x = \cos 42°16'39" * 120.56$$

1.8.33. To finish solving this equation it must be kept in mind that the 'cos' is not a multiplication, instead it is a function of the angle and must be evaluated before the multiplication of the distance 120.56' is performed. Therefore, the evaluation of the equation proceeds as follows;

$x = \cos 42.277500000° * 120.56$, converting the angle from HMS format to HDD format.

$x = 0.739895329 * 120.56$, applying the cosfunction to the decimal degree of Angle A.

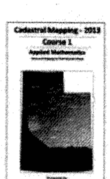


Example 1.03.02

$x = 89.201780905$, performing the multiplication. The result states that variable x is equal to approximately 89.20 feet.

1.8.34. All the angles and 2 of the 3 sides of the triangle are now calculated. The only remaining part of the triangle to calculate is side y.

1.8.35. **Side y**. Again, because we know all of the interior angles of the triangle to solve for variable y either Equation 1.03.01 or Equation 1.03.02 can be used. To use Equation 1.03.01 angle A must be used for angle θ, however, to use Equation 1.03.02 angle C must be used for angle θ. Solve for variable y (being side BC) using Equation 1.03.01 and angle A



$$\sin\theta = \frac{y}{r}$$ Equation 1.03.01

1.8.36. Angle A equals 42°16'39" and is the value for θ in the equation 1.03.01 and distance r can also be filled in.

$$\sin 42°16'39" = \frac{y}{120.56}$$

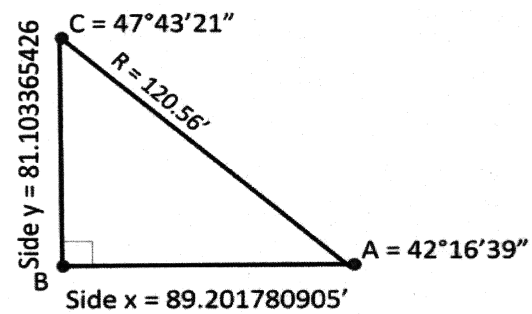
Using algebraic rearrangement and solving for the variable x the equation becomes

$$\sin 42°16'39" * 120.56 = y$$

or,

$$y = \sin 42°16'39" * 120.56$$

1.8.37. To finish solving this equation it must be kept in mind that the 'sin' is not a multiplication, instead it is a function of the angle and must be evaluated before the multiplication of the distance 120.56' is performed. Therefore, the evaluation of the equation proceeds as follows;



Example 1.03.03

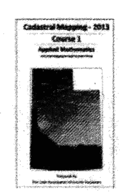
y = sin 42.277500000° * 120.56, converting the angle from HMS format to HDD format.

y = 0.672722009 * 120.56, applying the sin function to the decimal degree of Angle A.

y = 81.103365426, performing the multiplication. The result states that variable y is equal to approximately 81.10 feet.

1.8.38. The triangle ABCA is solved.

1.8.39. **Use of Functions.**It should be noted that in the hierarchy of mathematical operations we know that multiplication and division precede addition and subtraction. Also, the use of trigonometric functions are to be evaluated before multiplication or division. If it makes



visualizing how to evaluate an expression with a function easier you can think of the function as being bracketed as in the following example:

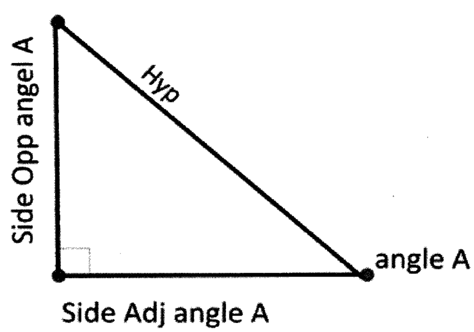
$$y = (\sin 42^\circ 16' 39'')\ (120.56)$$

Note: This is the same expression that was used above but by using the brackets it clearly shows that the function is to be evaluated before the multiplication.

1.8.40. **The Terms Opposite, Adjacent, and Hypotenuse.**The example of solving a right triangle to this point has used a triangle that has two sides coincident with the x and y axes of a coordinate system. The next example will examine the solution of a triangle that does not have sides that are coincident with either of the x or y axes. Using the following labeling and equations will make it easier to solve a larger variety of triangles that the limitation of using the variables of x, y and r to define the sides of the triangle. To do so we need to define the following terms, **Opp, Adj,** and**Hyp.**

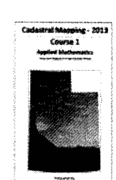
1.8.41. The term **Opp** is used to identify the side of the triangle which is opposite from any angle which is not 90° and is shown in Example 1.8.45-a as being the side opposite from angle A.

1.8.42. The term **Adj** is used to identify the side of a triangle which is opposite from any angle with is not 90° and is shown in the same example as the side adjacent to angle A. It can also be seen that angle A has two sides that could be seen as being adjacent to the angle, however, in respect to a right triangle, one of the sides will always be termed the hypotenuse.

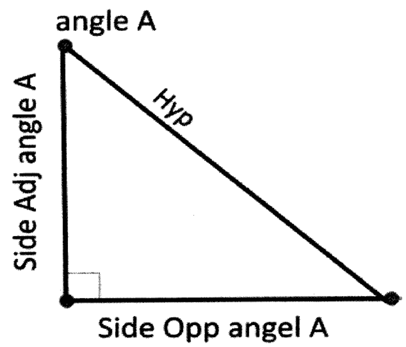


Example 1.03.04

1.8.43. The term **Hyp** is used to identify the side of a triangle which is opposite the 90° angle of a right triangle. Another way to identify the hypotenuse is that it is the side adjacent to any angle in the triangle that is not 90° with the characteristic of being the longest side of any triangle whether it is a right triangle or an oblique triangle. Example 1.8.45-a shows the relationship of all the sides as they relate to angle A.



1.8.44. Example 1.8.45-b shows that we have called a different angle as angle A but the **Opp** (opposite) side is still opposite the angle no matter what we call the angle. The **Adj** (adjacent) side is still adjacent to the angle and the **Hyp** (hypotenuse) side is still the side opposite the 90° angle of a right triangle or the longest side of any triangle.



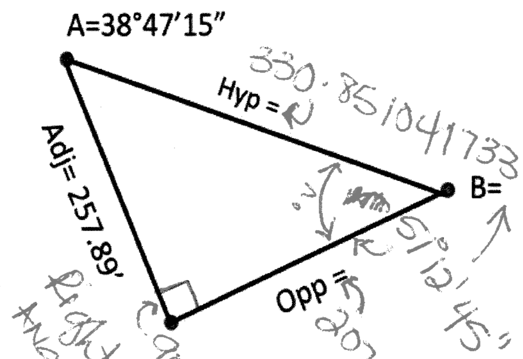
Example 1.03.05

1.8.45. Right Triangle Equations. These definitions lend themselves to developing the following equations for solving right triangles. The variable θ represents any of the angles that is not 90°. MEMORIZE

$$\sin\theta = \frac{\text{Opp}}{\text{Hyp}} \qquad \textbf{Equation 1.8.45-a}$$

$$\cos\theta = \frac{\text{Adj}}{\text{Hyp}} \qquad \textbf{Equation 1.8.45-b}$$

$$\tan\theta = \frac{\text{Opp}}{\text{Adj}} \qquad \textbf{Equation 1.8.45-c}$$

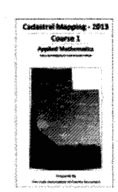


Problem

1.8.46. The sides of the triangle shown in the Problem have been labeled with respect to angle A. Remember to solve a right triangle, only 2 of the 6 parts of the triangle are required to be know and one of the 2 must be a side to give scale to the triangle.

1.8.47. Problem: Solve the triangle shown in the problem using Equation 1.8.45-a, Equation 1.8.45-b or Equation 1.8.45-c.

1.8.48. Angle B is solved simply by adding 90° to angle and subtracting the result from 180°, or in the case of a right triangle angle A can be subtracted from 90° and the result will be angle B.



128°47'15"

$90° + 38°47'15" = \cancel{51°12'45"}$

180° - 128°47'15" = 51°12'45"

Angle B = 51°12'45"

1.8.49. A Since angle A and the Adj side are known, by examination it is found that using Equation 1.8.45-b will solve for the Hyp side of the triangle. Fill in the known information in the equation and evaluate as follows;

$$\cos 38°47'15" = \frac{257.89}{Hyp}$$

rearrange and solve for Hyp;

$$Hyp = \frac{257.89}{\cos 38°47'15"}$$

begin evaluating the fraction by converting HMS to HDD;

$$Hyp = \frac{257.89}{\cos 38.7875°}$$

apply the cos function to the angle in decimal degrees;

$$Hyp = \frac{257.89}{0.779474650}$$

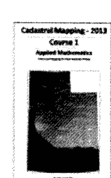
and make the division;

Hyp = 330.851041733'

Distance

1.8.50. With the angles known along with the Adj and Hyp sides of the triangle the Opp side is all that is left to solve for. There are several ways to accomplish this solution, the options are;

1.8.51. Using Equation 1.8.45-a with angle A and filling in the known information;



$$\sin 38°47'15'' = \frac{\text{Opp}}{330.851041733}$$

rearrange and solve for Opp;

$$\text{Opp} = \sin 38°47'15'' \,(330.851041733)$$

convert HMS to HDD, evaluate the sin function and performing the multiplication the solution is;

$$\text{Opp} = 207.256265806'$$

1.8.52. Using Equation 1.8.45-c with angle A and filling the known information;

$$\tan 38°47'15'' = \frac{\text{Opp}}{257.89}$$

rearrange and solve for Opp;

$$\text{Opp} = \tan 38°47'15'' \,(257.89)$$

convert HMS to HDD, evaluate the sin function and performing the multiplication the solution is;

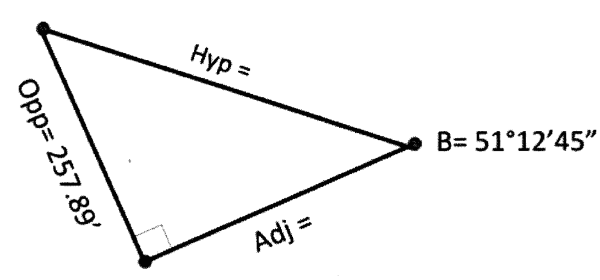
$$\text{Opp} = 207.256265807'$$

Notice the difference in the result in the 9th place. This is due to rounding.

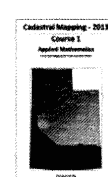
1.8.53. Since angle B has been calculated we could identify the sides as Opp and Adj with respect to angle B and use the appropriate equation to solve for what would be the Adj side to angle B.

1.8.54. If sufficient decimal accuracy is used in the calculation procedure all the values should be able to be checked by calculating the sides and angles.

1.8.55. **Problem:** Using the information shown in Problem rearranged calculate the Hyp and Adj of the triangle. Choosing to calculate the Hyp first, by inspection it can be seen that Equation 1.8.45-a can be used to obtain the value for Hyp. Fill in the



Problem rearranged



information in the equation and evaluate as follows:

$$\sin 51°12'45'' = \frac{257.89}{\text{Hyp}}$$

rearrange and solve for Hyp;

$$\text{Hyp} = \frac{257.89}{\sin 51°12'45''}$$

convert HMS to HDD, evaluate the sin function and performing the division the solution is;

$$\text{Hyp} = 330.851041733'$$

This value should check from the calculations previously done for the Hyp.

1.8.56. Solving for the Adj side by inspection it can be seen that Equation 1.8.45-c can be used to obtain the value for Adj. Fill in the information in the equation and evaluate as follows:

$$\tan 51°12'45'' = \frac{257.89}{\text{Adj}}$$

rearrange and solve for Adj;

$$\text{Adj} = \frac{257.89}{\tan 51°12'45''}$$

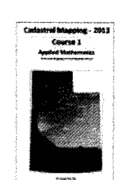
convert HMS to HDD, evaluate the sin function and performing the division the solution is;

$$\text{Adj} = 207.256265806'$$

This value should check from the calculations previously done for the Adj.

1.8.57. The second type of triangle to be examined is the Oblique Triangle and is the subject of the next section.

1.8.58. **Latitudes and Departures**. This subject will be expanded on during the



discussion of calculating rectangular coordinates. Rectangular coordinates are also used in the calculation of Area. So a brief discussion here will be valuable to give a basic understanding of what Latitudes and Departures are so you may become familiar with the principle.

1.8.59. Each line of a parcel will have a coordinate (x, y value or Northing and Easting) to define the line. The coordinate will be associated with some coordinate datum. Datum's will be defined in more detail in Course 2.

1.8.60. To demonstrate coordinates and the calculation of them for a line we will begin by examining a line that has coordinates of 0,0 at one end with a bearing and distance to define the other end. The task is to calculate the x, y value (and the Northing and Easting) of the other end. To calculate the coordinates a right triangle is created and solved for the x and y components.

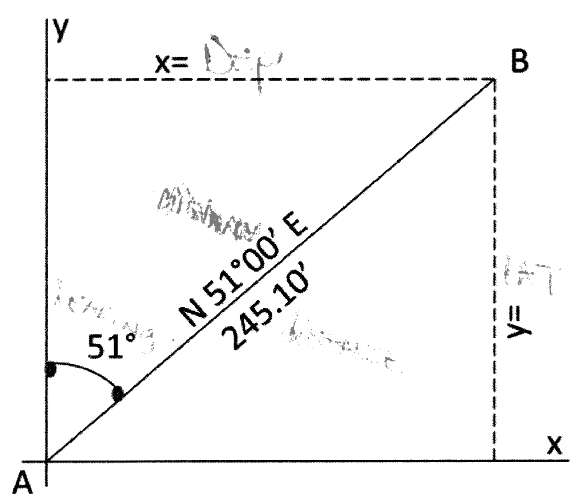
1.8.61. In the example we have a line that is a bearing of N 51° 00' E 245.10 feet long. We are going to assign Point A the x, y coordinates of x=0, y=0.

1.8.62. The equations for calculating coordinates are:

Equ. 1. y = Latitude = Cos (bearing angle) * (distance)

Equ. 2. x = Departure = Sin (bearing angle) * (distance)

1.8.63. With the information given the calcualtion for the Lat. and Dep. are calculated by filling in the bearing and distance each equation.



Latitude and Departure

1.8.64. Latitude is calculated as follows:

Lat = Cos 51° (245.10)

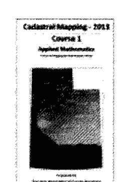
Lat = 0.62932039105 (245.10)

Lat = 154.246427846

1.8.65. Departure is calculated as follows:

Dep = Sin 51° (245.10)

Dep = 0.77715496146 (245.10)



Dep = 190.478475153

1.8.66. Looking at coordinates as ordered pairs (x , y) and the coordinates of Point A of (0 , 0) the coordinates of Point B are (190.478475153 , 154.246427846). Coordinates are easily transferred from one end of a line to the other by simple addition once the Latitude and Departure are known. For instance:

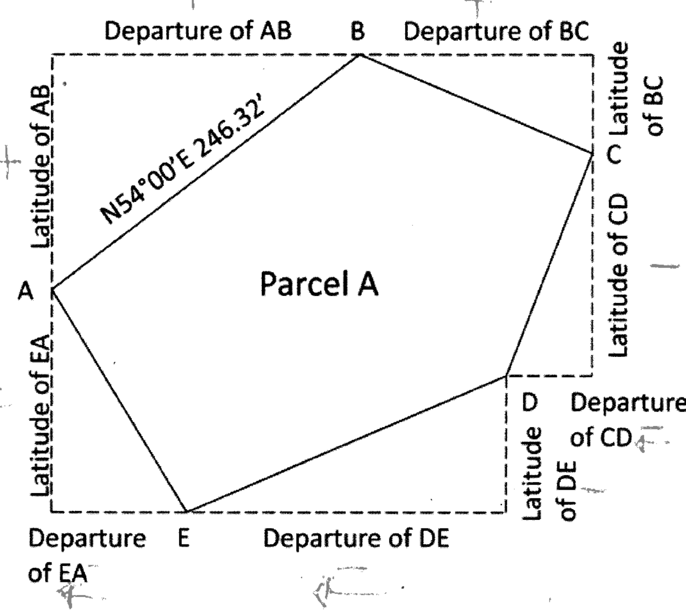
1.8.67. Assigning Point A coordinates of (1000 , 1000) the coordinates of Point B are (1190.478475153 , 1154.246427846).

1.8.68. Or if Point A has coordinates of (1258.46759 , 2357.14685) the coordinates of Point B are

x = 1258.46759 + 190.478475153 = 1448.94606515 and

y = 2357.14685 + 154.246427846 = 2511.39327785, resulting in an ordered pair of (1448.9406515 , 2511.39327785).

1.8.69. Every line of a traverse/parcel has a related Latitude and Departure. Some latitudes and departures are positive and some are negative. The bearings and labels of Parcel A are shown to run clockwise, therefore, examining Parcel A the Positive Latitudes are; line AB, and line EA. Negative Latitudes are; line BC, line CD, and line DE. Also, the positive Departures are; line AB, and line BC. Negative Departures are; line CD, line DE, and line EA.

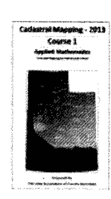


1.8.70. If the traverse/parcel closes mathematically then the sum of the North latitudes equals the sum of the South latitudes. It stands to reason that the sum of the East departures equals the sum of the West departures.

1.8.71. This subject will be more fully developed in Course 2 in calculating areas of parcels.

Northings = y

Eastings = x

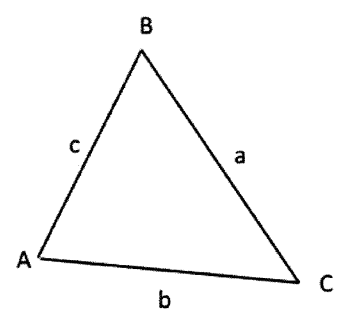


Acute Angle = All angles less than 90°

1.8.72. **OBLIQUE TRIANGLES** are triangles that no single angle equals 90°. An oblique triangle can be solved as long as three (3) parts of the equation are known with one of the 3 being a side to provide scale. While an oblique triangle can be solved by forming two right triangles within the oblique triangle it is easier to use two basic trigonometric laws to solve them. Later, when the area of a triangle is discussed the method of creating two right triangles will be examined.

1.8.73. **Acute and Obtuse Triangles.**The equations used to solve oblique triangles are known as the Law of Sines and the Law of Cosines. It is the use of these laws that will be examined. Oblique triangles have two types, the **Acute** triangle and **Obtuse** triangle (they are defined in Appendix 5 under Triangle). Both types will be discussed beginning with the Acute triangle.

1.8.74. From the discussion relating to right triangles naming conventions for the triangle were used to help identify the equations to be used in the solution. Another naming convention that is used to identify the angles and sides of a triangle is shown in the example to the right. This naming convention is used more often with oblique triangles but it can be just as effective with right triangles.



Acute Triangle

1.8.75. The remainder of the discussion for oblique triangles will use the naming convention shown in the example.As can be seen capitol letters are used to identify the angles of the triangle and small letters are used to identify the sides.

Inverted 1.8.76. **The Law of Sines.** The Law of Sines is defined by the following relationship.

same

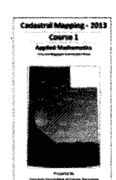
$$\frac{\sin A}{a} = \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

When using the Law of Sines only 2 of the 3 fractions are actually used. The selection of which 2 fractions will be used will be shown when specific problems are evaluated.

1.8.77 **The Law of Cosines.** The Law of Cosines is defined by the following equation which is written in general form.

$$c^2 = a^2 + b^2 - 2ab \cos C$$

1.8.78. Software developed for the solution of triangles designed for use on computers or calculators use what is called the Case Solution. There are 4 basic Cases which will be examined



Any function of an Acute Angle is equal to the cofunction of its complementary angle
Sine & Cosine are cofunctions

Law of Sines

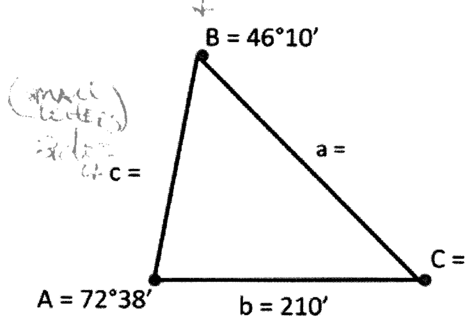
here; the Side-Angle-Angle (SAA) Case, the Side-Side-Angle (SSA) Case, the Side-Angle-Side (SAS) Case, and the Side-Side-Side (SSS) Case.

Law of Cosines

1.8.79. The Case makes reference to what parts of the triangle are known and in what order they are evaluated. As we examine each Case we will introduce the Trigonometric Law that is used to solve them.

1.8.80. **The SAA Case.**Example 1.03.06 is a SAA Case and will be solved by the Law of Sines. If the equation is examined and the values that are known are filled into the equation the following is observed.

$$\frac{a}{\sin 72°38'} = \frac{210'}{\sin 46°10'} = \frac{c}{\sin C}$$



Example 1.03.06

1.8.81. This examination shows that two of the three fractions contain data. This means that the fraction without data can be dropped and the other two fractions will be evaluated. The resulting equation is;

$$\frac{a}{\sin 72°38'} = \frac{210'}{\sin 46°10'}$$

1.8.82. This problem can be evaluated an solved for 'a' by cross-multiplying and then dividing or by algebraic rearrangement where 'a' is isolated on the left side of the equation. The algebraic rearrangement solution will be examined which will show that the cross-multiplication and division process works as well.

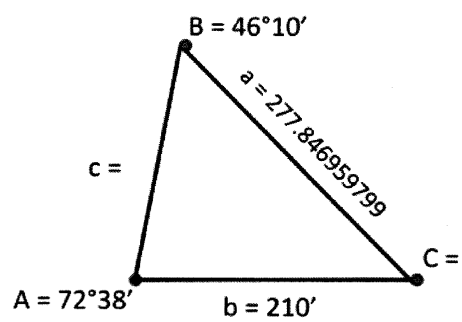
1.8.83. Multiplying both sides by the reciprocal of the sin 72°38' the resulting equation is:

$$a = \frac{210' \sin 72°38'}{\sin 46°10'}$$

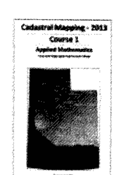
evaluating the equation

$$a = 277.846959799'$$

resulting in the length of side 'a' of Example 1.??



Example 1.03.07



1.8.84. Side c and angle C are the only remaining parts of the triangle left to calculate. From inspection of the equation it can be seen that to solve for side c the angle C needs to be known and vice-versa. Angle C can be derived because we know the values of the other two angles as follows,

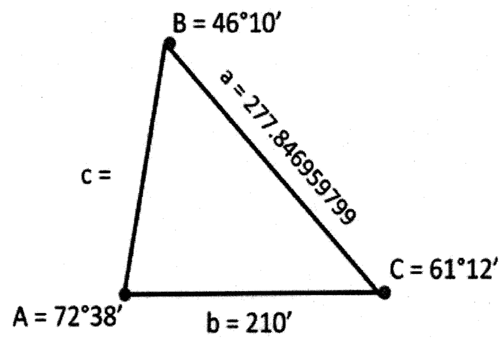
$$\text{Angle } C = 180^\circ - (72^\circ 38' + 46^\circ 10')$$

evaluating the equation Angle C equals,

$$\text{Angle } C = 61^\circ 12'$$

1.8.85. Having solved for all the parts of the triangle except for side c it is now possible to calculate the length of side c by using the fraction from the equation that solves for side c and angle C as follows,

$$\frac{c}{\sin 61^\circ 12'} = \frac{\qquad}{\qquad}$$



Example 1.03.08

1.8.86. For the right side of the equation, although it does not matter which fraction is used to solve for side c, in selecting whether to use the fraction that has side a and angle A or using the fraction that has side b and angle B it is remembered that angle B and side b are both original values of the triangle and that side a is a calculated value.

1.8.87. With this in mind it is better to use the original values when ever possible so in completing the equation lets use angle B and side b which derives the following equation,

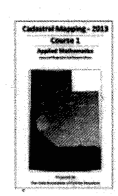
$$\frac{c}{\sin 61^\circ 12'} = \frac{210'}{\sin 46^\circ 10'}$$

cross-multiplying and dividing results in the value for side c of,

$$c = 255.108486151'$$

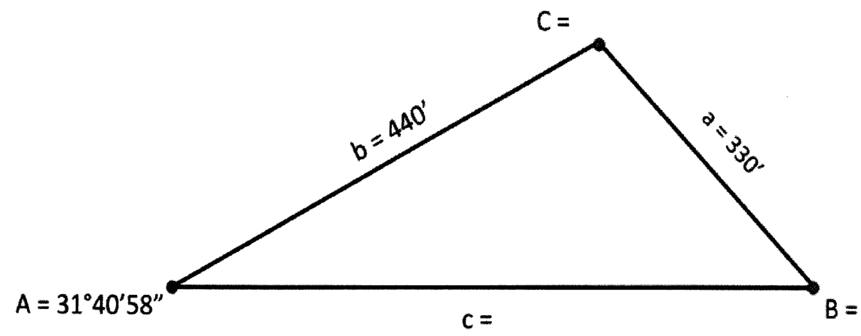
which completes the solution for the acute triangle.

1.8.88. **The SSA Case.** Example 1.03.09 is a SSA Case and will be solved by the Law of Sines. In surveying this solution is known as a Bearing-Distance Intersection, the reason that it is



called this will become clear as the problem is evaluated. The practical application of a Bearing-Bearing Intersection will be introduced later.

1.8.89. Examining the triangle and the sine equation the values that are known can be filled into the equation resulting in the following equation,



Example 1.03.09

$$\frac{330'}{\sin 31°40'58''} = \frac{440'}{\sin B} = \frac{c}{\sin C}$$

1.8.90. From this observation we can see that the fraction containing side c and angle C has no known values and the other two fractions do. So we can drop the fraction from the right side of the equation,

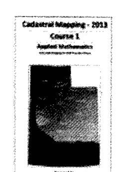
$$\frac{330'}{\sin 31°40'58''} = \frac{440'}{\sin B}$$

a. rearranging the equation to place the unknown value in the upper left side the equation would be inverted and the fractions would switch sides resulting in the following equation that can be solved by cross-multiplication and division

$$\frac{\sin B}{440'} = \frac{\sin 31°40'58''}{330'}$$

b. the result of cross-multiplying and dividing is

$$\sin B = 0.700287849$$

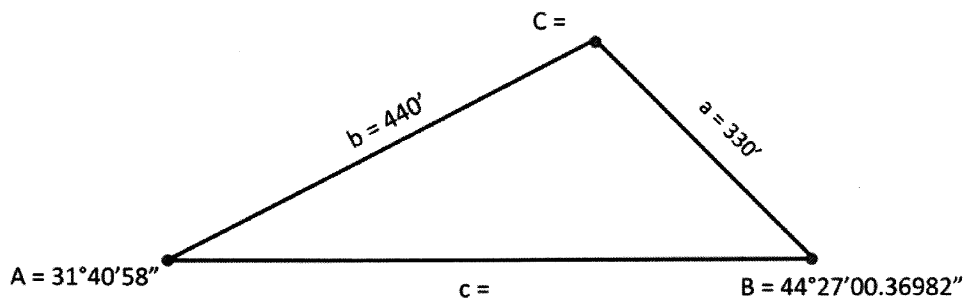


c. solve for angle B alone as follows,

$$B = \sin^{-1} 0.700287849$$

$$B = 44.450102728°$$

d. converting to HMS,



Example 1.03.10

$$B = 44°27'00.36982''$$

1.8.91. To complete the solution of the triangle the following steps would be employed. First solve for angle C

$$\text{angle } C = 180° - (31°40'58'' + 44°27'00.36982'')$$

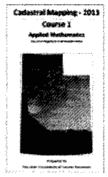
$$\text{angle } C = 103°52'01.63018''$$

a. Knowing all the parts of the triangle except side c at this point, side c is obtained using the Law of Sines,

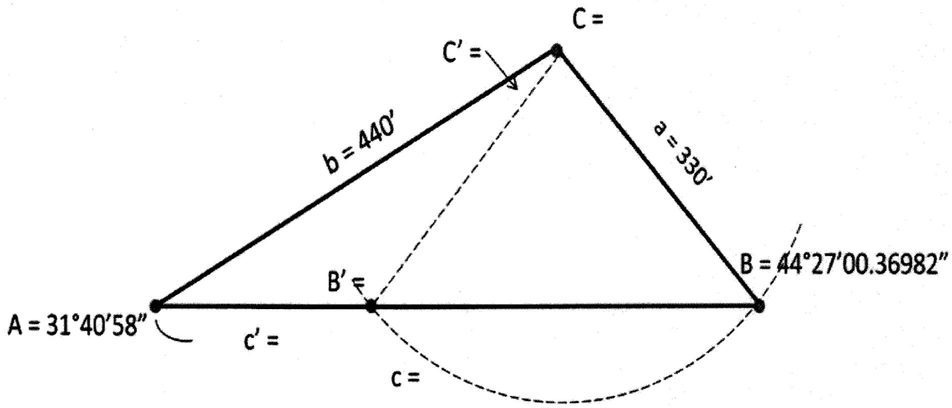
$$\frac{c}{\sin 103°52'01.63018''} = \frac{330'}{\sin 31°40'58''}$$

b. solve by cross-multiplication and division,

$$c = 610.000360243'$$



1.8.92. While this completes the solution of this obtuse triangle it does not recognize the fact that in a SSA Case there are two solutions for the triangle. This is because side a in the example can swing in an arc and intersect side c in two locations. This is shown in Example



Example 1.03.11

1.03.11 where the intersections create angle B' and angle C'. It also creates a side c' which identifies a new triangle of A B' C' A. To solve the new smaller obtuse triangle the process would begin where angle B had been calculated. So the known information would be as shown in Example 1.03.11.

1.8.93. A **Principle of Trigonometry** that needs to be discussed at this point is that when applying trigonometric functions to angles the resulting angle will be an Acute Angle. So angle B' is not calculated when the Law of Sines is utilized and another method must be used to obtain the value for angle B'.

1.8.94. Another aspect of the geometry of this solution is that the triangle B B' C B is an isosceles triangle. Knowing this angle B' can be calculated by subtracting angle B from 180°, therefore, angle B' equals,

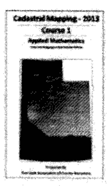
angle B' = 180° - 44°27'00.36982"

angle B' = 135°32'59.63018"

1.8.95. Having solved for angle B' the remaining side c' and angle C' can be calculated as follows,

angle C' = 180° - (31°40'58" + 135°32'59.63018")

angle C' = 12°46'02.36982"



a. using the Law of Sines to calculate side c',

$$\frac{c'}{\sin 12°46'02.36982''} = \frac{330'}{\sin 31°40'58''}$$

b. cross-multiply and divide,

$$c' = 138.852377022'$$

1.8.96. This process now completes both solutions of the SSA Case triangle solution.

LAW OF COSINES

1.8.97. **The SAS Case.** Example 1.03.12 is a SAS Case and will be solved by the Law of Cosines. If the general form of the equation is examined and the values that are known are filled into the equation the following is observed.

$c^2 = a^2 + b^2 - 2ab \cos C$:general form of the Law of Cosines

$c^2 = 346^2 + 458.06^2 - 2\,(346)\,(458.06) \cos 122°38'$

$c^2 = 60^2 + 50^2 - 2(60)(50) \cos 120° =$

solve the equation, remember to apply the trigonometric function to the angle before performing the multiplications,

(120)(2500)

$c^2 = 119716 + 209818.9636 - 316977.52 \cos 122.633333333333$

3600 + 2500.00 − 300,000. ~~120.008~~

$c^2 = 329534.9636 - 316977.52\,(-0.539260813)$

4100. − (−300,000.) ~~0.5000~~

$c^2 = 329534.9636 - (-170933.555054)$

6100 − ~~(150,000.)~~ (−3000)

$c^2 = 500468.518654$

~~156,100.~~ 9100.

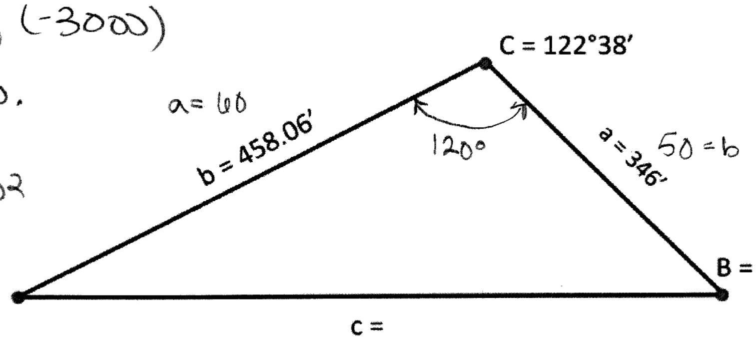
$c = \sqrt{500468.518654}$

~~156,100.2~~ 91002

$c = 707.437996332'$

95.393920142

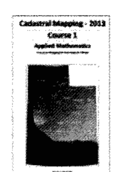
2 sides touch angle



Example 1.03.12

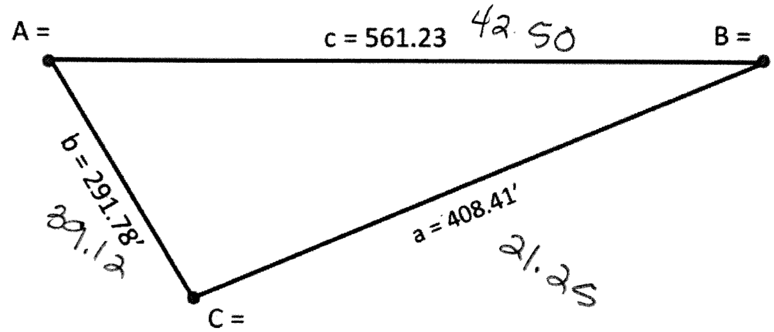
1.8.98. Having solved side c using the Law of Cosines there is enough information known about this obtuse triangle that the remaining angles, A and B, can be solved for by using the Law of Sines for one of the angles and finishing by subtracting the two known

c= 95.3939201417



angles from 180°.

1.8.99. **The SSS Case.** Example 1.03.13 is a SSS Case and will be solved using the Law of Cosines first. Again, starting with the equation for the Law of Cosines in general form and filling in the information which is known about this triangle it can be seen that the missing variable is angle C.



$$c^2 = a^2 + b^2 - 2ab \cos C$$

Example 1.03.13

1.8.100. It will be easier solve the equation for angle C before filling in the known distances. This algebraic rearrangement can begin by transposing

$$c^2 - a^2 - b^2 = 2ab \cos C$$

a. then multiply both sides by the reciprocal of the term 2ab.

$$\frac{c^2 - a^2 - b^2}{2ab} = \cos C$$

b. and finish the isolation with the reciprocal of the cos as shown,

$$\cos^{-1} \frac{c^2 - a^2 - b^2}{2ab} = \cancel{\cos}^{-1}\cancel{\cos}\ C$$

c. canceling on the right side of the equation isolates angle C, then the equation can be written as,

$$C = \cos^{-1} \frac{c^2 - a^2 - b^2}{2ab}$$

d. Now the variables can be filled in with the lengths of the sides.

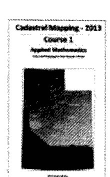
$21.25^2 + 39.12^2 - 42.50^2$

$$C = \cos^{-1} \frac{561.23^2 + 408.41^2 - 291.78^2}{2\,(408.41)\,(291.78)}$$

$2(21.25)(39.12)$

4.346148132

1662.6



e. Evaluating the expression angle C is found to be,

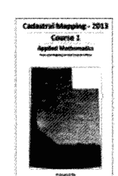
$C = 74°39'40.47011"$ $83°56'03."$

1.8.101. Now knowing angle C the remaining parts of the triangle can be found using the Law of Sines.

* Law of Sines: 1 side & 2 Angles (SAA)
  2 sides & the Angle opposite one (SSA)

* Law of Cosines: 2 sides & the included angle (SAS)
  3 sides (SSS)

$c^2 = a^2 + b^2 - 2ab \cos C$



*offset

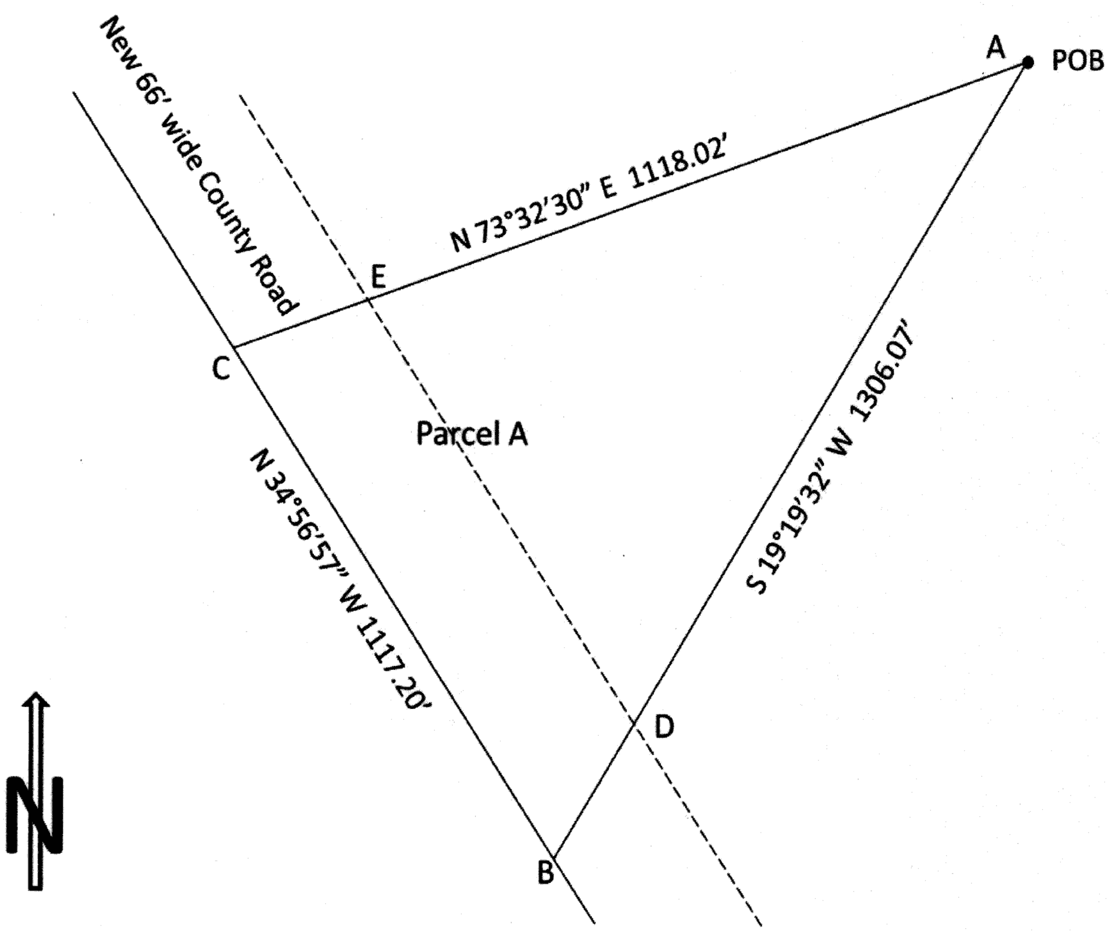
1.8.102. SAMPLE PROBLEMS:

a. In this sample problem a triangular shaped parcel shown as Parcel A is having a new 66 foot wide county road built along the western border. The description of record for the parcel prior to the county road taking is:

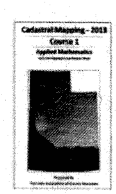
b. Starting at the POB, some Section, Township and Range; RUNNING thence South 19°19'32" West 1306.07 feet; thence North 34°56'57" West 1117.20 feet; thence North 73°32'30" East 1118.02 feet to the point of beginning. Containing 13.6 acres more or less.

c. The record bearings and distances shown are from point A to B to C and back to A. The purpose of examining this problem would be to calculate the area of the remainder for the tax rolls. To do that the distance labeled DE will need to be calculated. What is the bearing and distance from point D to point E?

d. The bearing of line DE is parallel to line BC.**Step 1:**
   i) Calculate the interior angles of the triangle and check to see if they summate to 180°.



Sample Problem: 1.03.070.01

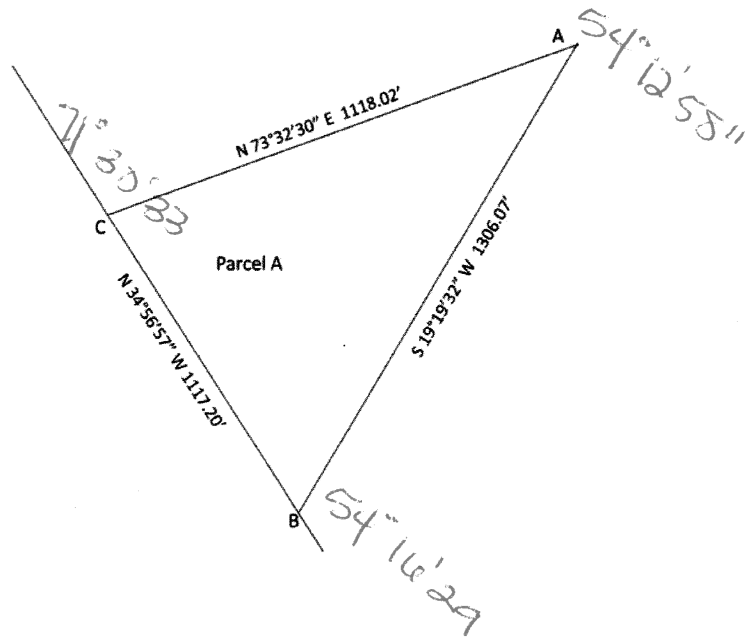


ii) So angle A equals:

$A = 180° - (AB + AC)$

$A = 180° - (19°19'32" + 73°32'30")$

$A = 54°12'58"$



iii) Angle B equals:

$B = BA + BC$

$B = 19°19'32" + 34°56'57"$

$B = 54°16'29"$

iv) Angle C equals:

$C = 180° - (CA + CB)$

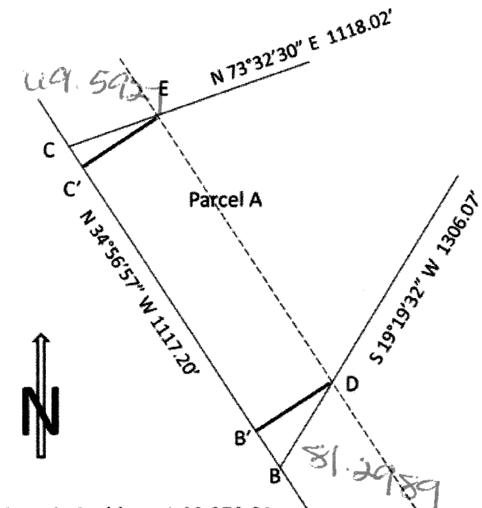
$C = 180° - (73°32'30" + 34°56'57")$

$C = 71°30'33"$

$\sin B = \frac{opp}{Hyp}$

v) Summate the angles to verify closure of the triangle.

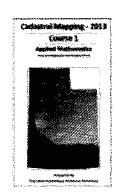
54°12'58" HMS +
54°16'29" HMS
<u>71°30'33"</u> HMS +
HMS 180°00'00" closure checks.



Sample Problem: 1.03.070.01

e. **Step 2:** Calculate the lengths of BD and CE by creating two right triangles as shown.

f. **Step 3:** Begin the calculations with triangle DBB'D. The value for angle B has already been calculated and equals 54°16'29". Because this is a right triangle we know that angle B' equals 90°. We also know that the length of side B'D equals 66 feet which is the width of the New County Road. Side B'D is Opposite angle B and side BD is the Hypotenuse of the triangle and the side that we want to know the length of and can be calculated using the following equation:



$$\sin B = \frac{B'D}{BD}$$

$$\sin 54°16'29'' = \frac{66}{BD}$$

rearrange and solve for BD:

$$BD = \frac{66}{\sin 54°16'29''}$$

[ENTER] [4 HMS] [sin] [÷]

BD = 81.2982'

g. **Step 4:** Calculate triangle CC'EC. The value of angle C has already been calculated and equals 71°30'33". Because this is a right triangle we know that angle C' is equal to 90°. We also know that the length of side C'E equals 66 feet which is the width of the New County Road. Side C'E is Opposite angle C and side CE is the Hypotenuse of the triangle and the side that we want to know the length of and can be calculated using the following equation.

$$\sin C = \frac{C'E}{CE}$$

$$\sin 71°30'33'' = \frac{66}{CE}$$

rearrange and solve for CE:

66

$$CE = \frac{}{\sin 71°30'33''}$$

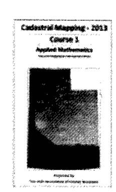
[ENTER] [4 HMS] [sin] [÷]

CE = 69.5928'

h. **Step 5:** Calculate the distances of AE and AD. This is accomplished by simply subtracting the distances that we just calculated.

AD = AB - BD

AD = 1306.07 - 81.2982



AD = 1224.7718

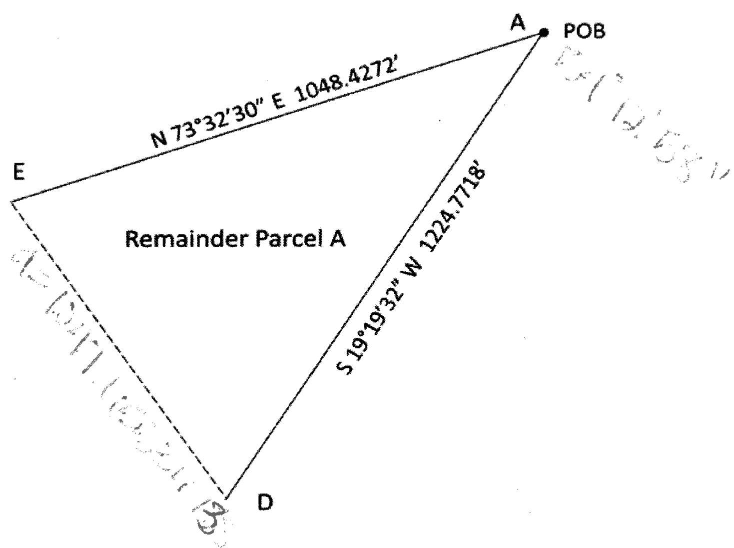
then

AE = AC - CE

AE = 1118.02 - 69.5928

AE = 1048.4272

i. **Step 6:** Now we have the following information relating to the triangle ADEA from which we can calculate the distance DE. Angle A is still known to be 54°12'58". From inspection of the geometry of the road and Parcel A it is found that angle D and angle E are the same as angle B and angle C, respectively.



Sample Problem: 1.03.070.01

i) Knowing all three angles and the distances on two sides the Law of Sines could be used to solve for DE. However, it can also be seen that the Law of Cosines can be utilized to solve for the missing distance.

ii) Solve for distance DE using the Law of Cosines:

$c^2 = a^2 + b^2 - 2ab \cos C$:general or standard form of the equation.

iii) If the triangle is labeled with small letters that correspond to the angles A, D, and E the side DE would be small 'a', the side AD would be small 'e' and the side AE would be small 'd'. Rewriting the equation using this lettering it becomes:

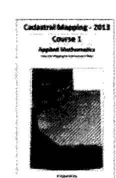
$a^2 = d^2 + e^2 - 2de \cos A$

iv) substituting the known values in the equation:

$a^2 = 1224.7718^2 + 1048.4272^2 - 2\,(1224.7718)\,(1048.4272) \cos 54°12'58''$

$a^2 = 2599265.55578 - 2568168.13783 \cos 54.2161111111111$

$a^2 = 2599265.55578 - 2568168.13783\,(0.584729587)$



$$a^2 = 2599265.55578 - 1501683.89525$$

$$a^2 = 1097581.66053$$

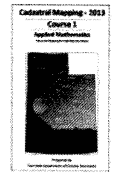
$$a = \sqrt{1097581.66053}$$

CROSS multiply & divide

$$\mathbf{a = DE = 1047.6553'}$$

END EXAMPLE 1.03.070.01

1.8.103. From this point the area of triangle ADEA can be calculated, but it will not be covered in this chapter. Area's will be covered in Course 2.



APPLIED MATHEMATICS:
Course 1. Chapter 9. Part 1

1.9 Circles, Arcs, Curves:

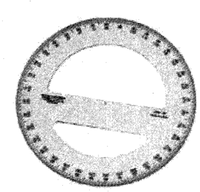
1.9.1. This subject is based on one of the basic geometric shapes that has not been discussed in detail up to this point, the circle. In Chapter 6, Section 1.6.40 Circles, this subject was introduced to show how geometric construction can be used to solve some aspects of the circle. The identification of the center of a circle by graphic means was shown, the construction of tangents to a circle, and how to construct a circle where only three points on the circle are known were also provided.

1.9.2. A knowledge of the use of circles, arcs, and curves becomes important when evaluating property or parcel descriptions. Incorrect usage of the curve can cause serious mapping errors especially when dealing with the spiral curve[1].

1.9.3. Each of the methods discussed in Chapter 6 to solve circles can be used in the CAD environment to solve the same problems. As this subject is developed we will be learning how to solve circles and arcs using mathematics. The focus, however, will be on the curve. Where appropriate we will discuss the CAD process to solve problems without using mathematics.

1.9.4. To understand the subject of Curves a discussion relating to the techniques of Surveying and the tools associated with Highway projects. The concepts of surveying and those of mapping cannot be separated to completely understand the Curve.

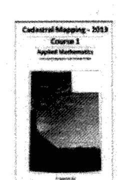
1.9.5. **Circles.** There are many aspects of the geometry of a circle that can be used in solving problems relating to arcs and curves. When bearings of lines are derived by the surveyor the use of the Graduated Circle is an essential part of the measurements made in the course of the survey. These measurements are then transcribed into deed description for use by the mapper. (An example of the Graduated Circle is shown in Chapter 5, Section 1.5.3 where the protractor is pictured.)



1.9.6. The protractor is a type of graduated circle. As mentioned, surveying instruments have graduated circles. Modern instruments no longer have visible circles, they are all internal parts which are electronically read. But the old instruments have a visible circle which were usually graduated to the minute of arc. Drafting equipment has also experienced a dramatic change in the technology of mapping parcels.

---

1 . The subject of the spiral curve will be dealt with in Course 2 in the appropriate chapter the focus of this material will be the Simple Curve.



1.9.7. The type of changes in mapping technology have evolved from the use of geometric principles demonstrated in Chapter 6, to the use of drafting arms (the drafting arm also went through technicalogical advances), to the use of electronic tools to create maps. The challange of this course is that budget inequities between Utah Counties has deprived some counties of the ability to keep up with technology. This inequity means that we will be learning hand drafting methods applied to the mylar plat and then relate that process to the CAD system. In Course 3 you will be learning how to use GIS mapping tools to produce the parcel maps so using ESRI products will not be dealt with in this material.

1.9.8. In Chapter 5, Section 1.5.3, the definition of Degrees, Minutes, and Seconds were given. This subject will assume that the student has a solid knowledge of the graduations of the circle. For example, 360° (degree) in a circle, 60' (minutes) in a degree, 60" (seconds) in a minute, and after that seconds are divided in units of tens.

1.9.9. **Circumference of a Circle.** The circumference of a circle is used in the equations for calculating the length of an arc of a curve. The circumference of the circle is found by the following equation,

$$C = 2\pi R$$

where C = the circumference, 2 is a constant, π = the value of PI (approximately 3.141592653589793238), and R = the radius of the circle (or radial). Therefore, the circumference of a circle that has a radius of 100 feet is,

$$C = 2\pi\, 100$$

$$C = 628.31853 \text{ ft.}$$

1.9.10. What would the radius equal of a circle which has a 100 foot circumference?

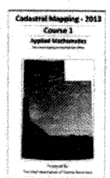
$$C = 2\pi R$$

solving for R,

$$\frac{C}{2\pi} = R$$

therefore,

$$R = \frac{C}{2\pi}$$



$$R = \frac{100'}{2\pi}$$

100 [ENTER]
2 [ENTER]
π [×]
ANSWER [÷]

R = 15.91549 ft.

1.9.11. **Area of a Circle.** Being able to calculate the area of a circle is also part of being able to calculate the area of a parcel which has curved boundaries. The practical application of this principle will be presented in Course 2. The area of a circle is found by the following equation,

$$A = \pi R^2$$

where A = the area of the circle, , π = the value of PI, and R^2 = the radius of the circle squared. Therefore, the area of a circle which has a radius of 100 feet is,

$$A = \pi\ 100^2$$

A = 31,415.9265 sq. ft.

1.9.12. What would the radius of a circle equal that has 1,000.00 sq. ft.?

$$A = \pi R^2$$

solving for R,

$$\frac{A}{\pi} = R^2$$

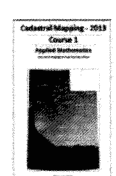
and

$$\sqrt{\frac{A}{\pi}} = R$$

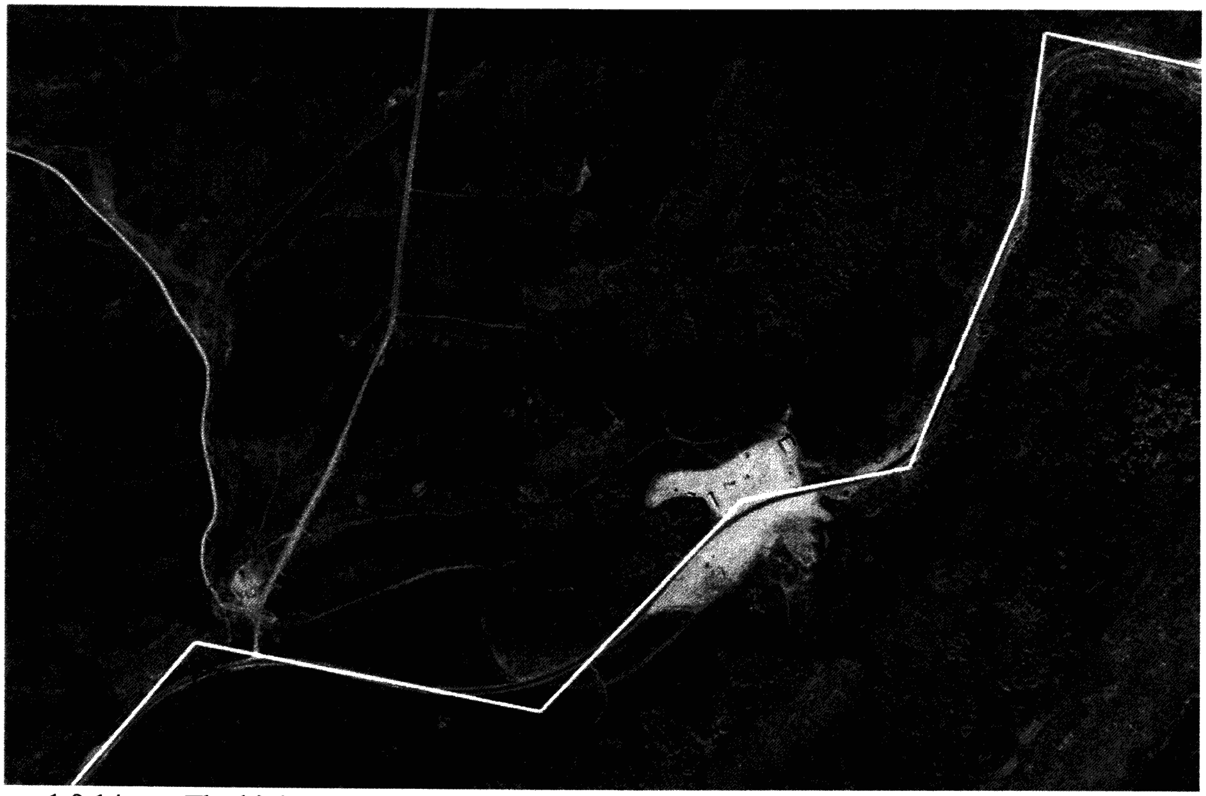
$$\sqrt{\frac{1000}{\pi}} = R$$

1000 [ENTER]
π [÷]
[√x]

R = 17.84124



1.9.13. **The Simple Highway Curve.** This type of curve is use most often in the development of the modern subdivision. It is also used in the construction of low speed highways. This type of curve is also being used in the construction of new Interstate Highways where there is room to widen the traffic lanes sufficiently enough to not warrant the need for designing a spiral curve.

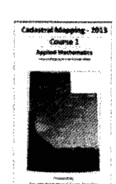


1.9.14. The highway project consists of straight lines connected at an angle point. The straight lines are called Tangents and the angle point is called the Point of Intersection (PI).
sections

1.9.15. The tangent lines are then connected with the curve. The aerial photo shows tangent lines laid on top of a section of highway. The highway curves are visible to show how the PI points and tangent lines relate to one another on the ground.

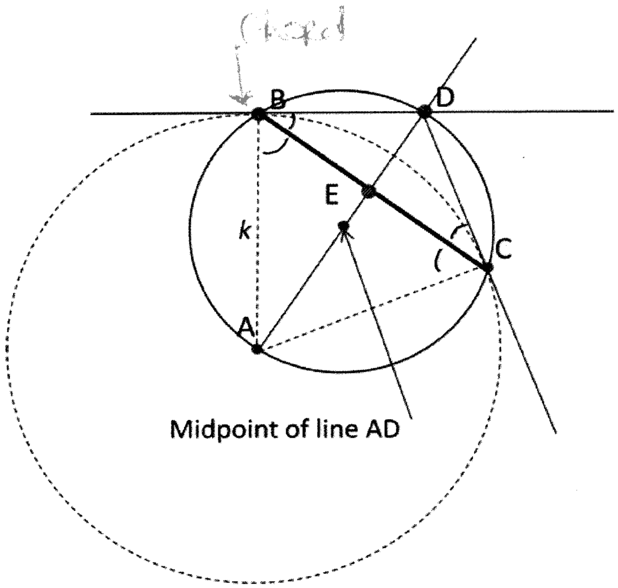
1.9.16. **Inversion of a Circle.** The relationship between circles and the geometry of a triangle can be seen when two circles are drawn along with the corresponding triangles as in the example. Understanding the mathematical relationships associated with the Inversion of a Circle (see graphic titled Inversion of a Circle) will help in understanding the geometry and mathematics of this principle.

CIRCULAR ARCS - KNOWN AS SIMPLE CURVES
SUPERELEVATION - tilting the roadway



1.9.17. First, the following equation will help to begin demonstrating the relationship and is directly related to the graphic.

$$AE * AD = AB^2$$



Inversion of a Circle

1.9.18. Each of these terms are distances as shown in the graphic. Point A is the center of a circle of any radius. Point E is the midpoint of the chord BC, or where the Chord and the Bisector of the Central Angle intersect. AE is the distance from point A to point E (which is also the height of the isosceles triangle ABCA).

1.9.19. Point D is the Point of Intersection of the two tangent lines, B to D and D to C. AD is the distance from point A to point D. The line AD is the perpendicular bisector of the chord.

1.9.20. Point B is the point where the radius of the circle and the tangent line touch. AB is the distance from point A to point B and is also known as the radius of the circle. This variable can be called k^2.

1.9.21. If we rewrite the equation with the variable k the equation becomes,

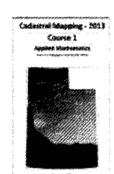
$AE * AD = k^2$ Equation 1.9.21-1

1.9.22. Another equation that relates to this principle is,

$$\frac{AE}{k} = \frac{k}{AD}$$ Equation 1.9.21-2

1.9.23. To understand these relationships consider the following example. Given a circle of having a radius 500 feet and a chord of 450 feet, construct a line tangent to the circle at each end of the chord. This can be done graphically (using basic geometry or CAD drafting tools) or mathematically.

1.9.24. These steps will be the mathematical solution of the problem along with the graphical constructs.



a. To begin, identify the circle whose radius point is at point A and draw the circle at the specified radius, AB = k =500 ft. (This can be done using an appropriate scale.)

b. Next solve a right triangle whose hypotenuse is the radius, k, and one of the sides is half the distance of the chord (BC), therefore, BE = (450 / 2).

c. Calculate the length of AE using Pythagorean theorem.

$$c^2 = a^2 + b^2$$

therefore,

$$k^2 = AE^2 + BE^2$$

$$500^2 = AE^2 + (450/2)^2$$

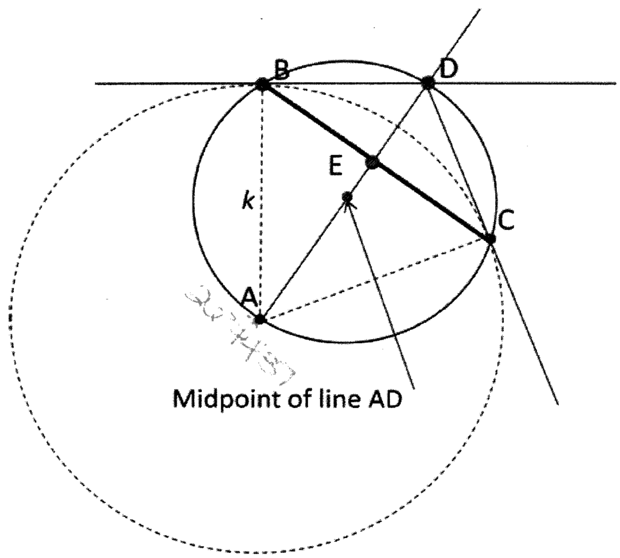
$$AE^2 = 500^2 - 225^2$$

$$AE^2 = 199375.00$$

$$AE = \sqrt{199375}$$

$$\mathbf{AE = 446.514277487}$$

d. Calculate angle BAE, which we will call angle A as follows (angle A is actually half of the total angle which is typically called Delta and designated by the symbol Δ and would be angle BAC in this graphic).



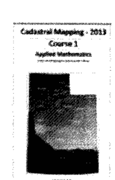
Inversion of a Circle

$$\text{Sin } A = \frac{BE}{k} \quad \text{or it could be written,} \quad \text{Sin } \Delta/2 = \frac{\text{Opp}}{\text{Hyp}} \quad \text{since this is a right triangle,}$$

solve for A and filling in the known variables,

$$A = \text{Sin}^{-1} \frac{(450 / 2)}{500}$$

$$\mathbf{A = 26°44'37.26''}$$



angle ABC (or just B) is then solved by subtracting angle A from 90°.

B = 90° - 26°44'37.26"

B = 63°15'22.74"

Note: The subtraction from 90° is the same as adding the two known angles and subtracting from 180°. This short cut only works when using a Right Triangle.

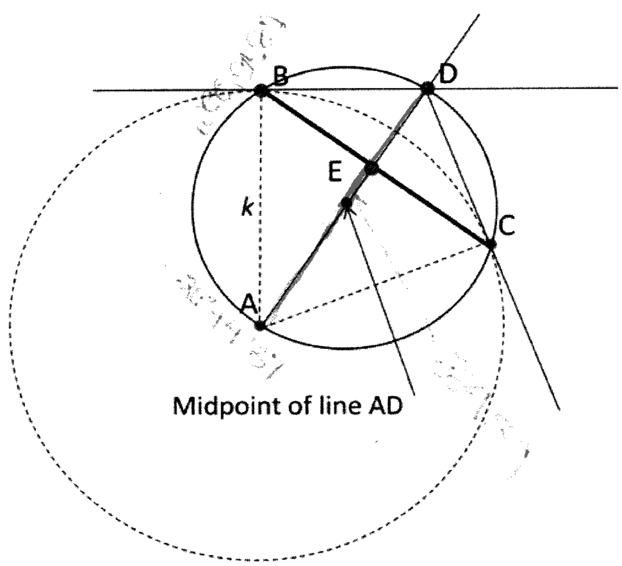
e. To get to this point graphically,

   i. a circle having a radius of 500 feet has been drawn.

   ii. From any point on the circle (in this example use point B) scribe an arc having a radius of the chord length, 450 feet (creating point C).

   iii. Draw the chord from point B to point C.

   iv. Construct a line perpendicular to the chord through point A. (If you need a refresher on how to do this refer to Chapter 6, Section 1.6.61 - A Line Perpendicular to a Line Thru a Point Not on a Line.)



Inversion of a Circle

f. Calculate AD using equation 1.9.21-1 (page 6) as follows.

$$AE * AD = k^2$$

distance AE was previously calculated on page 7,

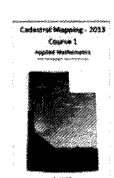
$$446.514277487 * AD = 500^2$$

$$AD = \frac{500^2}{446.514277487}$$

AD = 559.892510956

g. To graphically establish point D,

   i. construct a circle through the three points A, B, and C. (If you need a refresher on



how to do this refer to Chapter 6, Section 1.6.78 Three Point Arc. The process is the same to draw a circle as it is in connecting the three points with an arc.)

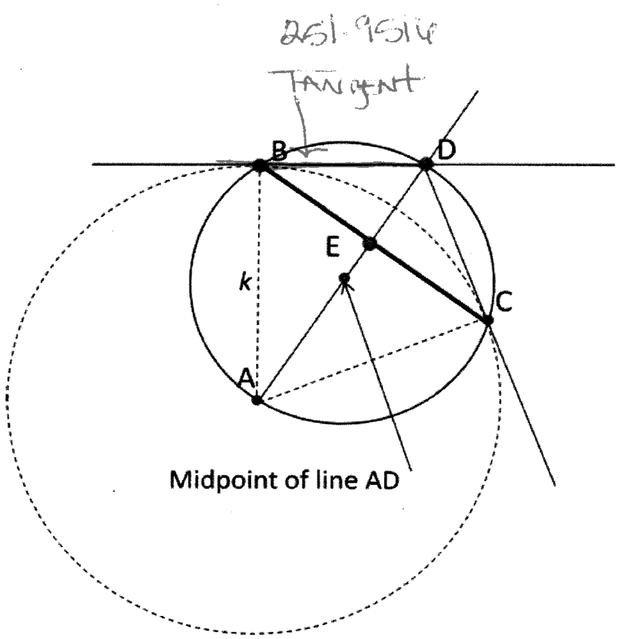
ii. next draw a line from point A through point E extending to the edge of the circle just drawn using three points, this creates point D.

h. With this information the right triangle ABDA can be solved which will result in the angle BDA and the length of BD making BD tangent to the circle.

BDA = 90° - A

BDA = 90° - 26°44'37.26"

BDA = 63°15'22.74"



Inversion of a Circle

Note that the angle BDA and angle ABC are equal and angle DBC and angle BAE are equal. Next calculate the length of BD. This can be done using Pythagoreans theorem in which the general form is;

$c^2 = a^2 + b^2$ and where

c = AD (which was calculated on page 8),

a = k (which is the given radius of the circle), and

b = BD.

the equation can be written as

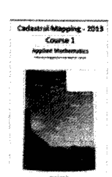
$AD^2 = k^2 + BD^2$

$559.892510956^2 = 500^2 + BD^2$

$BD^2 = 559.892510956^2 - 500^2$

$BD = \sqrt{63479.623825}$

BD = 251.951629931



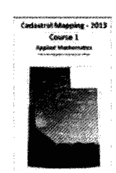
This distance of BD is also known as the Tangent distance of a curve and usually labeled as T.

i. The right triangle ABDA has now been mathematically solved.

j. To complete the graphical solution point D has already been drafted it is simply a matter to draw the tangent lines BD and DC by,

   i. draw a line through point B and point D and another through point D and point C.

1.9.25. It should be noted that because there are no directions (bearings) established on any of the lines this entire geometric process can rotate about the initial circle (whose center point is at A) without changing the results.

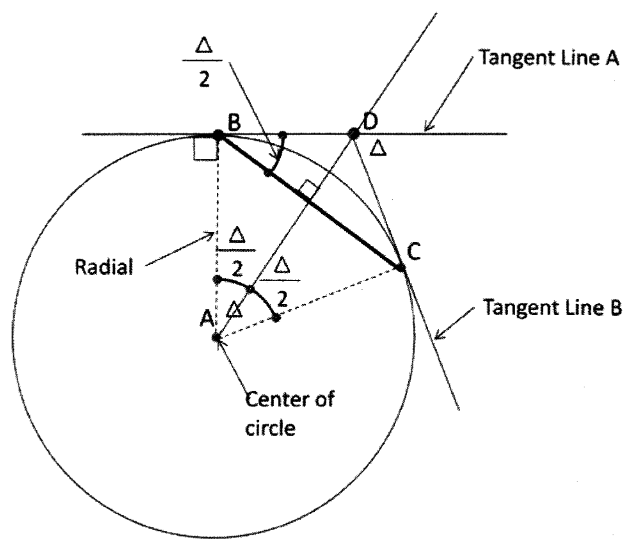


1.9.26. Arcs and Curves. In this subject we will build on the principles of the circle and apply them to the arc or curve. The curve being discussed in this subject is the simple curve. Other curves will be discussed in Course 2. To begin this discussion we will return to the geometric diagram "Inversion of a Circle" used above. However, we are going to add to the geometry of the circle and change the names of the points previously used.

1.9.27. **DEFINITIONS and SYMBOLS of the CURVE.** In this section the definition of the points of a Highway Curve are going to be given and used throughout the remainder of the text. When referring to curves whether they are curves of a highway, street, canal, curb, or any other feature which may be defined by an Arc or the Curve these abbreviations or symbols are standards in the Engineering and Surveying professions. That, however, does not mean that some individual circumstances could not be found that would violate these standards.

1.9.28. **Chord of a Circle.** The construction of the Chord and the associated angles becomes useful when evaluating highway curves and identifying stationing related to the curves whether the stationing is expressly written into the description or whether examining a set of State Highway drawings. Some common symbols or notations will be introduced as we discuss some of the relationships.

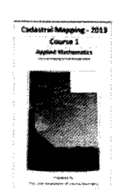
a. There is an angular relationship with the chord of a circle (line BC which is typically noted in relation to curves as C), the radius (R), and tangent (T) lines. If the chord line is constructed between point B and C a new angle at point B is created between point D and C (angle DBC). The angle DBC is equal to one half the Central Angle (Δ) and one half the Deflection Angle (shown as Δ/2).



Chord of a Circle

b. The same relationship is also true of the angle DCB because the triangle created by the chord (DCBD) is an isosceles triangle. Thus, angle DCB is also Δ/2. When the Deflection Angle (Δ) changes, the angle that is supplementary (angle BDC) also changes. Thus, when angle BDC changes the angles at B and C also change but because this is an isosceles triangle and angles B and C are equal when angle BDC changes angles B and C change equally.

c. The triangle ABCA is also an isosceles triangle and when the Central Angle (Δ) varies the angles at B and C also change equally.



d. The chord also sets up another relationship which will become useful in developing the equations related to arcs or curves. This is the construction of a line from point A to and through point D.

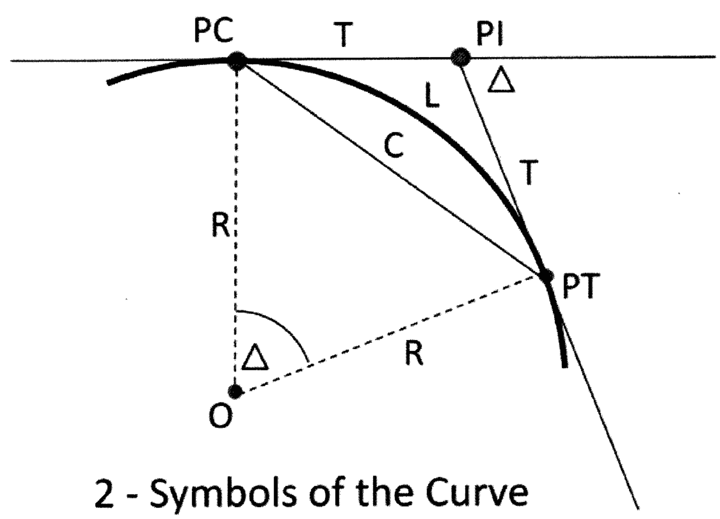
e. By establishing this line we can see that the angle BAD and angle DAC are also equal to $\Delta/2$.

f. Creating the chord and the bisector of the chord (line AD) will reveal another basic principle of the circle which is known as Inversion of a Circle, previously discussed.

g. **C = is the chord distance.** This is the straight line distance between the PC and PT of the curve. Sometimes the chord is termed the Long Chord (LC). There is also a term of Short Chord which will be defined in the section relating to the layout and calculation of station points on the curve.

h. The length (L), tangent (T), and chord (C), are parts of the geometry of the curve that are used in highway plans and route location. Sometimes legal descriptions use some of these symbols or terms to aid those interpreting the deed to be able to properly plot the curve as it relates to the boundary of a parcel.

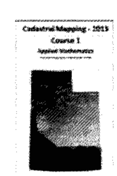
1.9.29. **Δ = the name for this symbol is the Delta and represents two angles of the curve.** The Central Angle and the Deflection Angle. In the text we will use the symbol Δ to represent the Central Angle and the symbol Δ_D to represent the Deflection Angle. Normally, when the word Delta is used it is making reference to the Central Angle even though the symbol is interchangeable with the Deflection Angle.



2 - Symbols of the Curve

a. The Deflection angle is identified by extending the tangent line from the PC of the curve past the PI of the curve and then starting the next tangent line from the PI of the curve running through the PT of the curve. The angle at the PI from the forward extention of the tangent to the PT of the curve has the same angular value as the Central Angle which is identified as point O.

b. **O** = this is the letter "O" not the number zero and represents the radius point of the curve.



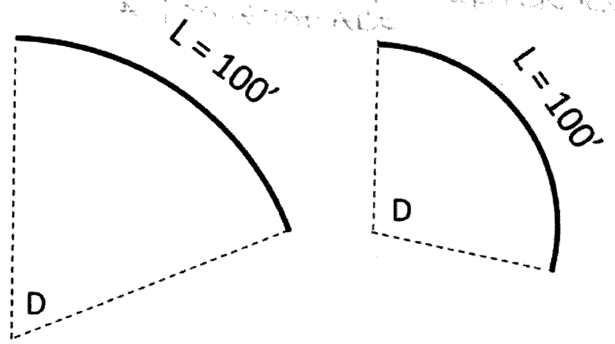
Sometimes this point is labeled RP. The use of RP would be recommended to help avoid confusion between 'O' (the letter) and 0 (zero).

c. The Central Angle (Point O in this diagram) is identified as the included angle between two Radii beginning at the center of the circle and extending to the circumference of the circle at points which are tangent to the Tangent Lines at the PC and PT of the curve.

1.9.30. **D = Degree of Curve.** The degree of the curve is a primary design parameter used in the engineering of highways and freeways. There are two basic definitions, the Arc definition and the Chord definition. Generally speaking, the Arc definition is used in highway design and the Chord definition is used in railroad design. The student should be aware that there are exceptions to this statement so it should be verified which definition is actually being used, if possible.

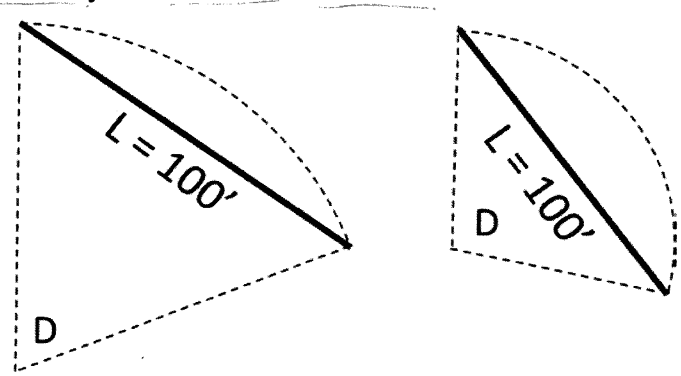
a. **Arc Definition.** An angle subtended by an arc length of 100 feet. As the radius of a curve changes so does the value of the Degree of the curve. The effect of the change in the degree of curve will flatten or sharpen a curve and allow a vehicle to negotiate a curve at specific rates of speed. The smaller the angular value of the degree of curve, the longer the radius becomes and the faster a vehicle can travel around the curve.

Central angle that subtends a [illegible] arc



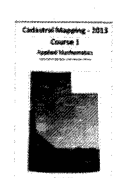
Arc definition

b. **Chord Definition**. An angle subtended by a chord distance of 100 feet. The same geometric relationship which is associated with the Arc definition also pertains to the Chord definition, that is, as the angular magnitude of the Degree becomes smaller, the radius of the curve becomes larger and the flatter the curve becomes allowing a train to negotiate the curve at a higher rate of speed.



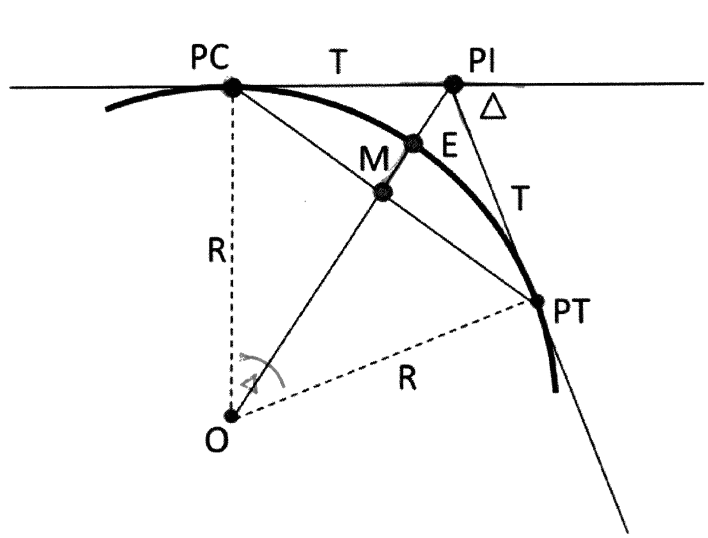
Chord definition

c. Again, the degree defines the sharpness of a curve and is selected in conjunction with the design speed of the curve. Many times this parameter is one of the data parameters



which highway plans include in the listing of curve data. In the design of highway curves there are three basic design parameters which are the primary parameters used in fixing the shape of the curve, the Deflection Angle, the Design Speed, and the Degree of curvature which is actually a function of design speed.

1.9.31. **E = is the External distance.** This is the distance along a line which bisects the central angle from the PI to a point on the arc of the curve. The point on the arc of the curve is at the midpoint of the curve. The line is shown between the PI and point E.



3 - Symbols of the Curve

1.9.32. **L = length of the curve.** This is the distance from the PC to the PT of along the arc of the curve. It is also a key design parameter for the curve and is used to derive the Stationing of the PT in highway projects.

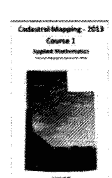
1.9.33. **M = is the Middle Ordinate distance.** This is the distance along a line which bisects the central angle from a point on the arc of the curve to a point on the chord of the curve. Both points are on the midpoint of both the arc of the curve and the chord.

a. The External and Middle Ordinate are generally used in the engineering and design of a highway or road. Other than how to calculate them this text will not discuss the utility of these geometric parts. They are seldom used in property descriptions.

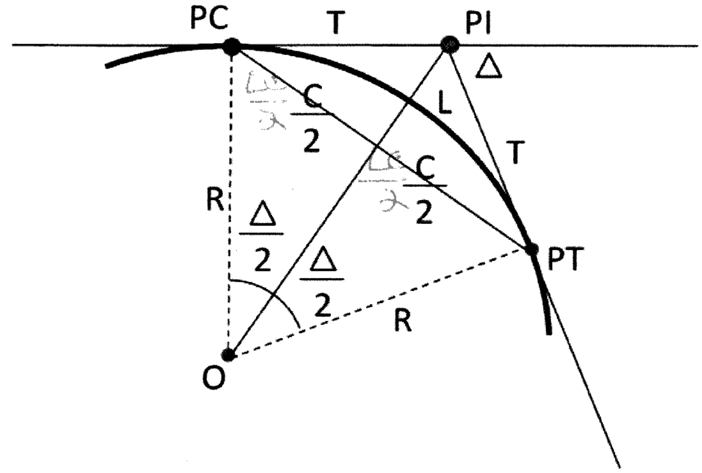
1.9.34. **PC = Point of Curvature.** This is the point where the tangent line and the arc of the curve are coincident with one another. This point is sometimes referred to as BC or Beginning of the Curve. BROKEN BACK CURVE

1.9.35. **PI = stands for Point of Intersection.** This is where the two tangent lines intersect.

1.9.36. **PT = stands for Point of Tangency.** This is the point that the curve ends and is coincident with the next tangent line. This point is sometimes referred to as EC or End of the Curve. EC - END OF CURVE



a. In labeling a curve the symbols PC and PT always denote the direction that the Stationing of a highway project will be increasing.[2] In highway drawings and plans the symbol PC always indicates the point where the incoming tangent line ends and the arc of the curve begins and will be labeled with the project station for that point.



4 - Symbols of the Curve

b. The symbol PT always indicates the point where the arc of the curve ends and the tangent line begins and will be labeled with the project station for that point. The station number is obtained by adding the length of the arc of the curve to the station of the PC.

c. The PI of the curve is the point where the two tangent lines, both the incoming tangent and the outgoing tangent, intersect. Project stationing for the PI is labeled in the forward direction using the tangent distance. To obtain the station of the PI the tangent distance is added to the station number of the PC. (Note, the PT station is never obtained by adding the tangent distance to the PI station.)

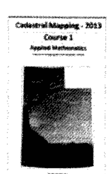
d. When route surveys are being conducted in the field (surveys to identify the alignment and location of a highway) the tangent lines are established first by locating the PI points on the ground. The survey will then measure the deflection angle (Δ_D) at each PI location.

1.9.37. **R = the radii of the curve** which is also called the radius distance of the curve. This is the distance from the center of the curve to the circumference of the circle. This is also a key design parameter of the highway curve. center point of circle

1.9.38. **T = the tangent distance.** This distance occurs twice in the figure. First, the distance from the PC to the PI and Second, the distance from the PT to the PI. In all cases the tangent distances of the simple curve are equal. equal in length

a. When a line is constructed tangent to a circle the tangent line (Tangent Line A) is also perpendicular to a line from the center of the circle (point A) to the point where the circle

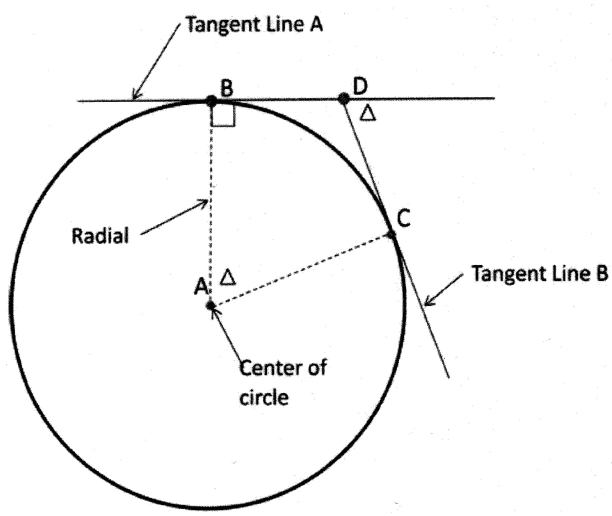
2 . See Appendix for sample highway drawings and how stationing is notes and shown.



and Tangent Line A touch (point B) which is known as the radial.

b. If two lines are tangent to the circle (Tangent Line A and Tangent Line B) they are both perpendicular to the radial lines (radial AB and radial AC).

c. The tangent lines and the radial lines create an angle and the angular relationship between the two radial lines (AB and AC) and the two tangent lines (BD and DC) are equal and noted with the Delta symbol (Δ).



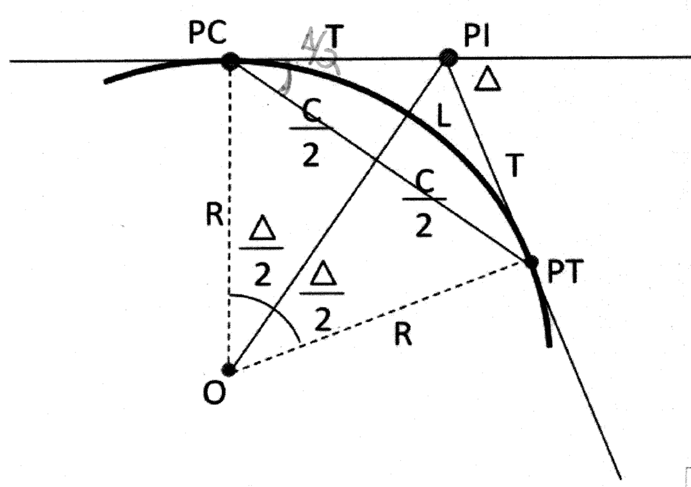
Tangent To A Circle

d. Angle A, designated by the symbol Δ, is known as the Central Angle of the circle. Angle D, which is also designated by the symbol Δ, is known, in surveying, as the Deflection Angle. Thus the angle at point D from point B to point C is supplementary to the Deflection Angle (being angle BDC). See the definition of Delta Angle above.

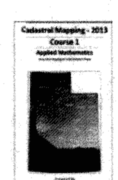
e. The distance of the radial lines (AB and AC which are typically noted in relation to curves as R or r) are not the same as the distance between points B and D or point D and C, although, the distance between points B and D are equal to the distance between points D and C (these tangents BD and DC are typically noted in relation to curves as T). There is only one exception to this statement and that is when the Central angle and the Deflection angle are 90° then the distances just mentioned (R and T) are equal.

1.9.39. **$\Delta/2$ = is Half Delta** or Delta divided by 2 and is the angular value of the Central Angle divided by two.

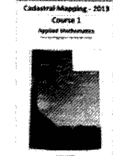
1.9.40. **C / 2 = is Half Chord** or Chord divided by 2 and is the length of the Chord (or long chord) divided by two.



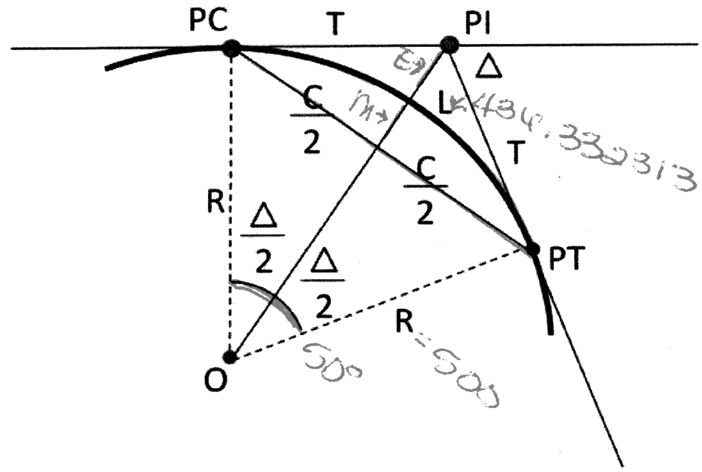
4 - Symbols of the Curve



1.9.41. The use of these two parts of the curve (**$\Delta/2$** and **C / 2**) become useful in solving some parts of the curve especially when working with highway drawings and plans. These relationships may become useful when evaluating a deed description that indicates a Non-tangent Curve beginning on a specific station of a highway which station is in the middle of a curve. This circumstance will be discussed in Course 2.



1.9.42. **Simple Curve Formulas – Arc and Chord definition.** The basic formulas used in calculating the parameters or components of a simple horizontal curve will include the symbols previously defined. As the equations are examined, refer to the definition diagrams shown here.



4 - Symbols of the Curve

a. Δ = Central angle for **Arc** Definition.

Length of rec? + degree of curve

$$\Delta = \frac{L\,D}{100} \qquad \text{Equation: 1.9.39-1}$$

b. Δ = Central angle for **Arc** Definition. Δ= 50°

$$\Delta = \frac{180\,L}{\pi R} \qquad \text{Equation: 1.9.39-2}$$

RE-ARRANGEMENT of 1.9.40-6

c. Δ = Central angle for both **Arc and Chord** Definition curves.

$$\Delta = \tfrac{1}{2}\,\mathrm{Sin}^{-1} \frac{C}{2R} \qquad \text{Equation: 1.9.39-3}$$

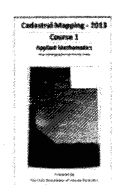
d. D = Degree of curve **Arc** definition. D= 112732.961249

$$D = \frac{36000}{2\pi R} \qquad \text{Equation: 1.9.39-4}$$

e. D = Degree of curve **Arc** definition.

$$D = \frac{100\,\Delta}{L} \qquad \text{Equation: 1.9.39-5}$$

f. D = Degree of curve **Arc** definition.



$$D = \frac{5729.573}{R}$$ Equation: 1.9.39-6

g. D = Degree of curve **Chord** definition.

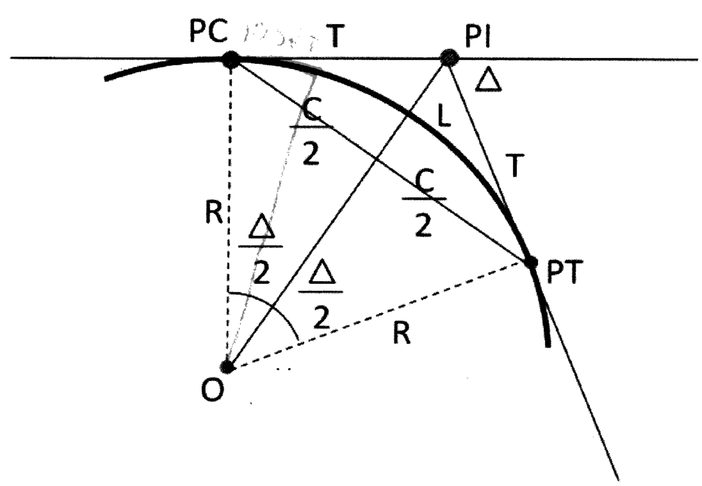
$$D_C = 2 \sin^{-1} \frac{50}{R}$$ Equation: 1.9.39-7

h. D = Degree of curve **Chord** definition.

$$D_C = 2 \sin^{-1} \frac{D}{2}$$ Equation: 1.9.39-8

Where D_C equals the degree of curve for the chord definition and D equals the degree of curve for the arc definition.

1.9.43. **Simple Curve Equations – Arc definition.** The remaining equations are being used to calculate the components of the Arc definition curve. Unless you have railroads running through private property in your county the arc definition curve will be the curve that you use the most.



4 - Symbols of the Curve

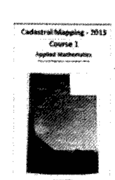
1.9.44. **R** = Radius of a curve.

$$R = \frac{36000}{2 \pi D}$$ Equation: 1.9.40-1

$$R = \frac{T}{\tan \Delta / 2}$$ Equation: 1.9.40-2

$$R = \frac{C}{2 \sin \Delta / 2}$$ Equation: 1.9.40-3

a. **C** = Chord distance (Long Chord).



$C = 2\ R \sin\ \Delta / 2$ Equation: 1.9.40-4

C = Chord distance for Degree of curve. This equation calculates the chord for a 100 foot arc. 100ft SECTION

$C = 2\ r \sin D / 2$ Equation: 1.9.40-5

b. **L** = Length of Arc. L= 436.332313

*

$$L = \frac{\pi\ R\ \Delta}{180} \quad \text{or} \quad L = \frac{2\ \pi\ R\ \Delta}{360}$$ Equation 1.9.40-6

$$L = \frac{100\ \Delta}{D}$$ Equation: 1.9.40-7

c. **T** = Tangent distance. T= 233.153829

$$T = R \tan \frac{\Delta}{2}$$ Equation: 1.9.40-8

d. **M** = Middle Ordinate. M= 46.84610482

$M = R \text{ vers } \Delta / 2$ Equation: 1.9.40-9

To use the calculator trigonometric functions equation 1.9.40-9 substitute equation 1.8.6 (see Course 1, Chapter 8) which results in the following equation.

$M = R\ (1 - \cos\ \Delta / 2)$ Equation: 1.9.40-10

e. **E** = External distance. E= 51.68891591480

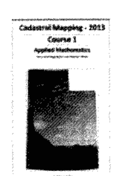
$E = R \text{ exsec } \Delta / 2$ Equation: 1.9.40-11

To use the calculator trigonometric functions equation 1.9.40-11 substitute equation 1.8.7 (see Course 1, Chapter 8) which results in the following equation.

$E = R\ (\sec\ \Delta / 2 - 1)$ Equation: 1.9.40-12

Then substituting equation 1.8.5,

$E = R\ [(1 / \cos\ \Delta / 2) - 1]$ Equation: 1.9.40-13

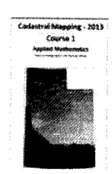


1.9.45. **The Value of PI.** PI (π), an approximate value for PI is found by the following fraction.

$$\pi \cong \frac{1146408}{364913} \qquad \text{Equation: 1.9.40-14}$$

1.9.46. The value of π to 30 decimal places is, $\pi = 3.14\ 159\ 265\ 358\ 979\ 323\ 846\ 264\ 338\ 327\ 9$

1.9.47.

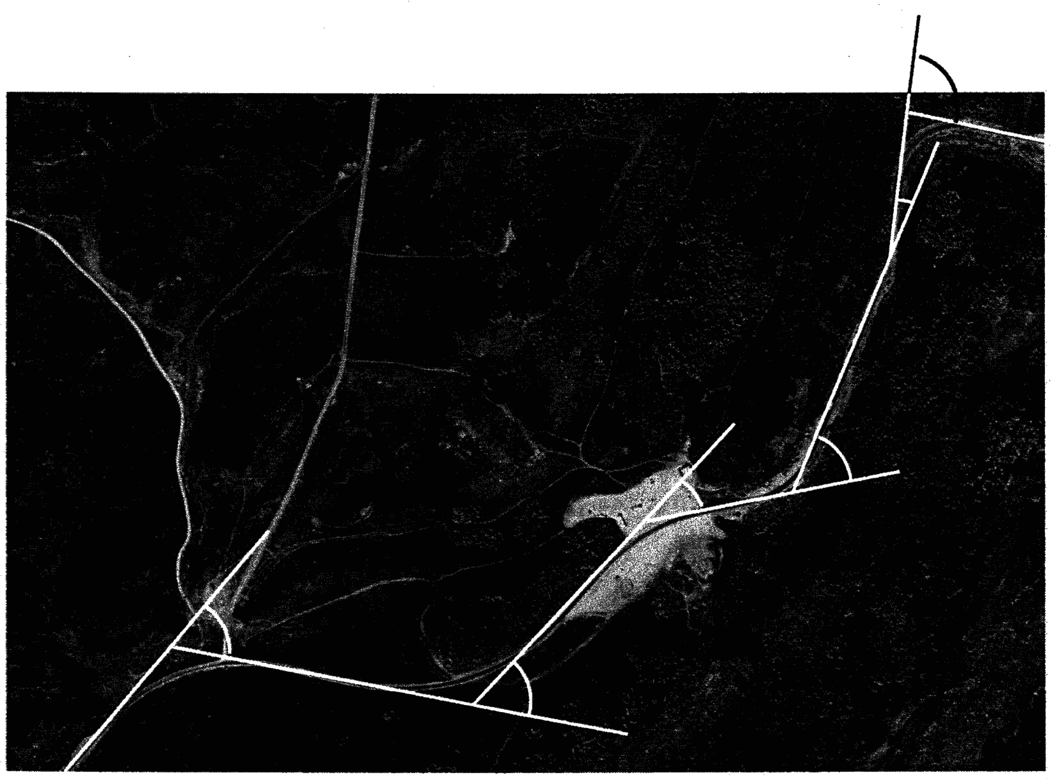


APPLIED MATHEMATICS:
Course 1. Chapter 9.
Part 2

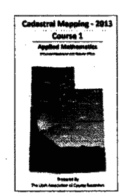
1.9 Circles, Arcs, Curves:

1.9.1. **Working with Simple Curves.** The simple horizontal curve is the basic curve which is used in highway design and contained in property descriptions. Highway rights-of-way are designed to be a continuous line related to the center line alignment of the road. Sometimes the right-of-way is an offset line of a specific distance and parallel to the center line of the highway. Other highway right-of-way lines are not parallel to the center line alignment and are designed to accommodate the needs of the highway construction.

1.9.2. The geometry of a simple curve is such that only two components of the curve are required to solve the remainder of the components of the curve. The two design components are selected based on the terrain that the highway must traverse and on the speed at which the vehicles will be allowed to travel on the highway.



1.9.3. The first component, being the terrain considerations, is obtained by exanining possible alignments and selecting a traverse line that establishes PI points of the alignment. This

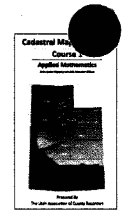


initial traverse line is surveyed and the Deflection Angles are obtained. Measuring the deflection of the tangent lines will give by direct observation the size of the Central Angle or Delta Angle.

1.9.4. The other design component, that being the speed of travel or design speed, is an engineering determination in which many factors are considered. Generally, the Degree of Curve is selected to determine how sharp or flat the curve will be. Once these two parameters are selected, Central Angle and Degree of Curve, the remainder of the curve parameters can be calculated.

1.9.5. During the design process the tangent alignment may be adjusted or altered several times to better accomodate economic considerations relating to the cost of construction.

1.9.6. In Part 1 of the material the definitions of the parts of the curve have been discussed along with the equations that will help the mapper solve curve problems. In Part 2 we will examine some sample problems and calculate the needed parts of the curve.



1.9.7. **Example 1.**

1.9.8. A highway curve is shown to have a Central Angle of 45° ($\Delta = 45°$) and a Degree of curve of 1°. Calculate the following components; R, T, L, C, E, and M. As the values are calculated the geometry of the curve will be shown in the corresponding figures.

1.9.9. R (Radius of the Curve):

1.9.10. This equation is derived from components of a circle. Calculating the circumference of a circle is done with the equation, $Cir = 2\,\pi\,R$. The value of 36000 is a derivative of 360° in a circle being multiplied by 100. Remember that the definition of a Degree of curve is the angle subtended by the arc length of 100 feet. Therefore, if one degree (1°) sweeps 100 feet of arc, in a 360° sweep of arc (one full circle) a distance of 36,000 feet will have been traveled and one full circle will have been rotated.

$$R = \frac{36000}{2\,\pi\,D} \qquad \text{Equation 1.9.40-1}$$

$$R = \frac{36000}{2\,\pi\,1°}$$

$$\mathbf{R = 5729.57\ 795\ 131}$$

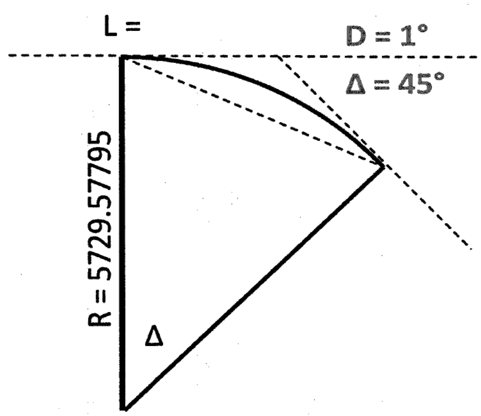


Figure L-1

1.9.11. So if D = 2° the resulting radius is half that of that shown. Using equation 1.11.40-1 calculating the value of R equals ~~2373.26889'~~.

1.9.12. T (Tangent of the Curve):

1.9.13. Calculate the Tangent of the curve by creating a right triangle with the geometry of the curve. The tan of Half the central angle is equal to the Opp side (T) divided by the Adj side (R). Rearranging and solving the equation for T the result is equation 1.11.40-8.

$$T = R \tan \frac{\Delta}{2} \qquad \text{Equation 1.9.40-8}$$

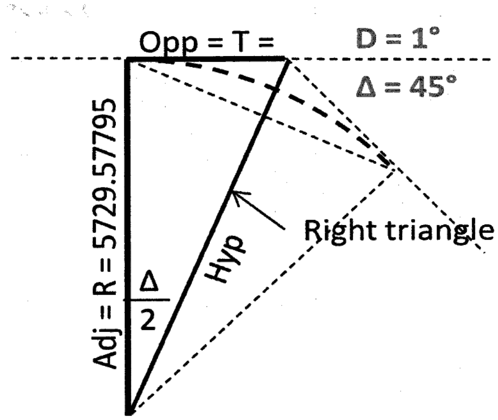
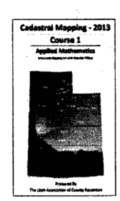


Figure R-1



$$T = 5729.57795131 \tan \frac{45°}{2}$$

T = 2373.26 889 411

1.9.14. L (Length of the Curve):

1.9.15. Examining equation 1.11.40-6 in the numerator 2 π R results in the circumference of a circle. Delta (Δ) divided by 360° is a ratio of the portion of the circle represented by the Delta angle. Reduce the equation to lowest terms and fill in the known components and complete the equation.

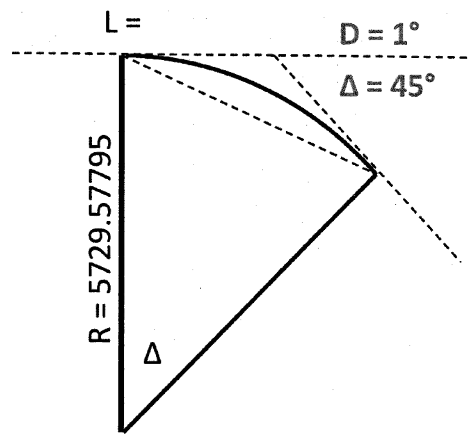


Figure L-1

$$L = \frac{2\ \pi R \Delta}{360} \quad \text{Equation 1.9.40-6}$$

Reducing to lowest terms.

$$L = \frac{\pi R \Delta}{180}$$

$$L = \frac{\pi\,(5729.57795131)\ 45°}{180}$$

L = 4500.00 000 000

1.9.16. C (Chord or Long Chord of the Curve):

1.9.17. Examining equation 1.9.40-4 it can be seen that it is similar to equation 1.9.40-8 (calculating the Tangent distance). In this circumstance the right triangle is derived as shown in the figure to the right. Once the Opp side is calculated the result is multiplied by 2.

$$C = 2\,R \sin\ \Delta / 2 \quad \text{Equation 1.9.40-4}$$

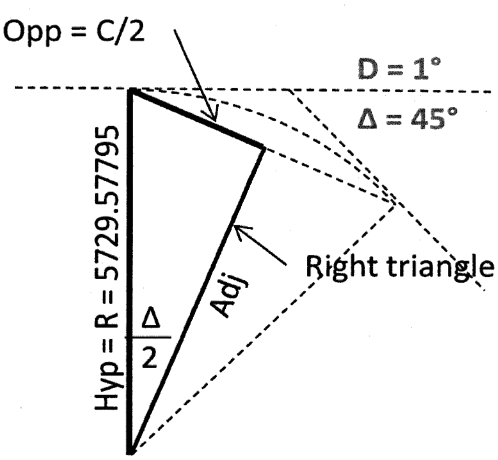
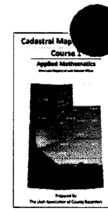


Figure C-1



C = 2 * 5729.57795131 * sin 45°/2

C = 4385.22 911 282

1.9.18. E (External Distance of the Curve):

1.9.19. Examining equation 1.9.40-13 the cos of half the central angle is the ratio associated with the Adj and Hyp sides. Dividing by one results in the recipicol of the ratio. By subtracting the value of one the remaining portion of the ratio when multiplied by the radius results in the External distance.

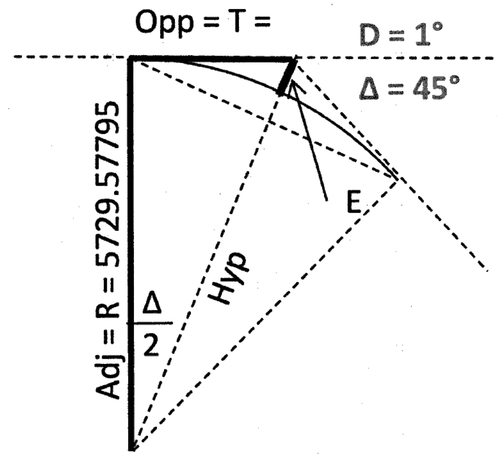


Figure E-1

E = R [(1 / cos Δ / 2) – 1] Equation 1.9.40-13

E = 5729.57795131 [(1 / cos 45°/2) – 1]

E = 5729.57795131 [(1 / cos 22.5°) – 1]

E = 5729.57795131 (1.08239220029 – 1)

E = 5729.57795131 (0.08239220029)

E = 472.07 253 414

1.9.20. M (Middle Ordinate of the Curve):

1.9.21. Examining equation 1.11.40-10 the cos of half the central angle is the ratio associated with the Adj and Hyp sides. The ratio is the same in this calculation as in the calculation for E. The difference is that subtracting one from the ratio and multiplying by the radius will give the short portion of the radius.

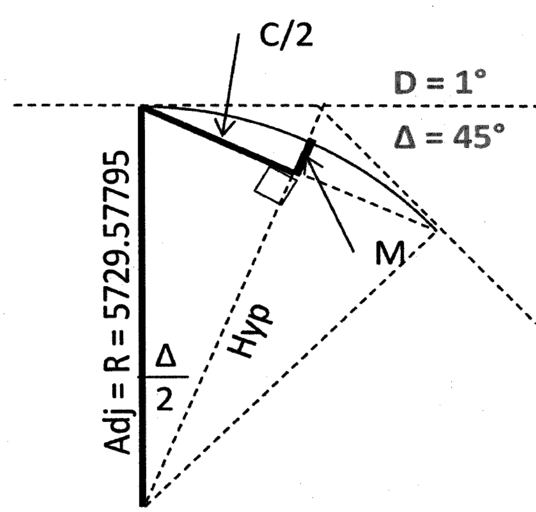
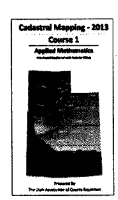


Figure M-1

M = R (1 – cos Δ / 2) Equation 1.9.40-10

M = 5729.57795131 (1 – cos 45°/2)



M = 5729.57795131 (0.07612046749)

M = 436.13 815 216

1.9.22. Stationing of the Curve:

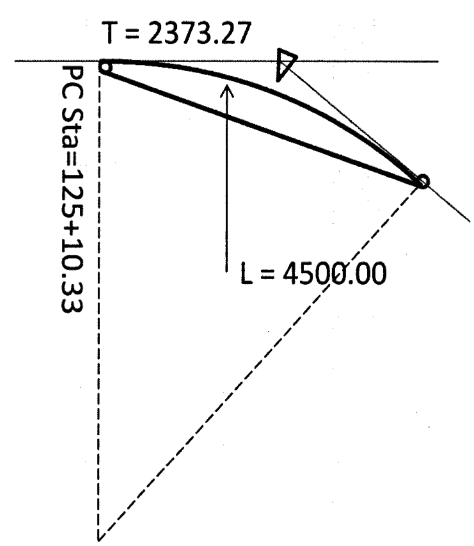
1.9.23. We can now calculate the Engineering Stations of the curve. These stations can be key in being able to properly draft a deed description. Many deed descriptions use Stations as a means of identifying the location of a property line. For instance, a description may read "beginning at a point located 50 foot perpendicularly distant to the right of Station 125+10.33 of Highway number...". Knowing how stations are calculated will help in identifying the proper locations of points in descriptions.

1.9.24. From our example the **PC Station is identified as 125+10.33**. In this number there is a plus (+) which we are not accustomed to seeing in numbers. The plus divides Even Stations from decimal parts of stations. For instance, if we said that the point being identified was at station 10, the station would be written as 10+00.00 and equates to 1000 feet from station zero. Station zero is also zero feet.

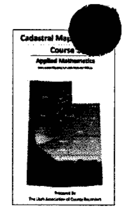
1.9.25. So when we say, station 125, we are saying the point is 12,500.00 feet from station zero. To calculate the PI and the PT of the curve the Tangent Distance and the Length of the curve are needed.

1.9.26. When calculating the Stationing of a curve the values used are always rounded to the hundredth of a foot. Since Stationing is never shown with greater precision than the hundredth of a foot, it is appropriate to round our calculation for T (Tangent Distance) and L (Length of Curve) to the hundredth of a foot and use these rounded values in the calculations.

1.9.27. In Example 2 we will see how rounding to tenths of a foot will introduce error into the problem of curve solutions when State Highway maps (or any other highway maps) are used to solve a description problem. The T and L distances are shown in the diagram as rounded values for the purposes of calculating the Stations for this curve and will be used in the following two calculations.



1.9.28. The PI Station is calculated by adding the Tangent Distance in feet to the foot equivalent of the PC



Station.

PC Station 125+10.33 converted to feet = 12,510.33

PC Station = 12,510.33
T Distance = 2,373.27
PI Station = 14,883.60

Convert result in feet 14,883.60 to **PI Station 148+83.60**

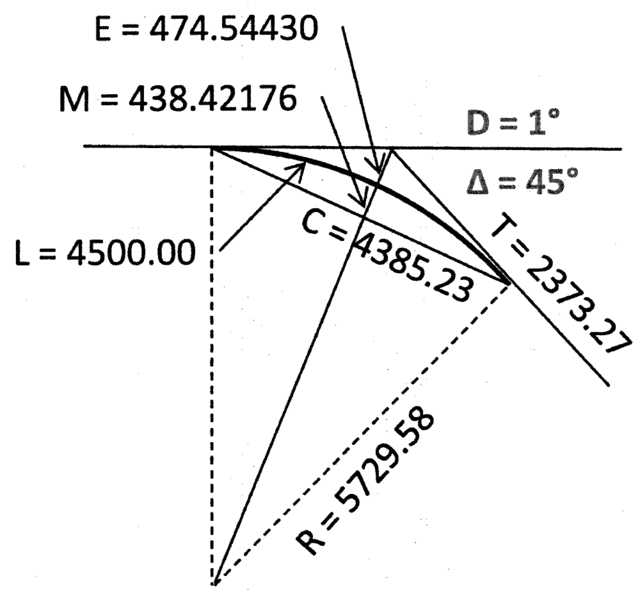
1.9.29. The PT Station is calculated by adding the Length of the curve to the PC Station.

PC Station 125+10.33 converted to feet = 12,510.33

PC Station = 12,510.33
L Distance = 4,500.00
PT Station = 17,010.33

1.9.30. Convert result in feet 17,010.33 to **PT Station 170+10.33**

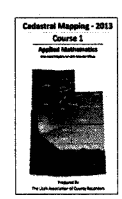
1.9.31. Remember, in a curve stations are always calculated along the arc of the curve. The only exception is the Station of the PI which is calculated along the tangent line from the PC Station of the curve only. ***Do not calculate the PI Station using the PT Station.***



Example 1 - Completed

1.9.32. Also notice that in the final solution some parameters of the curve are shown with rounded values and some are not. The E (External Distance) and M (Middle Ordinate) are not rounded although they may be rounded on some highway maps (if they are noted) but because they are a value that the Cadastral Mapper may never use they are not being rounded in this example problem nor will they be included in the tabular data.

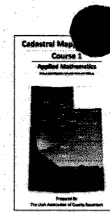
1.9.33. The other curve parameters are shown as rounded and after the calculation processes have been completed the final values of the curve can be noted as rounded values. These



rounded values will be listed on the highway map. The information will result in a curve table as follows.

Final Curve data:		
Symbol:	**Calculated Value:**	**Final Rounded Value:**
Δ (Central Angle and Deflection Angle)	Given	45°00'00"
D (Deree of curve)	Given	1°00'00"
R (Radius)	**5729.57 795 131**	5729.58
L (Length)	**4500.00 000 000**	4500.00
T (Tangent distance)	**2373.26 889 411**	2373.27
C or LC (Chord or Long Chord)	**4385.22 911 282**	4385.23
PC Station	Given	125+10.33
PI Station	14,883.60	148+83.60
PT Station	16,895.56	168+95.56

1.9.34. The Highway map shown below is a typical example of how curve data is shown and noted.



1.9.35. From Example 1 it can be seen that solving the components of the curve is directly related to triangles and trigonometry.

1.9.36. METRIC STATIONS.

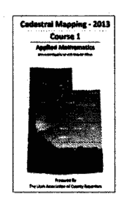
1.9.37. There are also metric station equivalents and they are written with the plus in a different position than when written in feet. When you see a station written using 3 digits as 15+123.000 you are looking at a metric station. Notice that there are three digits after the plus and after the decimal point. The plus has been moved specifically to show stations in terms of meters so that they are not mixed up with feet. In the example above the station represents 15,123.000 meters from zero.

1.9.38. Since the meter is the base unit of the metric system and of metric Stationing, the decimal values are also listed with 3 digits to express thousandths of a meter. In Appendix 4 Conversion Factors, the names of the 3 digits are, 0.1m = decimeter, 0.01m = centimeter, and 0.001m = millimeter.

1.9.39. The reason that the millimeter is used in these values can be demonstrated by the following conversions.

0.01m (or one centimeter) = 0.0328 feet
0.001m (or one millimeter) = 0.00328 feet

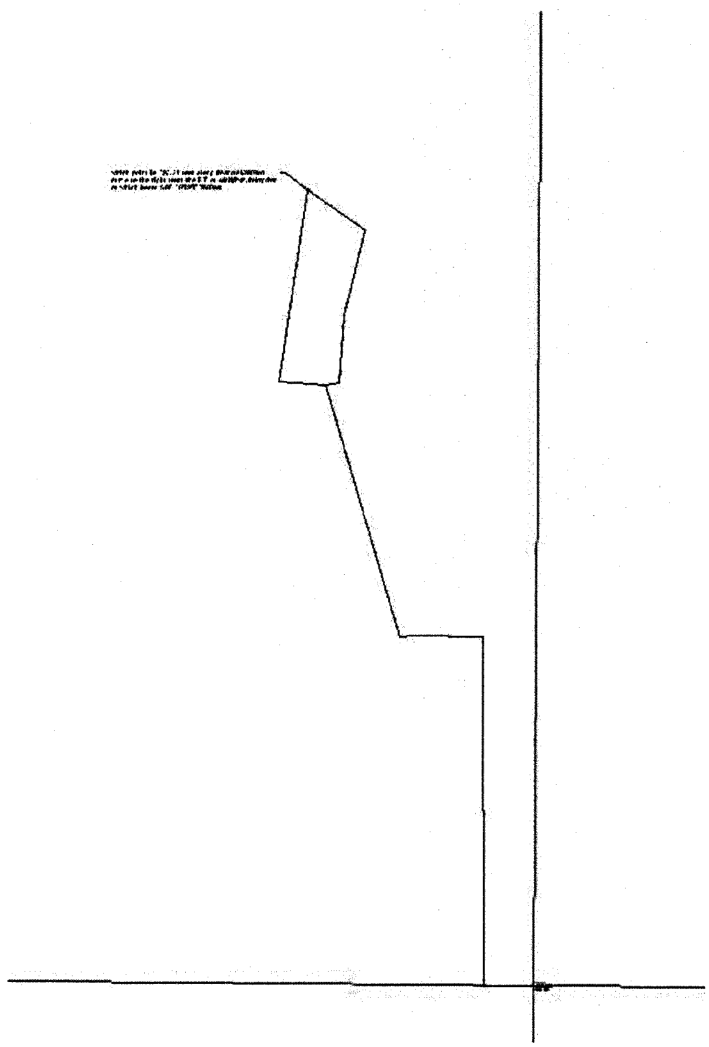
1.9.40. From the conversion we learn that 2 decimal places of a meter (which is one centimeter) is approximately equal to 3 hundredths of a foot. To retain accuracy to a minimum of one hundredth of a foot an extra digit is needed, the millimeter.



1.9.41. **Example 2.** In this problem we use the same calculation processes we have learned in Example 1 and apply them to a deed description. We will plot a description that has curve information in the bounds call which will require that we make some calculations to solve for part of a curve. The description is an actual description found in county deed records.

1.9.42. The description is as follows:

1.9.43. Part of SE 1/4, Sec 28, T7N, R1E, SLB&M, US Survey: Beginning at a point S89°34'20"W 110 feet, N0°10'W 756.05 feet, N88°43'W 150 feet, S82°10'W 29.09 feet and N16°17'W 567.31 feet from the SE corner of SE 1/4 of said Section 28; Running thence N85°22'W 100.00 feet; thence N.8°30'E. 418.29 ft, to South R/W of State Highway 162, which point is 493.21 feet along the arc of a 1399.69 foot radius curve to the right from the PT of said highway, the long chord of which bears S68°40'19"E 490.66 feet; thence along said arc 151.89 feet, the long chord of which bears S54°30'E 151.82 feet; thence S13°54'W 188.98 feet thence S4°04'W 146.38 feet; thence S82°W 29.09 feet to the point of beginning. Together with a 50 foot wide R/W adjoining said property on the West.[1]



1.9.44. Plotting this description from the SE corner of the section as called for yields the following configuration.

1.9.45. The NW corner of the property, being noted and pointed to is described in the deed as ***" thence N.8°30'E. 418.29 ft, to South R/W of State Highway 162, which point is 493.21 feet along the arc of a 1399.69 foot radius curve to the right from the PT of said highway, the long chord of which bears S68°40'19"E 490.66 feet;".***

1.9.46. There are several things in this call that need to be analyzed to determine the actual location of the point being described. The evaluation will most likely change

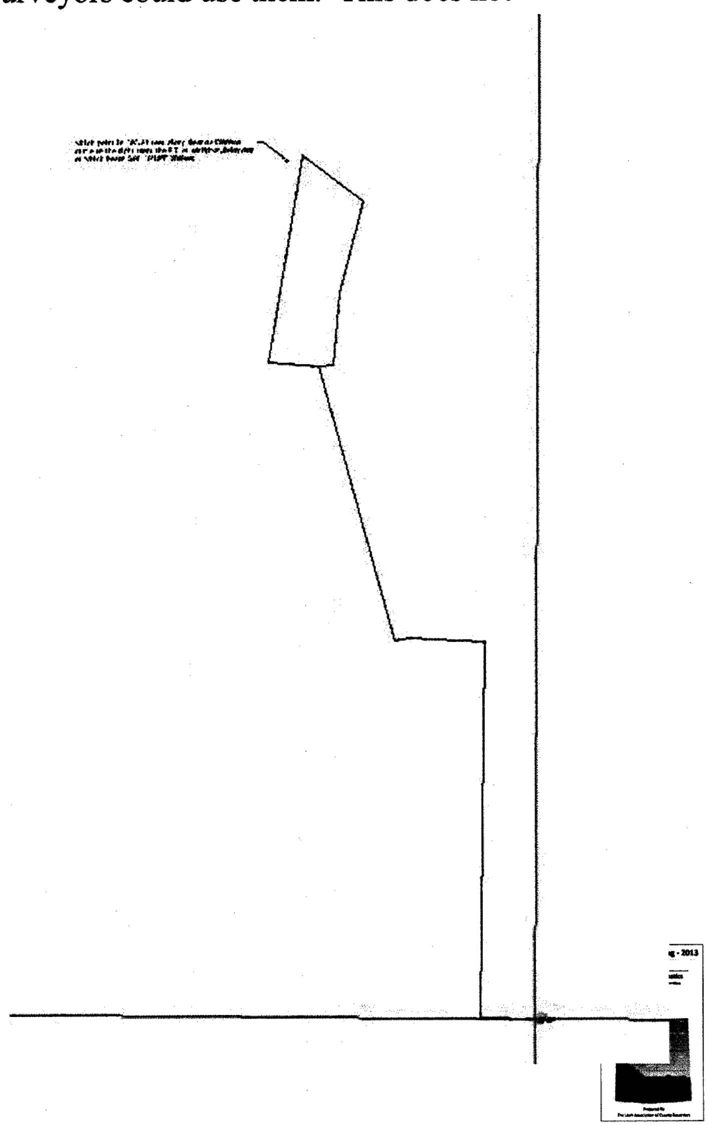
1 . Book 1377 Page 221 of the deed records of Weber County, Utah.

the bearing and distance which are given in this call as N.8°30'E. 418.29ft. Again, this adjustment is necessary because of the legal requirement that bounds calls take precedence over and will modify the metes of a description.

1.9.47. To start this evaluation we must first evaluate the basis of bearing that this description is written with, or at least see if there is a way for us to determine what that basis of bearing actually is. To fully understand this discussion the material in Chapter 10 should be reviewed at this point and then return here to continue the discussion on the basis of bearing for this description.

See Chapter 10 for discussion on Basis of Bearing.

1.9.48. The description for this property originated in 1980 and at this point in time Weber County had survey monuments in place so that surveyors could use them. This does not always mean that the description was written by a surveyor but in this case it seems like it was because of the use of the highway curve information. For a non-surveyor to make this type of calculation they would have to know a lot about the mathematics of curves, have copies of the State highway drawings, and know where the location of the PT of the curve is related to the end point of the line being described. It is unlikely that a layperson would have this type of knowledge. So the first assumption that could be made would be that the description was prepared by a surveyor that knew where the section corners were monumented in relation to the highway.



1.9.49. It is not easily visible in this drawing to tell that the first call of the deed is not running the same direction as the Section line. However, if we examine the county survey records and the deed we can see that the deeds first call is S89°34'20"W and the section line according to Weber County data runs

N89°23'41.58"W.

1.9.50. This means that if we ASSUME (because the basis of bearing is not expressly stated) that the section line and the first bearing of the description to be the same line we will be required to rotate the deed by 1°01'58.42" clockwise. This rotation yields the drawing shown on page 11. You can see that the arrow is pointing to the first location of the corner which was obtained by plotting the description without any rotation. After rotation the corner has moved in a northeast direction.

1.9.51. Again, by making this rotation we are ASSUMING that the first deed tie call and the section line are to be the same line. In other words we are assuming that the basis of bearing for this deed is the section line. Thus, necessitation a rotation to the common line.

1.9.52. Because the bounds call of the deed uses the highway as a tie point and the right-of-way line of the highway as the next call in the deed we need to determine where the highway actually is and in what manner the highway location may affect how the parcel is drafted.

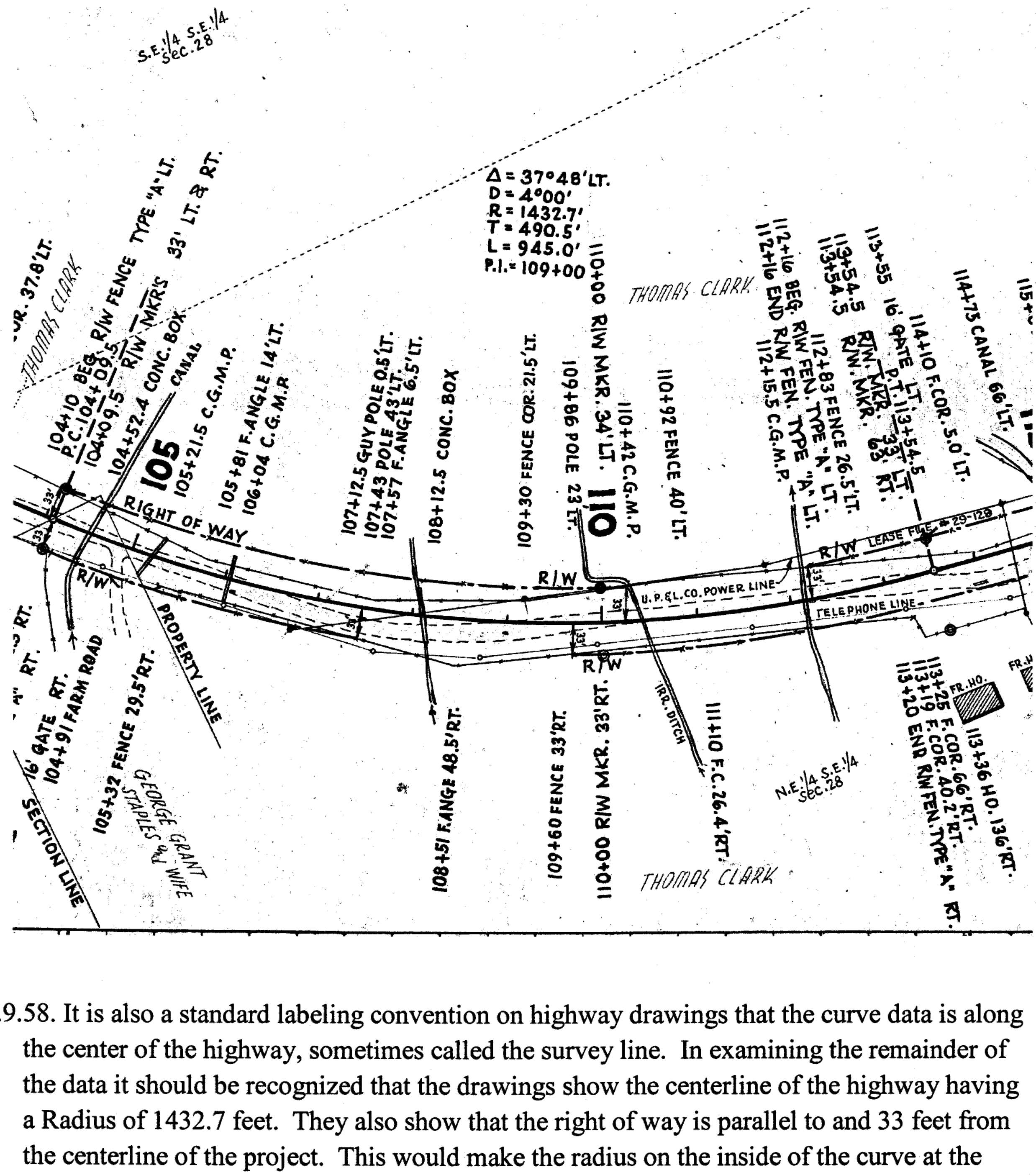
1.9.53. Returning to the description - notice that the call is to a point intended to be on the right of way line of Highway 162. It is also located at a point that is *493.21 feet from the PT of the curve*. So to properly identify this point we need to know some information about the highway. This will require that we obtain a copy of the State Road drawings for Highway 162 in the area that this deed is located.

1.9.54. The reason for obtaining this information and going through the process of calculating and drafting the highway is, One, to locate the PT of the curve, and Two, to verify that the location of the highway shown on the Ownership Plat is actually correct so that we know our use of the highway will result in the most correct location for the line of the parcel being mapped.

1.9.55. If your county does not have highway drawings you can contact your local UDOT Region office and they will provide you with copies of the drawings. Many of these drawings are now scanned and they can email them to you.

1.9.56. A section of this map has been enlarged to more easily see the curve information and is shown on page 13.

1.9.57. The curve data is shown near the top-center of the map and is listed in tabular form. If you will remember from previous discussions relating to highway design and surveying there are two pieces of this information that are used in the initial design of the road, the Delta or Central Angle (Δ) which is obtained from survey processes of measuring the Deflection Angle of the curve and the Degree of Curve (D) which is an engineering design parameter determined from such things as the design speed of the road and other safety considerations.



1.9.58. It is also a standard labeling convention on highway drawings that the curve data is along the center of the highway, sometimes called the survey line. In examining the remainder of the data it should be recognized that the drawings show the centerline of the highway having a Radius of 1432.7 feet. They also show that the right of way is parallel to and 33 feet from the centerline of the project. This would make the radius on the inside of the curve at the right-of-way 1399.7 feet. The parcel that we are working with is on the inside of the curve or the south side of the highway (notice where the north arrow is pointing).

1.9.59. The deed is indicating the radius to be 1399.69 feet. It is not clear why there would be a hundredth of a foot difference between the values. While the difference of 0.01ft is not really something that generally would be worried about in mapping there may other situations where the information has a discrepancy much greater and will need to be evaluated to see what the correct data would be. Therefore, this discrepancy will need to be evaluated to see if there is a reason that the deed would be different than highway drawings.

1.9.60. The drawing supplies directly several design parameters. From examining the numbers it will be seen that they are rounded values. The angles are rounded to the nearest minute of arc and the distances are rounded to the tenth of a foot. This will be a consideration when calculations are made to help determine which radius is the correct one.

a. Delta = 37°48' LT

   i. The LT refers to the direction the curve turns, Left when standing on the centerline looking up station. In survey terms this is the Deflection Angle and the direction the angle was measured at the PI.

   ii. This same nomenclature of left or right is used to describe the direction a curve is bending in deed descriptions. In deeds, however, the directions given do not make reference to the direction that the stationing of the road is increasing.

b. D = 4°00'

   i. This is the Degree of the curve Arc Definition.

c. R = 1432.7

   i. This is the Radius of the curve and has most likely been calculated from the Delta and Degree of curve.

d. T = 490.5

   i. This is the Tangent distance.

e. L = 945.0

   i. This is the Length of the curve. This length can also be found by subtracting the PC Station 104+09.5 from the PT Station 113+54.5, 1.9.61. First, let's check the curve data to make sure that there are not errors on the highway plans. The first check to make would be to verify the L (length of curve) and the difference between the PC Station and the PT Station should accurately reflect one another. This is done by subtracting the PC Station from the PT Station.

PT Station =	113+54.5
PC Station =	104+09.5
L by Stations =	945.0'

The highway plans show L = 945.0', therefore, they check.

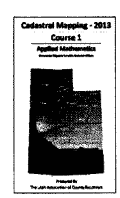
1.9.62. Next check the PI Station. This will verify the T (tangent length of the curve). This is done by subtracting the PC Station from the PI Station. *Do not subtract the PI Station from the PT Station.* Again, the PT Station is calculated along the arc of the curve and not from the tangent lines

PI Station =	109+00
PC Station =	104+09.5
T by Stations =	490.5

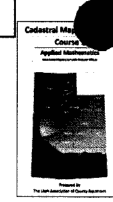
The highway plans show T = 490.5', therefore, they check.

1.9.63. Since L and T are verified by the Station numbers we can now calculate the other parameters of the curve to see if they check. Looking at the equations for solving curves you will see that there is not a single equation that uses both L and T. This means that we will have to start by using other data of the curve. Since the two parameters of the curve that the engineer and surveyor would have used in the initial curve design are the Δ and D it would be logical to begin with the given data for them.

1.9.64. Examining the equations R can be calculated using equation 1.9.40-1 and only D is needed to make this calculation. Note: some rounding in the displayed values is being done for format purposes, however, un-rounded values are being used in the calculations. The full final calculated value is being shown and would be the value you should obtain using un-rounded calculations.



Using $\Delta = 37°48'$ $D = 4°00'$		
Calculate R Using Equation 1.9.40-1 Record = 1432.7	Calculate L Using Equation 1.9.40-7 Record = 945.0	Calculate T Using Equation 1.9.40-8 Record = 490.5
$R = \frac{36000}{2 \pi D}$ $R = \frac{36000}{2 \pi\, 4°00'}$ $R = \frac{36000}{25.132741229}$ $R = 1432.39448783$ This calculation does **NOT** check with the plans. About 0.31 difference.	$L = \frac{100\,\Delta}{D}$ $L = \frac{(100)\, 37°48'}{4°00'}$ $L = \frac{3780.00}{4°00'}$ $L = 945.00$ **This calculation checks with the plans.**	$T = R \tan \frac{\Delta}{2}$ $T = 1432.7 \tan \frac{37°48'}{2}$ $T = 1432.7 \tan 18.9000000$ $T = (1432.7)\ 0.342376525$ $T = 490.522848412$ This calculation does **NOT** check with the plans. About 0.03 difference.
Using Equation 1.9.40-2	Using Equation 1.9.40-6	Using Equation 1.9.40-8
	$L = \frac{\pi R \Delta}{180°}$ $L = \frac{\pi\, (1432.39448783)\ 37°48'}{180°}$ $L = \frac{170100.0000000}{180°}$	$T = R \tan \frac{\Delta}{2}$ $T = 1432.39448783 \tan \frac{37°48'}{2}$ $T = 1432.39448783 \tan 18.900$ $T = (1432.39448783)\ 0.34237$



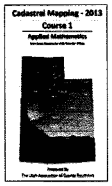
	L= 945.00000000 **Using the calculated R also checks with the plans.**	T = 490.418248217 Using the calculated R still does **NOT** check with the plans.
$R = \dfrac{T}{\text{Tan } \Delta / 2}$ $R = \dfrac{490.418248217}{\text{Tan } 37°48' / 2}$ $R = \dfrac{490.418248217}{\text{Tan } 18°54'}$ $R = \dfrac{490.418248217}{0.342376526}$ R = 1432.39448783 Using this equation and T calculated from using the calculated R this result checks, which indicates that we have not made an error in calculating T the second time.		

1.9.65. What this evaluation shows is that the radius (R) noted on the plans is incorrect. Even if we were to round the calculated R of 1432.39448783 to two decimal places it would still only be 1432.40 which is still 0.3 feet shorter than that noted on the plans. This still does not explain why the deed would only be indicating a radius 0.01 feet shorter.

1.9.66. If the calculations for each part of the curve had checked using the record values on the highway plans would be preferable over a value that disagrees with the plans.[2] In proceeding

---

2 The reason that the UDOT drawings would be used instead of the values in the deed is because the deed should be reflecting the highway right-of-way, that is what the bounds call states. It is possible that the deed was written in



with evaluating the location of the highway right-of-way the following values will be used for the centerline of the highway.

1.9.67. Record and Calculated curve data.

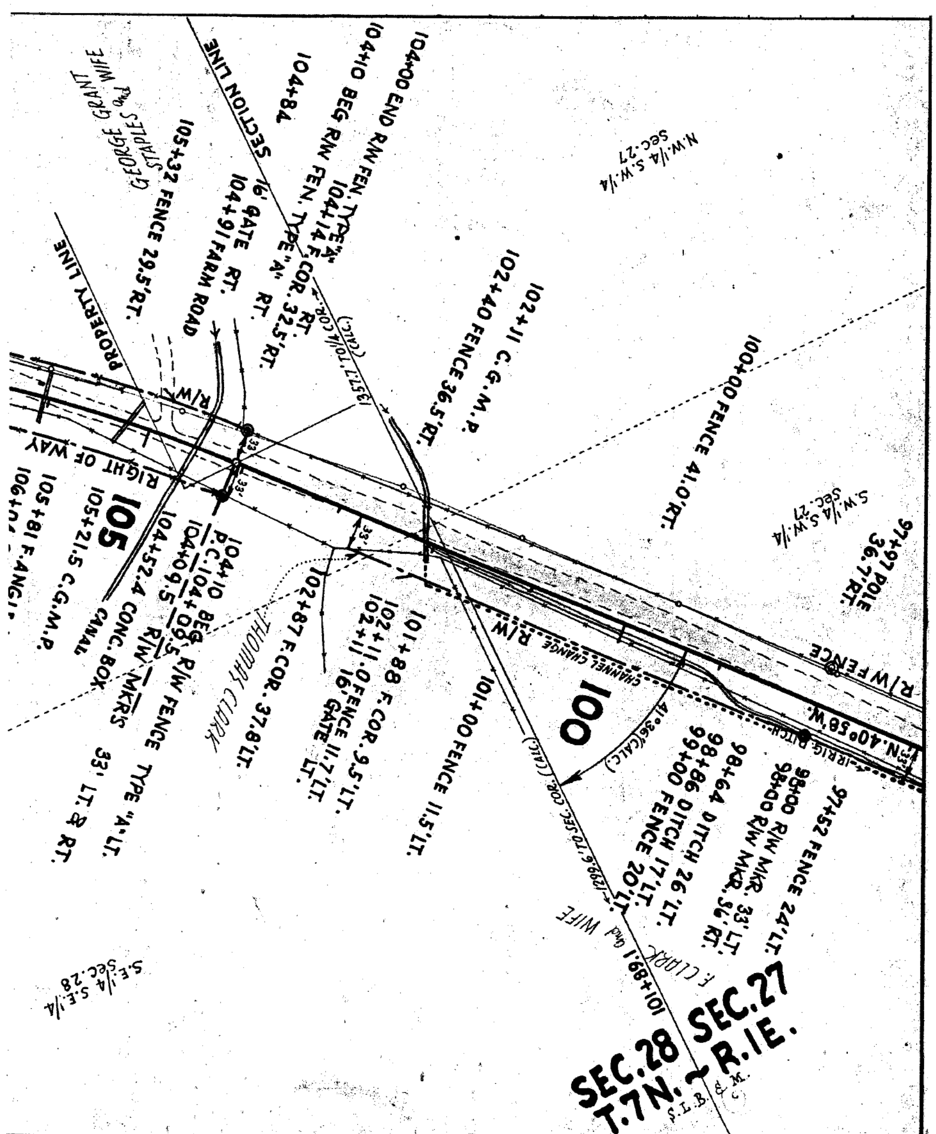
a. Delta = 37°48' LT

b. D = 4°00'

c. R = 1432.39 adj.

d. T = 490.5

e. L = 945.0



1.9.68. Examining the highway drawings between Station 101+00 and Station 102+00 they show the section line to cross the road. Looking at the information for the section line it is labeled with Station 101+89.1 which is the location of where the section line crosses the center line of the highway. Again examining the drawing there is a curved arrow just above the large bold station label of 100 which reads "41°36' (Calc)". This is an error. However, it is not safe to assume that the deed is what is in error. The error could also be on the highway plans but without verifying the plans (as has been done in this example) to determine if they contain some type of error the best source for the location of the highway would be the record produced by those who engineered and surveyed the highway. A mathematical check on the highway plans information should always be performed even if the deed does not disagree with the plans.

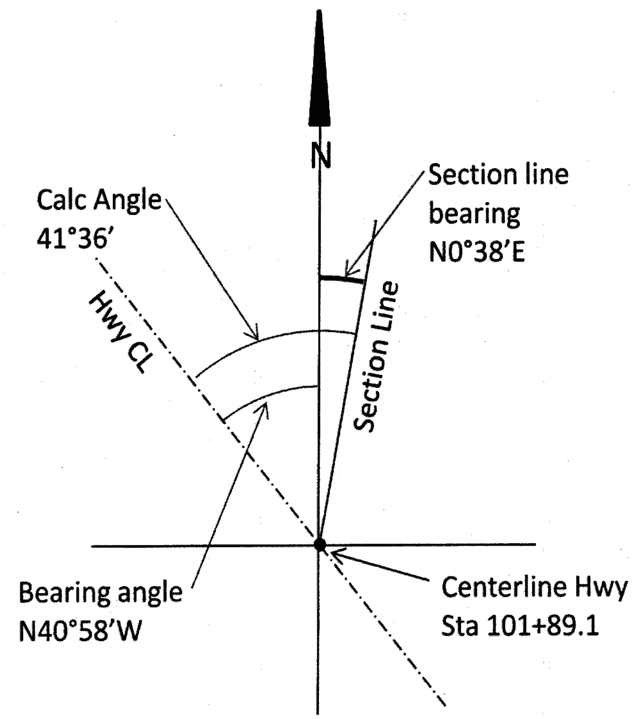
It should be noted that to properly place the highway it may be necessary to have your county surveyor locate the right-of-way monuments so that you can have an accurate location of the highway for your map.

angle that can be used to calculate the bearing of the section line as it relates to the highway plans.

1.9.69. To make the calculation for the bearing of the section line find the bearing of the highway centerline and proceed by drawing the compass and making the calculation.

Calc Angle = 41°36'
Bearing angle = <u>- 40°58'</u>
Section line Bearing = 0°38' in the NE Quadrant.

1.9.70. The plans also note the distance from the center line of the highway to the section corner as being 1299.6 which is the Southeast Corner of Section 28, T 7 N, R 1 E, SLB&M. This is the same corner that our deed description is tied to. With this information the highway can be drafted in relation to the section corner and section line information.



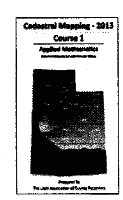
Section Line Calculation

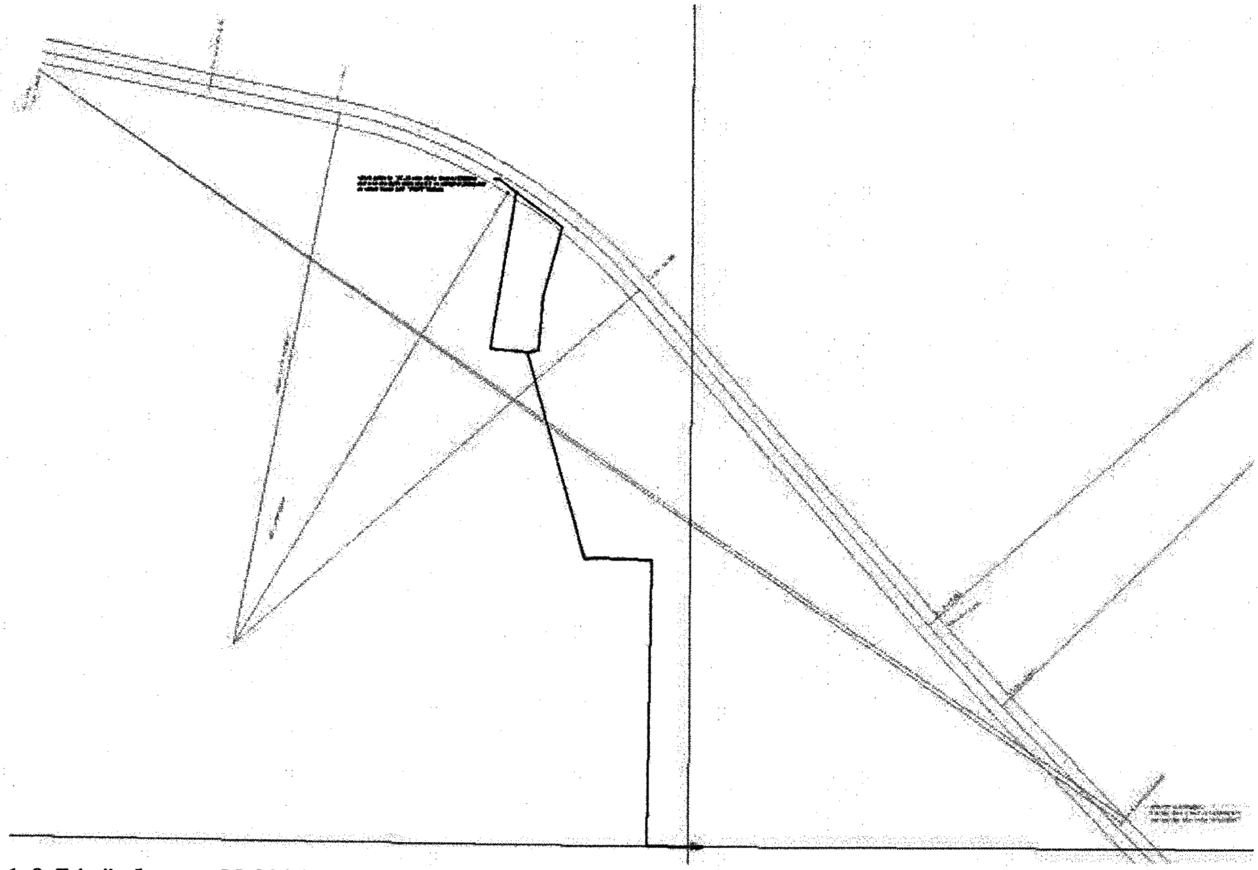
1.9.71. The county surveyor's data shows this section line to run N 0°21'14" E. From this we can calculate the rotation that is needed for the highway drawings to place them on the State Plane Bearing system. Since the bearings are both in the Northeast quadrant subtracting the bearings will result in the rotation angle.

Highway bearing of section line = 0°38'
State Plane (by county survey) = <u>- 0°21'14"</u>
Rotation Angle = 0°16'46" CCW

1.9.72. The direction of rotation is Counter-clockwise because the highway bearing is greater than the Section Line bearing.

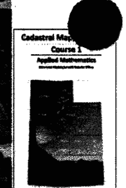
1.9.73. Adding the highway information to the map where we started the parcel drafting shows that the line being described as along the highway right-of-way is actually north of the right-of-way line. Returning to the description of the line we are mapping.





1.9.74. " ***thence N.8°30'E. 418.29 ft, to South R/W of State Highway 162, which point is 493.21 feet along the arc of a 1399.69 foot radius curve to the right from the PT of said highway, the long chord of which bears S68°40'19"E 490.66 feet;".***

1.9.75. Since Weber County has aerial photography available to use in mapping the location of the highway and the deed description can easily be plotted on top of the photos. Once the descriptions and highway have been rotated to the same basis of bearing as the aerial photos



they should give a good representation of the actual lines.

1.9.76. The bounds call describes the distance from the PT Station of the curve to the point as being 493.21 feet along the right-of-way. Because CAD requires additional information to identify this point. Even though we are given a radius (which we calculated to be 1432.39 - 33 = 1399.39) and the chord bearing and distance are also given it is a good practice to make sure that the chord bearing and distance actually identify the correct point being 493.21 feet from the PT Station.

1.9.77. The problem is that we don't know the bearing base for the chord bearing as it is noted in the description. It could be on the same bearing as the deed or on the bearing of the highway plans. If it is related to the highway plans then we are not able to use the bearing without rotating to the correct basis of bearing.

1.9.78. The best way to identify the point in the correct position and make sure that the point is positioned for the map and basis of bearing we are using on the map is to make the calculation for the Central Angle of the curve identified by the Arc Length of 493.21 and the Radius that we have calculated for the right of way curve of 1399.39.

1.9.79. We can make the calculation as follows:

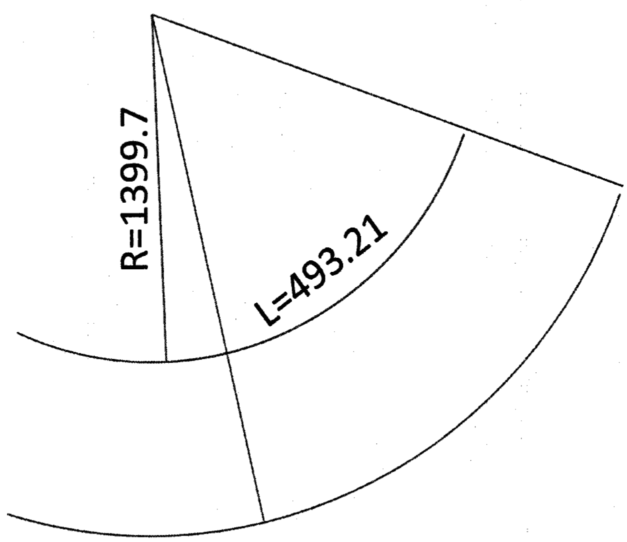
$$\Delta = \frac{180\ L}{\pi\ R}$$

$$\Delta = \frac{180\ (493.21)}{\pi\ 1399.7}$$

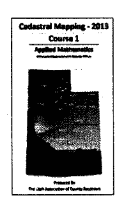
$$\Delta = \frac{88777.8}{4397.28723723}$$

$$\Delta = 20.1892201283°$$

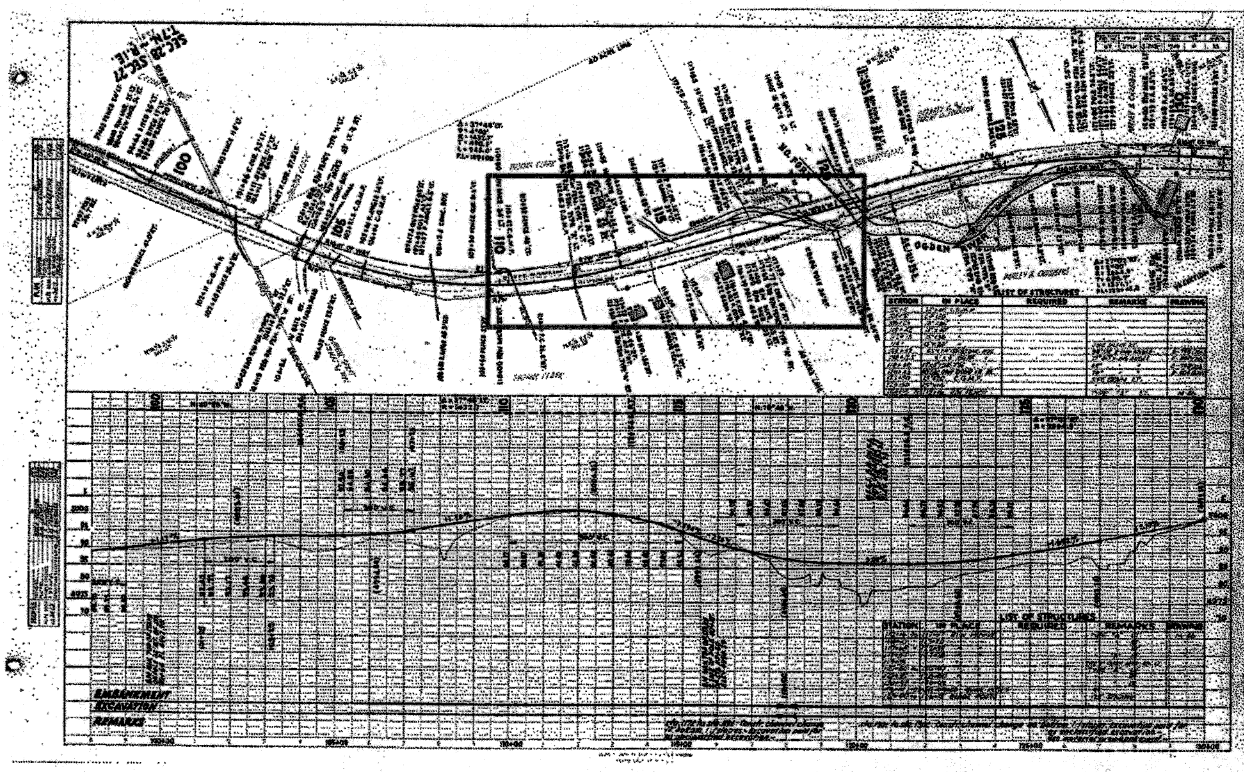
$$\Delta = 20°\ 11'21.192462''$$



1.9.80. Now that we have the central angle for the curve we just need to know the bearing from the PT of the curve to the RP (radius point of the curve). From our mapping we can calculate this bearing from our drawing. For the example that we are using I will be using the bearings of the highway drawings unrotated to demonstrate how to make the calculation. Just keep in mind if you have had to rotate the highway (as we would be doing in this project) the bearing would also have the same rotation.

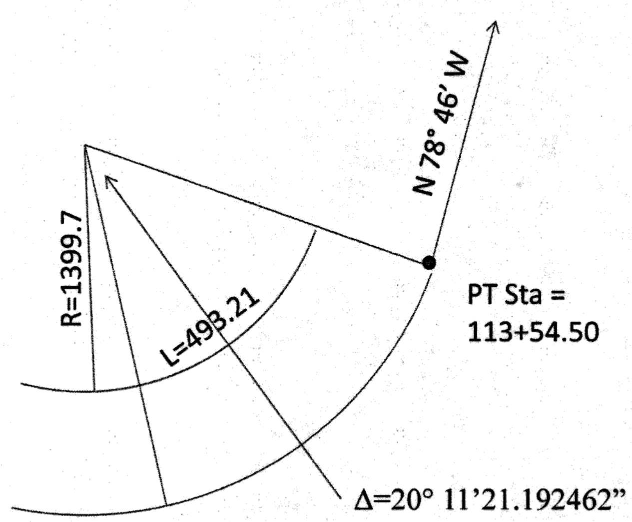


1.9.81. The bearing we will need is the bearing of the tangent line that runs westerly from the PT of this curve. It is found on the drawings in the red box.



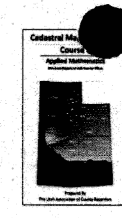
1.9.82. We will find it to be noted as N78°46'W.

1.9.83. We calculated the central angle and we now have the bearing from the PT Station to the radius point. To calculate the bearing of the radial line we will be using the principles of bearing calculations discussed in Chapter 7.



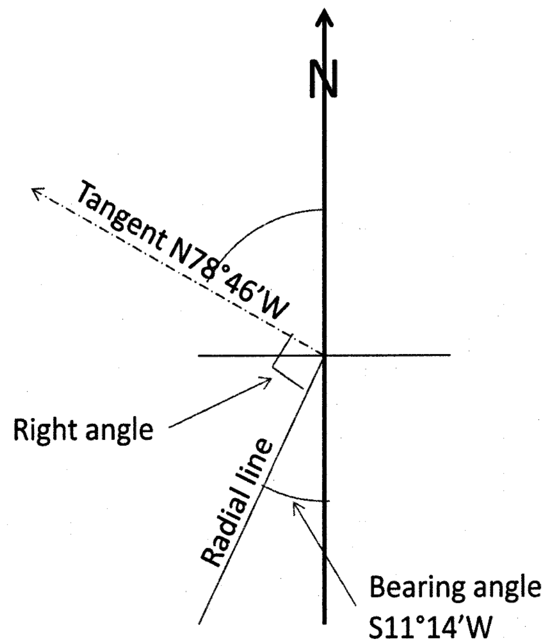
1.9.84. This is accomplished by starting with the compass.

1.9.85. Knowing the bearing angle of the centerline of the highway and that the angle to the radius point is a Right Angle, adding them together and subtracting from 180° will result in the bearing angle of the radial line. Knowing it is in the Southwest quadrant the bearing is also known.



$$\begin{array}{r} 78°46' \\ \underline{-90°00'} \\ 168°46' \end{array}$$

$$\begin{array}{r} 180°00' \\ \underline{-168°46'} \\ 11°14' \end{array}$$



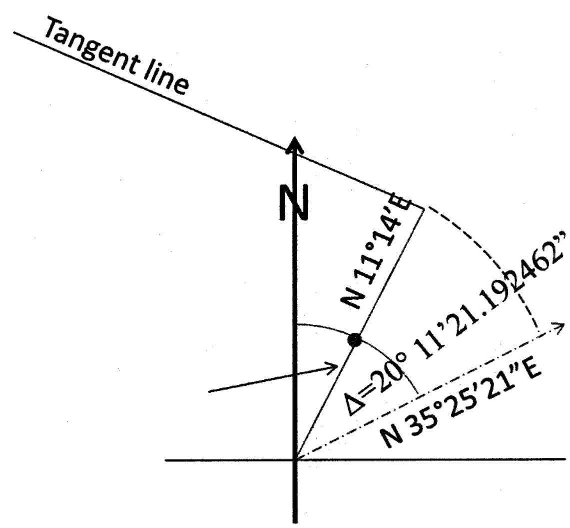
1.9.86. Therefore, the bearing of the radial line is S11°14'W.

1.9.87. Once again, this bearing is based solely on the record bearing of the centerline of the highway. This would not be the case if the highway had to be rotated to match the Datum of the Ownership plats.

1.9.88. The next part of the process is to calculate the radial bearing to the point we are trying to identify. This begins again with drawing the compass and identifying the angles that we know.

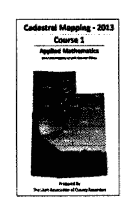
$$\begin{array}{r} 11°14'00'' \\ \underline{+20°11'21.192462''} \\ 35°25'21.192462'' \end{array}$$

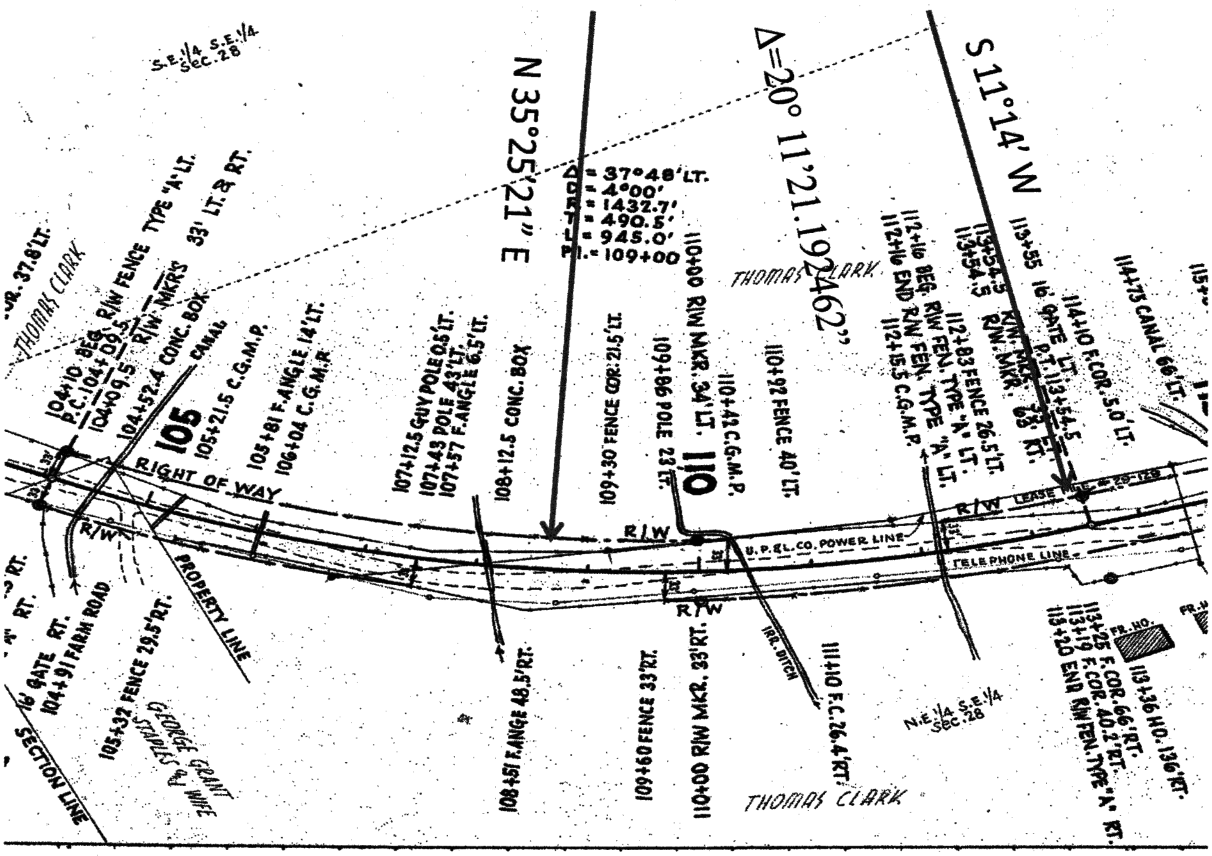
1.9.89. Reversing the radial bearing and plotting it on the compass identifies the bearing angle.



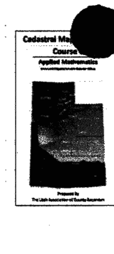
1.9.90. Adding the bearing angle of 11°14' to the central angle of 20°11'21.192462" then sums to the bearing N 35°25'21" E. It is appropriate to round for the bearing at this point because it is the final calculation.

1.9.91. The next part of the process it to plot the point using your drafting software or drawing board plotting the end of the line. Starting at that radius point drawing another radial line with the bearing of N 35°25'21" E for a distance of the radius (1399.39).





1.9.92. This process now identifies the point described in the deed. This location will probably not be located at the original bearing and distance of the deed. There will be a slight



adjustment to the location. So by using the bounds call of the deed both the original bearing and distance will change. This identifies that correct bearing and distance of the deed line.

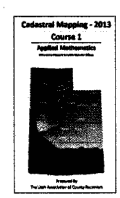
1.9.93. Placing this location on aerial photography along with the alignment of the highway. The true position of the point being described can be identified. The red line is now drafted correctly.

1.9.94. There is more information that could be evaluated for this description.

> Part of SE 1/4, Sec 28, T7N, R1E, SLB&M, US Survey: Beginning at a point S89°34'20"W 110 feet, N0°10'W 756.05 feet, N88°43'W 150 feet, S82°10'W 29.09 feet and N16°17'W 567.31 feet from the SE corner of SE 1/4 of said Section 28; Running thence N85°22'W 100.00 feet; thence N.8°30'E. 418.29 ft, to South R/W of State Highway 162, which point is 493.21 feet along the arc of a 1399.69 foot radius curve to the right from the PT of said highway, the long chord of which bears S68°40'19"E 490.66 feet***; thence along said arc 151.89 feet, the long chord of which bears S54°30'E 151.82 feet;*** thence S13°54'W 188.98 feet thence S4°04'W 146.38 feet; thence S82°W 29.09 feet to the point of beginning. Together with a 50 foot wide R/W adjoining said property on the West.

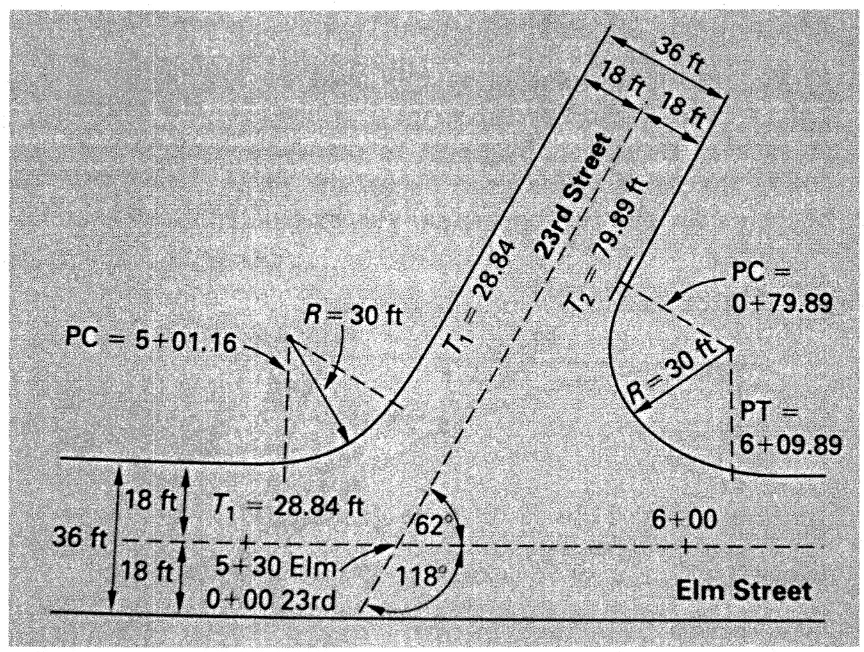
1.9.95. The next call along the highway right of way would be calculated in the same manner we just did for the end point from the PT of the curve. This would be the most accurate method of identifying the end of the highlighted call. Because the road and the deed have been rotated to match the Datum of our mapping system, simply using the chord bearing and distance as it is written will not identify the correct point.

1.9.96. This is a much larger subject that will be discussed in more detail in Course 2.



1.9.97. CURB RETURNS AT STREET INTERSECTIONS.

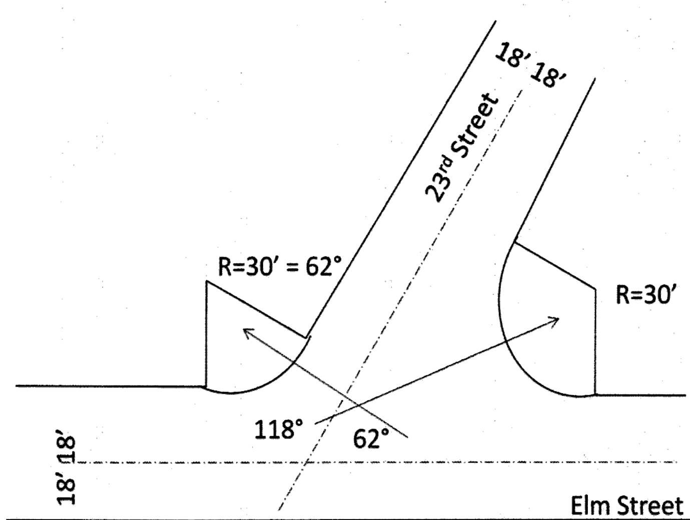
1.9.98. This subject will only demonstrate some of the geometry involved in the solution of the problems. The CAD solutions for these problems can be made using tools for offsetting lines and the fillet command. The geometry shown here is still the same when constructing these objects.



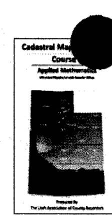
1.9.99. This graphic is from a text book[3] on surveying math. It shows the geometry associated with street intersections and how the angles correlate with one another. Notice the 62° angle location and the 118° location as they are referenced to the centerline of the roads.

1.9.100. Also notice that the street right of way widths are shown to be 18 foot half widths. So each line would be offset the 18 feet from the center line. If we remove much of the extra information we can se how the geometry works.

1.9.101. From this graphic you can see that how the angles relate to one another. If you go back to our discussion on the central angle and the Deflection angle you can see how the deflection angle of the curve along the centerline is the same as the central angle associated with that same curve.

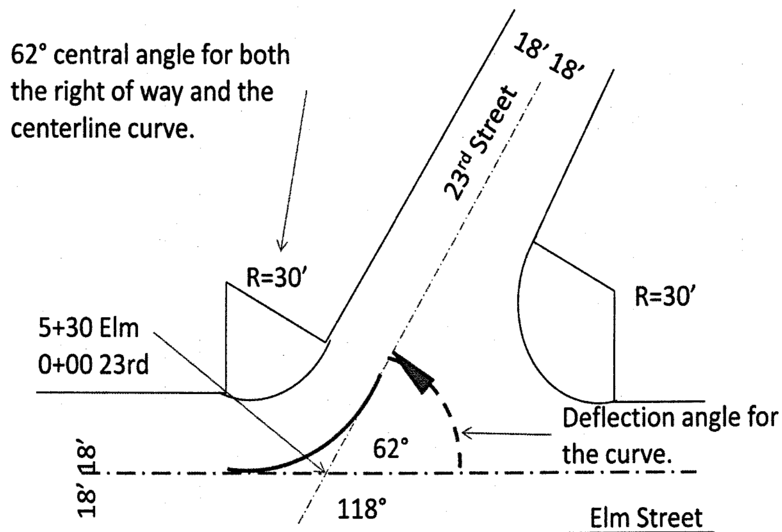


3 . Land Surveyors Reference Manual, Harbin.

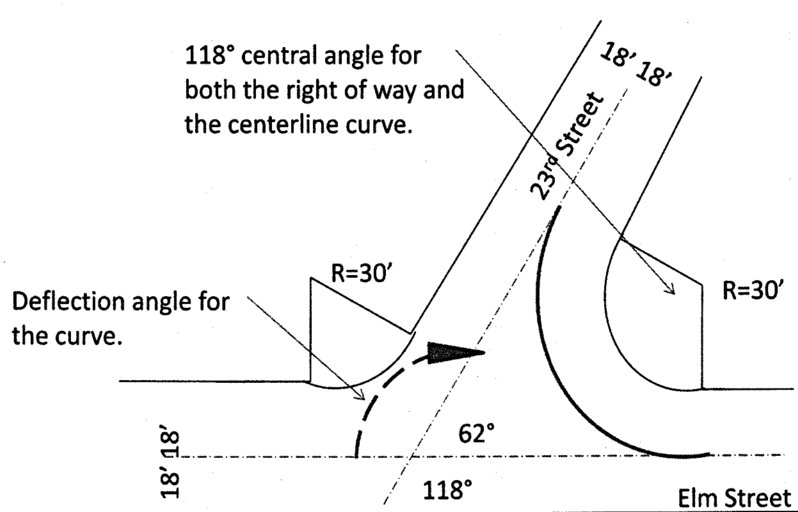


1.9.102. Perhaps the next two graphics will help to show how the angles relate.

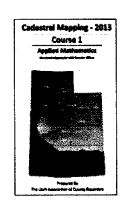
1.9.103. This one shows the deflection angle that described as starting on Elm Street and rotating counter-clockwise toward 23rd Street. If you remember from the discussion before the deflection angle and the central angle are equal.



1.9.104. In this graphic you can see how the deflection angle again begins on Elm street and rotates clockwise to 23rd Street.



1.9.105. This completes Course 1.



Course 1. Chapter 10. - APPLIED MATHEMATICS

1.10 What is North?
(An Introduction to Datums.):

1.10.1. The question may seem simple on its face, however, an accurate answer to, What is North?, depends on the context in which the phrase is being used. It can be a precise term as used in surveying and mapping or a general term as used in navigation. This subject deals with Basis of Bearing and is a critical component of producing an accurate map that reflects the truth of a description.

1.10.2. Every property description is written to some type of bearing base. It may or may not be obvious on the face of the document nor may it be obvious in looking at supplemental information, section maps, GLO plats, or survey documents. If fact, a single description may have more than one basis of bearing where some lines of the description use a basis different and separate from other lines. Generally, though perhaps not always, when a description of property is prepared by a surveyor the issue of basis of bearing, including multiple basis of bearing, are resolved so that the surveyed description only contains a single bearing base.

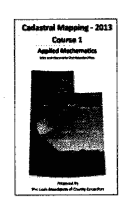
1.10.3. **GENERAL USE OF NORTH.** So what is North? When making a general reference to north one may say that Logan, Utah is North of Salt Lake City, Utah or that Texas is south of Maine.

1.10.4. Surveyors may use a form of general reference by saying that a road or line runs in a Northerly direction or it might be referred to as North. The Utah Department of Transportation (UDOT) uses these general terms in deeds and descriptions that they prepare for road and highway projects. UDOT descriptions and drawings will be dealt with as their own subject in Course 2. Appendix 6 contains a chart on how general directions are used in UDOT descriptions.

1.10.5. Many descriptions are written by non-professionals and this issue makes the determination of North more difficult for the mapper. These types of challenges may require that the mapper have additional resources to help identify directions properly.

1.10.6. For the term North to have meaning and context it is necessary to understand what a Datum is and how to use them. At this point only a general discussion on Datum's will be made but the details of Datum's will be given in Course 2 when the subject of GIS systems and surveying is developed.

1.10.7. **BEARING BASE.** To answer the question "What is North?", an understanding of what a bearing base is should be explored. The subject of bearing base begins with a general discussion on Datum's and direction of a line.

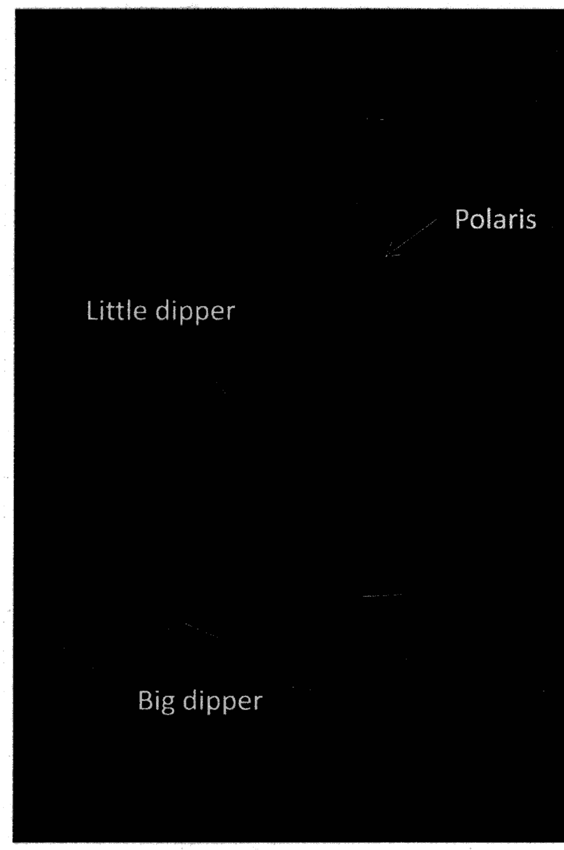


1.10.8. So when someone says the section line is North. The question in response should be, "**What North**?". You could continue the probing by asking, "Do you mean North related to Astronomical, Magnetic, True, Geodetic, State Plane Grid, Assumed, Record" (e.g. along a fence or in a deed), ... ?

1.10.9. The last question is really asking, "What Datum is "North" being referenced to?" Each type listed above is a separate Datum and is established with a given set of criteria. Some only differ slightly from one another but others have significant differences. It is important for the cadastral mapper to understand the different bearing bases that may be used in the preparation of a property description and how to apply that knowledge to the map.

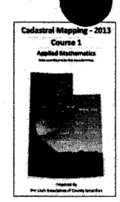
True North

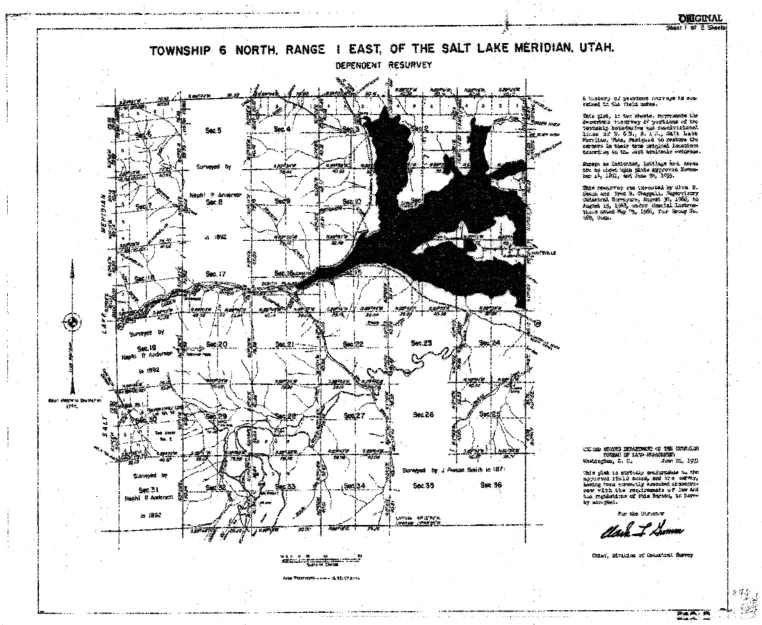
1.10.10. **Astronomical North**. This is a Datum related to the rotation of the earth on it's axis. Most people know what the North Star is but most don't realize that the North Star is not at the precise location where the rotational axis of the earth points when projected into space. There is currently about 47 minutes (0°47') of arc between the rotational axis point and the location of the North Star.



1.10.11. The rotational axis of the earth can be seen in a photograph that is taken with a long exposure pointed toward the North Star. This type of photograph leaves streaks where the star appears to have traveled across the film. In reality this phenomenon is a result of the earth spinning on it's axis. This causes the stars to appear to rotate (looking north) in a counterclockwise direction.

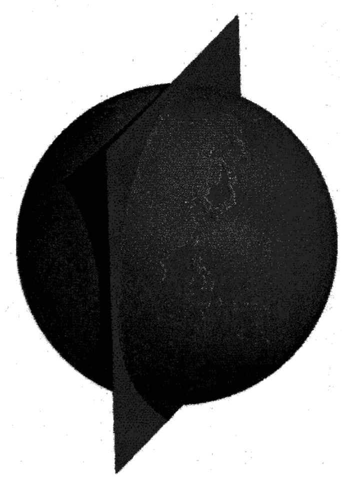
1.10.12. To determine where the point in space is located for Astronomical North a surveyor or astronomer can use the stars to precisely position the North Astronomical Point. Surveyors actually utilize data tables known as an Ephemeris (produced by the National Observatory) that will tell the surveyor the position of stars at a given time. With a few more pieces of information the surveyor can establish the north point of the rotational axis of the earth known as Astronomical North.





1.10.13. This is an important Datum because it is the basis for the Public Land Survey System (PLSS) which is the basis of all the tax plats (Cadastral maps or Ownership plats) when they were first drafted. As other Datum's were used in legal descriptions compensation for these different "North" directions needed to be made. We will explore the concept of compensating for differing bearing bases when discussing bearing rotation in Course 2. For now a basic understanding will suffice for this discussion.

1.10.14. As just mentioned Astronomical North was the basis of bearing used for the PLSS and is the basis on which all the "GLO" plats[1] were drafted. This includes more modern plats that are prepared by the BLM. This is the Datum that all the cadastral (Ownership) plats of the State had their origin and many are still reflective of this bearing base. However, as modern drafting and electronic systems are being used many of the maps begin to reflect either a complete change in the bearing basis which the map reflects or a hybrid showing multiple bearing bases in the maps or systems. This problem is a source of mapping error.



1.10.15. In fact, the modern electronic mapping system can not map correctly the GLO plats with the bearings as documented on the GLO plat without making a rotational correction from Astronomical North to the bearing system that the electronic mapping system is based on.

1.10.16. **Great and Small Circles.** To continue the discussion on Astronomical North and how it is determined a definition of Great and Small Circles comes into play.

1.10.17. Great Circles can be thought of as being a circle created on a flat plane which is passed through the earth and running through the north and south poles. As the earth spins on it's

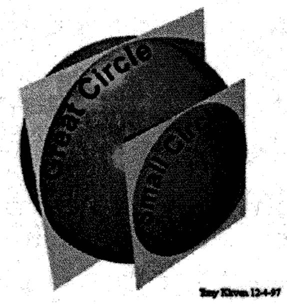
---

1 . The term GLO is used generically to identify the government cadastral plats. The GLO (General Land Office) was placed as a division of the Department of Interior and eventually given a name change to the BLM. The BLM is the custodian of the original federal survey plats for Public Land Survey States.

axis an infinite number of Great Circles may be created. In mapping, these Great Circles are known as Longitudes.

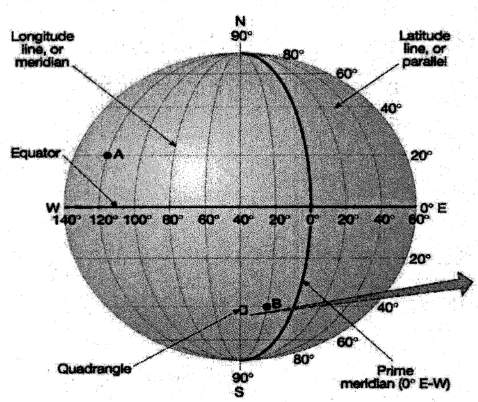
1.10.18. Because all Great Circles pass through the poles of the earth, with one exception, (the equator) they all converge at the poles. Every point on which a person may be standing on the surface of the earth has a corresponding Longitude that can be determined.

1.10.19. There is another type of circle that is known as Small Circles. This type of circle is used to create Latitudinal lines. From the Great Circles and Small Circles the Datum of Latitudes and Longitudes are developed for use in surveying and mapping.



1.10.20. Small circles can be thought of as a flat plane being passed through the surface of the earth at any location. However, this is not how a line of Latitude is developed. Latitudes are created using angular relationships between three points. The center of the earth, the equator, and the location of a point on the surface of the earth which is being identified.

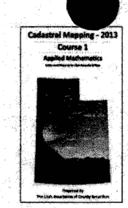
1.10.21. **Latitudinal Lines.** The angular relationship which creates a Latitude should be understood. The first line is created by starting a line on the equator and passing it through the center of the earth. The other line begins at the center of the earth and passes through the surface of the earth at any point (be that point the center of a street, building corner, easement angle, property corner), monument, or survey instrument. This creates an angle at the center of the earth.



1.10.22. Holding these two lines as fixed and allowing the earth to rotate around them causes another line to be traced on the surface, thus creating a Small Circle. This Small Circle also traces out a line of Latitude. The coordinate system of Latitude and Longitude will be discussed in more detail in Course 2.

1.10.23. **Longitudinal Lines.** When lines of latitude and longitude are drawn or traced on the surface of the earth they create a grid system. Longitudinal lines are traced from each person, monument, survey instrument, or any other position or point on earth and running through the poles a Great Circle will be created. There is only one Great Circle that does not pass through the poles. This Great Circle is known as the Equator.

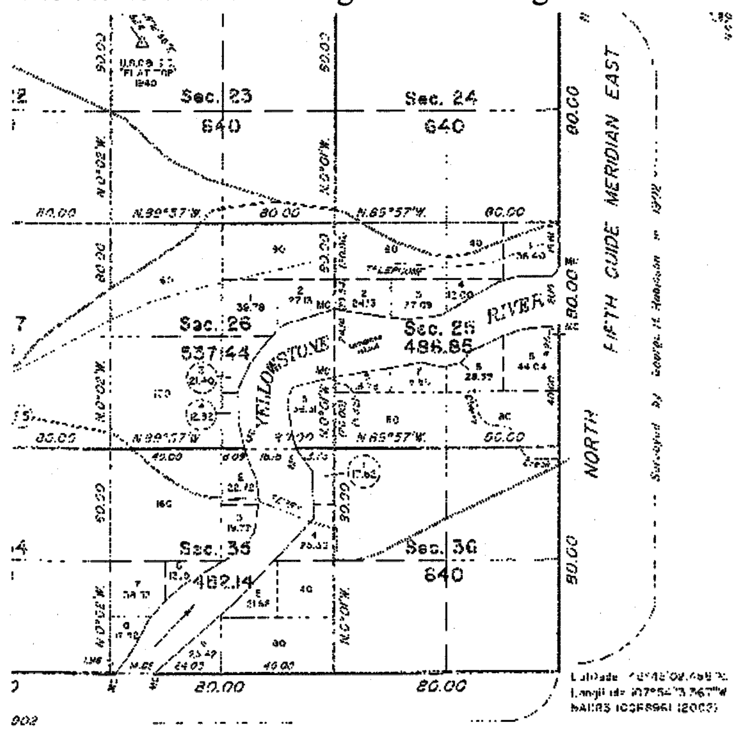
1.10.24. The convergence of these Longitudinal lines are also reflected on USGS Quadrangle Maps. When a close examination of the left and right sides of the maps are made it can be seen that the sides are not parallel to one another. This shows that the USGS Quadrangle



Maps are drafted on Astronomical North with converging Longitudinal lines or Meridians.

1.10.25. It should be noted that the Small Circles are also represented on the USGS Quadrangle Maps in that the top and bottom of the map are not straight parallel lines. They too are based on Astronomical North.

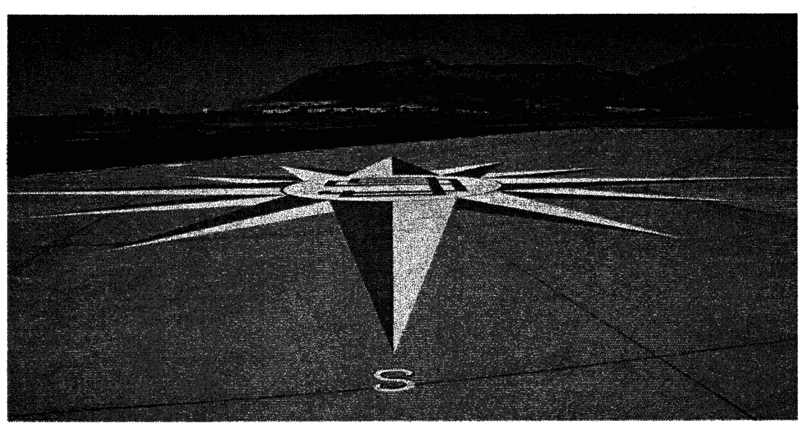
1.10.26. Because Latitudinal lines appear to be a curved line from any point on earth, not on the equator, then the top and bottom of the USGS Quadrangle Maps are also a curved line. In the Northern Hemisphere they are concave to the north and are tangent to the Longitude lines of the sides. Which means that the top and bottom of the map are concentric circles to one another.



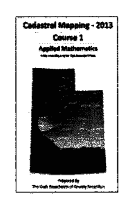
1.10.27. There is, however, a condition found on the GLO plats that should be understood to properly draft the section. This is that all bearings on the plat are "true" bearings which means that they are not on the grid – they are based on astronomical directions. The grid is what all mapping systems use for plotting bearings. In the astronomical system the bearings, related to true north. Each bearing shown on the map is corrected for convergence of the meridian.

1.10.28. Therefore, the bearings shown on the maps cannot be used as written to map the section lines. This principle will be developed more fully in Course 2.

1.10.29. **Magnetic North.** This is North as derived using the magnetic compass and is closely related to Astronomical North. However, Magnetic North is much more difficult to determine because direction based on the magnetic field of the earth does not draw a magnetized needle in the same direction at all times nor in all locations. Magnetic



1½ min



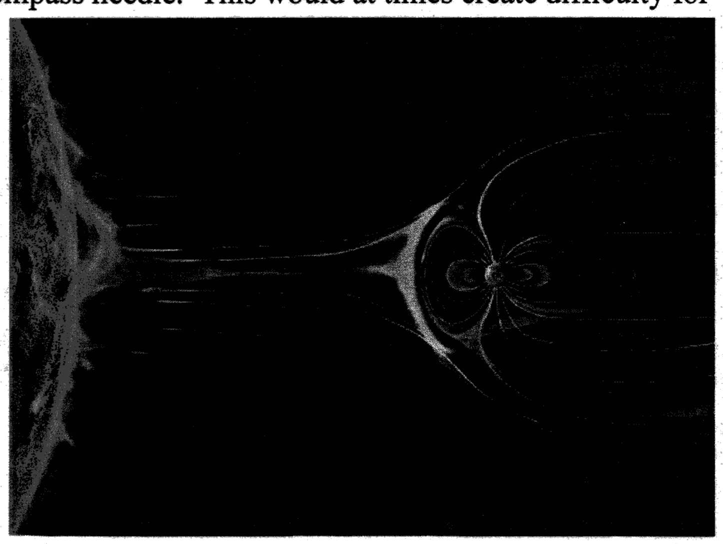
North is constantly changing. This constant change is why modern airports have what is known as a Compass Rose on the tarmac for the pilots of small aircraft to use.

1.10.30. Many small aircraft only have a compass for navigation aid. This compass must be constantly calibrated or adjusted to be able to determine North. To do this the pilot brings his aircraft up to the Rose so that the nose of the plane is lined up with the north point of the Rose. This North point is based on Astronomical North. Once the aircraft is lined up with North the pilot adjusts the compass to reflect where North is as identified by the compass and the relationship between Astronomical North and Magnetic North can be directly observed by the pilot.

1.10.31. This is essentially the same principle that Surveyor's used to layout the PLSS throughout the United States. For example, an observation would be made on the North Star to determine Astronomical North. Once Astronomical North was determined the survey instrument could be oriented to point north and the magnetic deviation from Astronomical North could be read directly on the survey instrument. This deviation would be noted on the plats as variance. The correct application of variance will allow the section to be properly drafted.

1.10.32. Many of the PLSS surveys would begin at the Southeast Corner of a Township by establishing the Latitude and Longitude of the monument at that location. An observation on another fixed point on the earth would be made and the instrument would then be pointed to the North Star. After making several observations and reducing the data through a calculation process Astronomical North is determined.

1.10.33. From the same location that the astronomical observation was determined and the survey instrument oriented so that it pointed Astronomical North the compass in the surveying instrument would show the difference between Astronomical North and Magnetic North. This difference is also called Magnetic deviation and is noted on the federal survey plats – this is Variance. Once this deviation or variance was determined much of the progress of the laying out of the Sections would be done with the compass needle. This would at times create difficulty for other surveyors trying to follow the exact line that was originally established on the ground.



1.10.34. The inaccuracies in surveying North would be evident because the compass is affected by many physical elements and circumstances. Some of which are ore deposits, especially iron ore, which would cause the compass to point more strongly toward the ore deposit causing a directional deviation. The federal surveyor would have no idea that the compass was giving a false direction.

1.10.35. Other effects which cause a false reading on a compass is the intensity of magnetic storms. We know more about magnetic storms in our modern age than was known 100 to 150 years ago. Sunspot activity will increase or decrease the magnetic fields strength or direction that the compass will point. We see the effects of Sunspot activity by how strong or active the Aurora Borealis appears to be.

1.10.36. So even though the record may say that the section line was run North, the method used, especially if magnetic compass was used, will have a significant effect on the actual direction of the line.

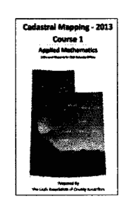
1.10.37. **True North**. The phrase True North has no legal or scientific basis. There is, however, a common definition for the term and it equates the phrase to Astronomical North. Many times when True North is talked about most will assume it to mean Astronomical North. For purposes of discussion in this text the phrase True North will be used in connection with Astronomical North and vice versa.

1.10.38. **Geodetic North.** Geodetic North is a direction which the National Geodetic Surveying service (NGS) has established through the process of making physical measurements of the physics of the earth. The earth is not a perfect sphere, in fact, it is far from round. Before the advent of modern mapping and surveying technology, when the earth was being mapped for navigation and cadastral purposes the fact that the earth was not round did not matter. Maritime navigation was done by the stars and ground navigation was accomplished by crude maps representing land features, trails, and roads.

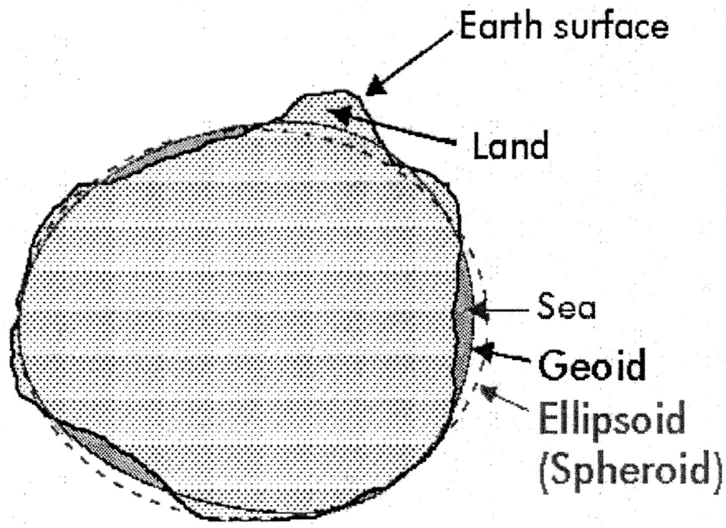
1.10.39. The ability to travel great distances over short time periods has brought a greater need for more accurate navigational ability. Geodesy is the study of the shape and size of the earth. Because of the need for greater accuracy in navigation the NGS has used Geodetic science to create a shape known as a Spherical Ellipsoid (Ellipsoid) which best approximates the shape and size of the earth.

1.10.40. The Ellipsoid has a major and minor axis just as any ellipse has and the intersection of these axes has been fixed mathematically to a point near the mass center of the earth. And the direction of North, represented by the minor axis, has been oriented to closely coincide to Astronomical North. Notice that I have not stated in either the location of the center of the earth or Astronomical North that the Ellipsoid conforms precisely to them.

1.10.41. Because the Ellipsoid is a mathematical shape it conforms to mathematical principles but the earth does not. So Geodetic North is not equal to Astronomical North, there is a slight difference. GPS (Global Positioning Systems) which surveyors use work on the



Ellipsoid system, not Astronomical North. It is important to know this difference in precise mapping systems such as GIS (Geographic Information Systems).



1.10.42. All GIS systems use some type of State Plane Coordinate System to map points as x and y values. The computer has the ability to relate the Plane Coordinate System to the Spherical System of Latitude and Longitude but when it does the Latitude and Longitude used are based on the Ellipsoid and not on the Astronomical system of Latitude and Longitude.

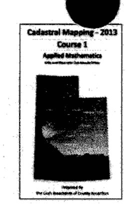
1.10.43. Knowing that there are slight differences between the Latitudes and Longitudes of the Astronomical and Geodetic systems can help the student of mapping understand that it is not an easy process to convert Astronomical bearings to Geodetic or Grid bearings.

1.10.44. The reason that these differences are important has been mentioned previously which is the fact that the original federal surveys of the PLSS were conducted on Astronomical bearings and the early cadastral maps were drafted on this same bearing system. The modern computerized systems that are used to perform cadastral mapping use the Geodetic system of coordinates and a conversion is necessary to properly create a cadastral map.

1.10.45. When it comes to the conversion of section corner locations the conversion can be done in the process of survey where the surveyor will make measurements on the section monuments and can provide the geodetic bearing data to the mapping system. What is not quite as easy is that of converting the deed descriptions from Astronomical bearings to Geodetic bearings. How to do this is discussed in detail in Course 2 and relates again to the subject of bearing rotation and parcel translation.

1.10.46. The modern GIS system is built on the basis of a Plane Coordinate System. There are many different plane systems of which two will be discussed in Course 2 in the surveying subject but for the continuation of this discussion on North we will introduce the basics of State Plane Coordinate Systems.

1.10.47. **State Plane Grid North.** State Plane Bearings are derived from a complex set of physics, mathematics, and earth science. Details of the system and how it functions will be discussed in more in Course 2 For this discussion a basic understanding of what Grid North is



will be given.

1.10.48. Grid North is a direction that has been fixed by legislation in reliance on science. In Utah the direction for North and the Grid System has been established in UCA 57-10. North is defined by establishing a Grid Coordinate System in which the origin of the coordinates occur "at the intersection of the meridian 111 degrees 30 minutes west of Greenwich ..." and a latitudinal line for the zone. This line of meridian or longitude is the same value for both NAD27 and NAD83 coordinate systems. What these systems are, as with other detail information on surveying systems, will be discussed in Course 2.

1.10.49. Suffice it to say that for the Utah State Plane Coordinate Systems the longitudinal line of 111°30' West has been selected as representing Grid North. Remember that longitudinal lines are created by Great Circles and converge at the poles of the earth (GLO section lines). Geodetic North is a mathematical direction based on the Ellipsoid and Grid North is established to equate to Geodetic North on the 111°30' west longitudinal line.

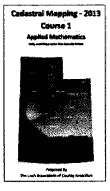
1.10.50. In the Grid Systems only one line of the grid is oriented to Geodetic North and that is the 111°30' west longitudinal origin (meridian) line. All other grid lines of the system are parallel to this origin meridian which is known as the Central Meridian.

1.10.51. The further in the east or west direction that a point is located from the Central Meridian the greater the angular difference becomes between Geodetic North and Grid North due to convergence of the meridians. The angular difference between Geodetic North and Grid North is known as the Convergence Angle. This angle can be calculated for any point on the surface of the Grid System.

1.10.52. Astronomical, True, and Geodetic North are systems which conform to a spherical shape of the earth. To work in these systems the use of Spherical Trigonometry is required. The State Plane Coordinate Systems, while they are related to Geodetic Systems, are a modified plane surface. It is this plane surface that works well with Cartesian coordinate systems which is what the computer systems, both GIS and CAD, work in.

1.10.53. Cartesian systems are based on and x axis, y axis, and z axis. Because State Plane Systems are Cartesian (x, y, z or Northing, Easting, and Elevation) plane trigonometry can be used in the process of computer mapping.

1.10.54. **Assumed North.** This type of north is not nearly as complex as the others previously discussed. This direction is as simple as assigning a direction between two points and then utilizing that direction for the remainder of the survey project, description, or map. Many surveys were conducted utilizing an assumed bearing especially prior the advent of GPS technologies.



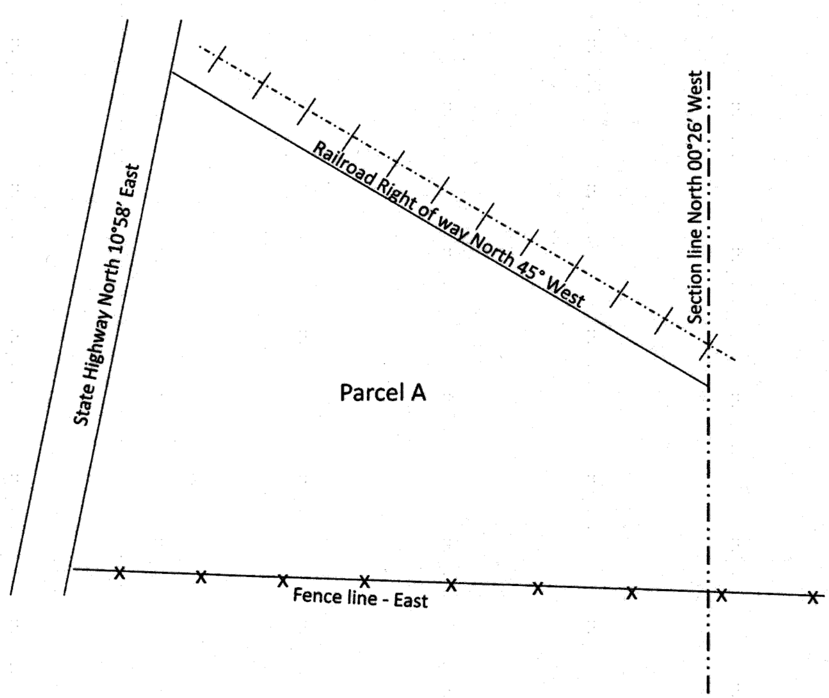
1.10.55. It can be as simple as saying that the direction between the two ends of a fence line is going to be assigned North. The same is true with survey monuments. Many times a local property survey would simply have assumed a direction between two section corners and the survey of the parcel was performed using that as the basis of bearing. Although this process of deriving a bearing base may still be used it is far less common as in the past in urban settings. This process is still used in the rural areas of the State.

1.10.56. The reason that a surveyor would have used an assumed North may not always be known but some of the reasons that I am aware of are 1) the lack of monumentation that has known coordinates readily available, 2) the complexity of determining True North, 3) a small local project or survey is costly to have North determined and may not warrant the expense, and 4) time dead lines may drive short cuts.

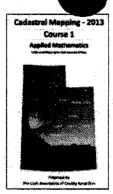
1.10.57. This type of process could mean that properties that have contiguous boundaries on the ground may have descriptions which are not contiguous on a map without knowing the basis that was used for each description. It also means that there could be thousands of individual bearing bases in a single section. Unless they are properly related to the basis of bearing of the map or section the drawing will show gaps and overlaps that do not exist.

1.10.58. **Record North.** Sometimes deeds will contain very definitive information on what the basis of bearing is that was used to prepare the description. More often than not, however, a description may be completely void of bearing base information or it may be hidden to those not trained in identifying such.

1.10.59. The deed of record can contain bearing information from, subdivision plats, survey maps, and other records that will help the cadastral mapper know what the basis of bearing is that a description is derived from. In a perfect world all descriptions of record would contain a positive statement regarding bearing base that can then be used to correctly orient the description to the map as described.



1.10.60. A single parcel may contain several different

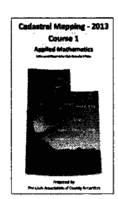


bearing bases within a single description if the scrivener does not understand the source of the bearings being used in the records or does not understand datums. Many times a layperson will use maps, other descriptions, surveys, etc. and combine them to describe a parcel. For instance; a railroad is shown on a map to run in a North 45° West direction, a state highway is described as running North 10°58' East, a section line is identified as North 00°26' West, and a fence line referred to in the adjoiners deed as East.

1.10.61. If the description of record uses these values it may seem on its face that there are no issues to be dealt with when mapping the description. However, when examining the documents that were used when each line was originally identified it may be found that the highway used State Plane Bearings on NAD27, that the railroad used a basis of bearing related to old GLO plat records, that the section line is using Astronomical North, and the fence is simply an assumed bearing. Therefore, a description using the bearings as noted would contain at a minimum 4 separate bearing bases.

1.10.62. To correctly map a description of this type the cadastral mapper must understand the differences of bearings and how to relate each document that originated the line of the parcel to the basis on which the cadastral mapper uses on their map. In this case the highway and railroad must be mapped before the parcel can be properly drafted. This is also true with the Section line and fence line.

1.10.63. **Conclusion.** This should give the student an understanding of the complexity of using bearings, that not all description bearings contained in a deed is an absolute, and that each line of a description may require an independent bearing rotation to correctly reflect the true line of the parcel.



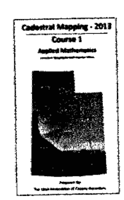
APPENDIX 1a

HALL et al.

v.

NORTH OGDEN CITY et al. (WYATT et al., Interveners)

No. 6834
Supreme Court of Utah
109 Utah 304; 166 P.2d 2211
February 14, 1946, Decided



APPENDIX 1:

HALL et al.
v.
NORTH OGDEN CITY et al. (WYATT et al., Interveners)
No. 6834
Supreme Court of Utah
109 Utah 304; 166 P.2d 221
February 14, 1946, Decided

DISPOSITION: Affirmed.

COUNSEL: Thatcher & Young and Parley E. Norseth, all of Ogden, for appellants. Samuel C. Powell and Derrah B. Van Dyke, both of Ogden, for respondents.

JUDGES: Turner, Justice. McDonough and Wolfe, JJ., concur. Larson, C. J., dissents. Wade, Justice (dissenting).

OPINION BY: TURNER

OPINION

Plaintiffs and appellants instituted this suit to enjoin the town of North Ogden and its officials from opening up as streets certain tracts of land indicated as streets by the plat of the survey of the townsite of North Ogden filed April 27, 1870. On August 2, 1872, by patent, the United States conveyed to Franklin D. Richards, County Judge of Weber County, Utah, "in trust for the several use and benefit of the occupants of the Town of North Ogden, Weber County, Utah Territory, according to their respective interests", certain lands in Township 7 North, Range 1 West, in Weber County, which lands were embraced within the area of lands platted as the townsite of North Ogden into lots, blocks and streets. The patent recited that it was executed in accordance with the provisions of the act of Congress of April 24, 1820, and the act of Congress approved March 2, 1867, entitled "An Act for the Relief of the Inhabitants of Cities and Towns upon the Public Lands," commonly known as the Federal Townsite Act. Since the plaintiffs and appellants claim title by adverse use to those portions of the plat designated as streets which have not been used for many years as public rights-of-way, if ever actually used for such purposes, it is necessary to make reference to the Federal and Territorial townsite acts.

The Federal Townsite Act, 14 Stat. 541, provides, inter alia:

> "That whenever any portion of the public lands of the United States have been or shall be settled upon and occupied as a townsite, and therefore not subject to entry under the agricultural pre-emption laws, it shall be lawful, in case such town shall be incorporated, for the corporate authorities thereof, and if not incorporated, for the judge of the county court for the county in which such town may be situated, to enter at the proper land office, and at the minimum price, the land so settled and occupied, [***3] in trust for the several use and benefit of the occupants thereof, according to their respective

interests; the execution of which trust, as to the disposal of the lots in such town, and the proceeds of the sales thereof, to be conducted under such rules and regulations as may be prescribed by the legislative authority of the State or Territory in which the same may be situated: ... ."

The act also provides for the amount of land which may thus be acquired, according to the number of inhabitants of such town. The amendment to the act, approved June 8, 1868, 15 Stat. 67, specifies that in addition to payment of the minimum price for such lands, "there shall be paid by the parties availing themselves of the provisions of said acts all costs of surveying ... by the United States, before any patent shall issue therefor."

The fact that patent issued in 1872 to the county judge in trust, is evidence that the costs of surveying were paid.

The Territorial Legislature by act approved February 17, 1869, adopted rules and regulations, known as the Territorial Townsite Act, C. L. Utah 1876, § 1166 et seq. The rules and regulations so enacted provide, inter alia, that "it shall [***4] be the duty of such corporate authorities or judge (as the case may be), and they are hereby directed and required to dispose of and convey the title to such land, or to the several blocks, lots, parcels or shares thereof, to the persons entitled thereto, to be ascertained as hereinafter prescribed."

The act provides for publication of notice, and for the presentation of claims within six months specifying in writing the parcel or parts of land in which the claimant asserts an interest, and the specific right, interest or estate therein; and that "all persons failing to make and deliver such statement within the time limited in this section shall be forever barred the right of claiming or recovering such land, or any interest or estate therein, or in any part, parcel or share thereof, in any court of law or equity" except in case good cause is shown for delay, the time may be extended not to exceed one year. Section 1168. Section 1170 provides for examination and adjudication of each claim, and section 1169 provides for appeal to the district court in the event of adverse claims if a party should feel aggrieved at the decision of the court.

Section 1173 clearly indicates that the Legislature never intended any title to be acquired to the streets laid out on the plat of a townsite, for in addition to requiring payment of the amount chargeable to such claimant, "before the corporate authorities or judge holding any such lands in trust as aforesaid, shall be required to execute, acknowledge or deliver any deed of conveyance thereof", the Legislature specified the method of computation of the amount to be paid, "by taking the whole amount of the cost of the land of which it is a part, and expenses stated in the account as prescribed in section 7 (1172) in the proportion which the area of such lot shall bear to the whole amount of land entered after deducting the area of all streets, alleys and public grounds in such city or town and the reasonable charges for preparing, executing and acknowledging such deed."

Section 1174 provides: "That whenever the title to such lands shall be held by the corporate authorities of any town or city, all lands designated for public use by such corporate authorities as streets, lanes, avenues, alleys, parks, commons and public grounds, shall vest in and be held by the corporation absolutely, and shall not be claimed adversely by any person or persons whatsoever; and the judge of probate who shall have entered any lands in trust for any town or city which may afterwards become incorporated, shall, under the same conditions, convey by deed to the corporation thereof the lands designated for the use of the public as aforesaid;" and that in case of death of the judge before complete execution of the trust, title to such lands shall vest in his successor in office who shall execute the trust in the same manner and under the same conditions imposed by the statute upon the judge or mayor receiving the trust in the first instance.

Section 1175 provides that if there is any unclaimed land after the expiration of the time for filing claims, the corporate authorities or the probate judge (if the town is unincorporated) "shall cause the same to be surveyed and laid out into suitable blocks and lots, and shall reserve such portions as may be deemed necessary for public squares, school houses or hospital lots, and shall cause all necessary streets, roads, lanes and alleys to be laid out through the same, a plot of which properly certified, shall be recorded in the recorder's office of the county in which the same may be situated;" and the mayor or the probate judge (as the case might be) "may sell the lots or blocks so laid out, and not reserved for public use in suitable parcels, to possessor of adjoining lands or to other citizens of such city or town", etc.

By the act approved February 18, 1876, the time for presentation of claims was extended, to relieve from default those who had not made timely application for the lands occupied by them. C. L. 1876, p. 385, § 1178.

At the trial, an abstract of title of two of the plaintiffs was introduced in evidence, and it was stipulated that such abstract is typical of the abstracts of title of the other plaintiffs. This abstract of title shows the plat and survey of North Ogden as filed in 1870, the United States patent to Franklin D. Richards, County Judge, "in trust for the several use and benefit of the occupants of the Town of North Ogden, Weber County, Utah Territory, according to their respective interests", etc.; and land claims filed under the Territorial Townsite Act. These land claims describe the lands entirely with reference to the plat of North Ogden survey, and the claims made are to specified lots in designated blocks. The adjudication of ownership is also according to the plat of North Ogden, and so are the judge's deeds, describing specified lots in specified blocks in plat "A" and in plat "B", of North Ogden Survey. All of the entries in the abstract of title show that the grantors in each case conveyed title by lot numbers, and a typical description in a deed is: The west half of lot 1, all of lots 2, 3, 4, 5, 6, and 7, and the west half of lot 8, block 38, plat "A", North Ogden Survey. The only variation occurs in the deed to two of the plaintiffs dated October 30, 1941, in which appears the following after the description of the lands conveyed with reference to lots in block 38: "Reserving to grantors a right of way over a roadway 1 rod wide adjoining said land on the South and being part of the land originally laid out as a street on the recorded plat."

As far as record title is concerned, the plaintiffs acquired fee simple title to specified lots in designated blocks as platted in the North Ogden Survey or the townsite survey. Where there is a recorded plat, the conveyance of land by designation of lot number and block number and name of the plat or subdivision passes the title of the grantors the same as if such lots had been described by metes and bounds.

None of the original settlers in North Ogden acquired the fee in the streets, in view of the express language of the Territorial Townsite Act, for the act specifically provides that the streets, lanes, avenues, alleys, parks, commons and public grounds shall vest in and be held by the corporation absolutely "and shall not be claimed adversely by any person or persons whatsoever; and the judge of probate who shall have entered any lands in trust for any town or city which may afterwards become incorporated, shall, under the same conditions, convey by deed to the corporation thereof the lands designated for the use of the public as aforesaid;" and in the event of death of the judge before complete execution of the trust, title vests in his successor in office who is charged with the duty of executing the trust -- that is, to convey to the corporate entity when the town is incorporated, such streets and other parcels reserved for public use. The fact that North Ogden was not incorporated until 1934 could not alter the effect of the statute, for provision was specifically made for future incorporation. Upon incorporation, the town became entitled to a seed of conveyance from the successor in office to the probate judge who received title in the first instance, to execute the trust and to vest in the municipal corporation the fee simple title to all streets, lanes, avenues, parks, commons and public grounds designated on the plat which were not vacated by proper authority. The district judge sitting in probate is the successor in office to the territorial probate judge.

The adjudication of all claims under the Territorial Townsite Act was with reference to lots and blocks in the plat of North Ogden, and such adjudications and the deeds executed pursuant thereto did not operate to vest in the owners of the lots, any fee in the streets. For any person to have acquired title to the streets, such acquisition of title would have necessarily been based on something apart from and subsequent to the adjudications of ownership under the Territorial Townsite Act. As stated in Holland v. Buchanan, 19 Utah 11, 56 P. 561, 562: "The officer who enters the land is the trustee, and the occupants are the cestuis que trustent, who are entitled to have the trust executed, and the land disposed of, under such rules and regulations as the state or territory where the land is situated may prescribe. The legislature of Utah has enacted the necessary rules and regulations for the disposal of the land which may be so entered, and has provided that the lots shall be conveyed to the rightful owner of possession, occupant or occupants, or to such person as might be entitled to the possession or occupancy."

No townsite could exist without streets, and all the settlers in a town are interested in having streets properly located. The statute not only prohibited adverse possession of the streets and precluded acquisition of title by adverse possession, but made it the duty of the officer entering the land to convey to the municipal corporation of such town upon incorporation, the title to such streets and lands reserved for public use. In the interim the title is held by such officer in trust for a public purpose or use. Since all citizens have a duty to protect public property from destruction and loss, title to such lands held in trust for public use can be acquired only in accordance with the statutes applicable thereto; and except where specially authorized by statute, a person may not acquire title by adverse user since whatever use is made must be presumed to be that use which citizens may properly make or such permissive use as will not defeat the objects of the trust which all citizens have a duty to safeguard.

The argument that streets could not properly be located on the plat of a townsite unless the street was already in use, must be rejected as unsound; for the Territorial Townsite Act contemplated that each town settled on the public domain would be planned with reference to future growth, and that not all of the lands would be occupied, but that some lots would be sold in the future. The Territorial Session Laws 1890, pp. 76-77, appearing substantially the same in U. C. A. 1943, §§ 78-5-1 to 4, codified an already existing practice of laying out subdivisions. By said act it was provided that "It shall be lawful for any owner or owners of any land, or any trustee or trustees selected by such owners, to lay out and plat such land into lots, streets, alleys, and public places"; and to make and record an accurate map of the same, setting forth: "All the parcels of grounds so laid out and platted by their boundaries, course and extent, and whether they are intended for avenues, streets, lanes, alleys, commons or other public uses, together with such as may be reserved for public purposes."

The act also specified (now section 78-5-4, U. C. A. 1943): "Such maps and plats when made, acknowledged, filed and recorded with the county recorder shall be a dedication of all such avenues, streets, lanes, alleys, commons or other public places or blocks, and sufficient to vest the fee of such parcels of land as are therein expressed, named or intended, for public uses for the inhabitants of such town and for the public for the uses therein named, or intended."

In lieu of "town" in the present statute appear the words "county, city or town." The Laws of 1894, p. 14, provided for vacating a plat or any portion thereof, or any street or alley in such plat, by petition of the owners of the land contained in the plat, and by owners of land contiguous or adjacent to any street or alley sought to be vacated, presented to the city council or to the county commissioners. No order could be made without hearing and no order vacating the plat or any portion thereof or any street or alley could be made without a finding that neither the public nor any person will be materially injured thereby. See U. C. A. 1943, §§ 78-5-6 to 8.

Appellants apparently recognize the rule that a complaining party must win on the strength of his own title, not on the defects of his adversary's title; for appellants attempted to show that North Ogden lost whatever title it had to streets which were either never opened or ceased to be used as streets, and they also claim title to the center of the streets adjoining the lots to which they established record title.

The only way in which North Ogden could have lost title or be barred by estoppel and laches from asserting title, if at all, would be some acts which would either vest title in others or which would give others the right to preclude North Ogden from asserting its title. In view of the unequivocal language of the Townsite Act, title could not be acquired by individuals through adverse user. As stated in the case of Tooele City v. Elkington, 100 Utah 485, 116 P. 2d 406, although a street or alley designated as such on the townsite plat has not been opened up, the officials of the municipality themselves could not convey the title without compliance with the statutes. It is necessary to find statutory authority for divesting the municipality or the public official holding title to the streets in trust for the town, of such title to the streets, in order for the appellants to prevail in this case. It is not claimed that any ordinance was adopted for vacating the streets in question, nor that the inhabitants of North Ogden ever filed any petition to vacate or alter any portion of the plat of North Ogden. The rule of estoppel could not apply against the town as to lands charged with a public trust, for the reasons hereinbefore indicated. Any abandonment or vacation of the land for street purposes, to discharge it of the public trust, would have to be in the manner provided by statute.

In 1886 the Territorial Legislature enacted legislation defining highways. As set forth in C. L. U. 1888, § 2065, "all roads, streets, alleys and bridges laid out or erected by the public, are highways."

Section 2067 provided that: "All roads, streets, alleys and bridges which are now used by the public, and have been declared to be highways by the county courts and municipal corporations within their respective jurisdictions, ... shall be deemed and taken to be public highways."

Section 2071 provided: "By taking or accepting land for a highway, the public acquire only the right of way, and incidents necessary to enjoying and maintaining it. A transfer of land bounded by a highway, passes the title of the person whose estate is transferred, to the center of the highway." U. C. A. 1943, § 36-1-7.

Could such statute operate to divest the probate judge of the fee simple title to the streets of North Ogden, as held by him in trust under the Townsite Act, and vest such fee in the land owners whose lots abutted on the streets? We think not.

The general legislation respecting highways could not alter the express terms of the Townsite Act. Such legislation related particularly to lands outside of cities and towns. The phrase, "By taking or accepting land for a highway, the public acquire only the right of way," has reference to acquisition of highways over privately owned land. HN3Where the State cuts a highway across a tract of land in private ownership, the State does not take the fee but only an easement. Many of the early roads established in this State were the result of adjoining landowners each giving half the amount of land necessary for the road. Such landowners already had the fee, and the Territorial Legislature and subsequently the State Legislature declared by such statute that such landowners would not be divested of the fee in such roadways. In the case of lots acquired under the Townsite Act, the persons awarded such lots never owned the fee simple title to the streets, and were prohibited by the statute from acquiring title by adverse use. The title to the streets was held by the probate judge to be conveyed to the town when it should be incorporated. The phrase, "A transfer of land bounded by a highway, passes the title of the person whose estate is transferred, to the center of the highway"; could not be applicable to the streets of North Ogden, for the reason the owners of the lots never did own the fee in the streets. The statute on highways did not give them title to the center of the streets. Therefore, the deeds of conveyance shown in the abstracts whereby title to lands was deraigned by lot numbers conveyed title to the grantees to only the area of those lots described, with such rights of access to the streets as are necessarily appurtenant to such lots. The title to the streets came to the probate judge under the United States patent, and he was required to hold such title to those streets in trust to convey to the town when it should be incorporated. It would be straining clear language to say that the highway statute was intended to alter the terms of such statutory trust.

By section 2070, C. L. U. 1888, it is provided in said same chapter, that: "A road not worked or used for a period of five years ceases to be a highway."

Said section appeared in the same statute, and referred only to those highways established over privately owned lands or over the public domain. If the highway was merely an easement, the fee would not be changed, and by abandonment of a highway as an easement, the burden in the form of a right of way ceased to exist. As to the streets in North Ogden as in other townsites, the owners of the lots who received deeds to such lots as they were adjudged entitled to own did not own the land within the boundaries of the streets in the first instance, and did not acquire the fee. They merely had an easement of access to the streets.

The case of Sowadzki v. Salt Lake County, 36 Utah 127, 104 P. 111, does not aid the claim of appellants, for the land involved in that case was not land patented under the Townsite Act. The plat therein mentioned was of land south of Salt Lake City. The land was in private ownership before the platting, and before the dedication of the streets. Tooele City v. Elkington, supra, on the other hand, applied to lands acquired under the Townsite Act, and is a more recent case and we see no occasion for overruling it.

Appellants have cited cases in which claimants under the Townsite Act of other states and territories were not bound by the surveys for the reason that they had occupied and were entitled to more land than was awarded to them. We are not concerned in this case what the claimants to lots in North Ogden might have done if they had asserted a right to the areas embraced within the limits of the streets on the plat made of the town. No complaint was made either within the time limited or at all, that any of the predecessors of plaintiffs were denied conveyance of all of the lands to which they became entitled. There is no evidence that they filed any claim to any lands within the area of the streets shown on the plat, and the time for presentation of applications for conveyance having expired over 70 years ago, the successors in interest of those persons who received their deeds from the probate judge, could not readjudicate the claims of the original claimants at this late date nor claim that the plat of lots, blocks and streets failed to recognize the claims of the original settlers. The original settlers were bound by the adjudications made on their applications and claims, and the recorded claims indicate that ownership and occupancy was declared with respect only to specified lots in designated blocks, no claim being made to any portion of the streets.

It is claimed that the public acquiesced in the extension of fence lines to the center of the streets in some cases and the erection of valuable improvements within the area of the streets now sought to be opened. Whatever use was made could only be deemed a permissive use and could not be claimed in perpetuity. While the persons who erected the improvements may be the owners of such improvements, they are not the owners of the lands on which they are erected, and the municipality has the right to terminate such permissive use and require the removal of such improvements or to compensate such owners of improvements if such improvements are taken for public use. The plaintiffs, however, cannot restrain the public authorities from opening up such streets, nor prevent the improvement of such streets, since plaintiffs do not have title to such streets.

The judgment is therefore affirmed. Costs to respondents.

WADE, Justice (dissenting).

I dissent. Material facts which are the basis of my opinion were not disclosed in the prevailing opinion, and so a statement of facts is necessary: On October 16, 1869, the predecessors in interest of plaintiffs' lands filed their claims thereto with the County judge under the Townsite Laws, and later made proof thereof and were adjudicated to be the owners and possessors of such lands, and received deeds through which plaintiffs derived their respective interests in such lands. The North Ogden Townsite had been previously entered in the Land Office by the County Judge of Weber County, it being an unincorporated town, under the Federal Townsite Act of 1867, and patent was issued to such judge on August 2, 1872, for the several use and benefit of the occupants thereof. A plat and survey of the North Ogden Townsite was made and filed in the Weber County Recorder's Office on April 27, 1870, in which the lands involved in this action were platted into square blocks, which were divided into lots. Between the blocks and on all sides of each of them was platted a street four rods in width. In all of the claims, adjudications, deeds and transfers in plaintiffs' chains of title the lands therein are described either as a stated lot or block, or a number of stated lots or blocks, or a described portion of a stated lot or block, in Plat "A", North Ogden Survey. Thus, none of the plaintiffs have ever received any deed or conveyance which expressly conveys any part of the lands which were platted as streets. Some of the streets as shown in the plat were opened and being used as such by the public at the time the Townsite was entered by the County Judge, but many of them were not.

Plaintiffs contend that at the time the County Judge entered this townsite their predecessors in interest not only occupied the lands which were platted as lots and blocks and which they claimed and were adjudicated to be the rightful owners and possessors of at that time, but that they also occupied the lands which were adjacent thereto which were platted as streets, which the city now proposes to open as such and which plaintiffs now occupy. At the trial it was stipulated that, except as to a portion of one street, none of the lands platted as streets, which the city now proposes to open as such, and which is now in the possession of the plaintiffs, has ever been used by the public as a roadway or for traveling purposes, and no evidence was introduced which showed that the portion of the street excepted from the stipulation was ever so used. Plaintiffs produced witnesses, among whom were some of the oldest residents of North Ogden who are still living, one of whom was eighty years old, who testified that they had been born, raised and lived in that neighborhood all of their lives; had known this property since they were old enough to know any property; that none of the lands which the city now proposes to open for use as streets had ever been used by the public as such but that the lands which plaintiffs now occupy had been fenced in, used and occupied by plaintiffs and their predecessors during all of the time they have known the property.

Thus, at the time of the entry of this townsite none of the lands now occupied by the plaintiffs was being used by the public for roadways or traveling purposes. Plaintiffs' predecessors occupied the lands adjacent to the lands now occupied by plaintiffs and which were platted as streets, and plaintiffs and their predecessors in interest have occupied such lands to the extent that plaintiffs now occupy them as long as a man eighty years old can remember. The foregoing facts require the conclusion that plaintiffs' predecessors in interest were occupying the portions of the lands which were platted as streets and which they now occupy, at the time the Townsite was entered by the County Judge for the benefit of the occupants thereof. Any opposing conclusion would be unreasonable.

The defendant, North Ogden City, now proposed to open up for public use as streets all of the lands which were originally platted as such, and to take from plaintiffs such lands which they have occupied for more than seventy-five years, without condemning the same and without compensation. Plaintiffs bring this action to enjoin the city from interfering with their use of such lands. The city, to succeed, must establish its right to so take such lands under the Federal Townsite Act of 1867, or show that plaintiffs or the predecessors have dedicated such lands for that purpose to the public.

The Townsite Act approved March, 1867, 14 Stat. 541, known as section 2387, Rev. St., 43 U. S. C. A. § 718, as it applies to this case, provides as follows: "Whenever any portion of the public lands of the United States have been or shall be settled upon and occupied as a town site, ... it shall be lawful, ... for the judge of the county court for the county in which such town [is] situated, to enter at the proper land office, ... the land so settled and occupied, in trust for the several use and benefit of the occupants thereof, according to their respective interests; the execution of which trust to be conducted under such rules and regulations as may be prescribed by the legislative authority of the State of Territory in which the same may be situated:"

The territorial legislature in 1869 adopted the Territorial Townsite Act, C. L. Utah 1876, § 1166 et seq., which provided for the publication of notice and the filing of a statement of claim within six months thereafter and provided further that: "All persons failing to make and deliver such statement within the time limited in this section shall be forever barred the right of claiming or recovering such land, or any interest or estate therein, or in any part, parcel or share thereof, in any court of law or equity."

Section 1175, C. L. U. 1876, provides that if there is unclaimed lands after the expiration of the time for filing claims, the probate judge (in case of an unincorporated town) "shall cause the same to be surveyed and laid out into suitable blocks and lots, ... and shall cause all necessary streets, roads, lanes and alleys to be laid out through the same, ..."

There are many cases which have interpretated this Federal statute, including some from the Supreme Court of the United States, in cases very similar to this one, and they have held without exception that at the time of the entry of the land of the townsite the occupants thereof have a vested right to the use of the streets and alleys then existing and being used, and that each of such occupants at that time becomes to the extent of the lands which he then occupied the beneficiary of the trust created by the act and was then vested with the equitable ownership of the land which he then occupied, and that while the execution of the trust was subject to the rules and regulations of the territorial Legislature, neither by such rules nor by any act of the trustee could the beneficiary be divested of his rights which accrued to him under the act of Congress at the time the entry of the townsite was made. Such cases further hold that the filing and recording of a map or plat of the townsite either in accordance with the territorial regulations or otherwise, which showed no street to exist where one existed and was then being used by the occupants at the time of the entry, or which showed a street or alley to exist over lands or a part thereof which was then occupied by a settler, could not divest the settlers of the right to use the street or alley which existed and were being used at the time of the entry, nor divest the occupant of his vested equitable ownership of all of the lands then occupied by him, and any attempt to do so would be null and void. Bingham v. Walla Walla, 3 Wash. T. 68, 13 P. 408; Parcher v. Ashby, 5 Mont. 68, 1 P. 204; same case Ashby v. Hall, 119 U.S. 526, 7 S. Ct. 308, 30 L. Ed. 469; City of Helena v. Albertose, 8 Mont. 499, 20 P. 817; Scully v. Squier, 13 Idaho 417, 90 P. 573, 30 L.R.A. N.S. 183; Id., 215 U.S. 144, 30 S. Ct. 51, 54 L. Ed. 131; City of Globe v. Slack, 11 Ariz. 408, 95 P. 126; City of Pueblo v. Budd, 19 Colo. 579, 36 P. 599.

In the cases of City of Helena v. Albertose, supra; City of Globe v. Slack, supra; [**229] City of Pueblo v. Budd, supra; and Treadway v. Wilder, 8 Nev. 91, the local statutes required the occupants to file their claims to such lots within a specified time and provided that a failure to do so would forever bar such claims, and also provided the manner of disposal of lands not claimed. The statutes in Colorado and Nevada are exactly the same in meaning as our statute above quoted, but the courts held that neither the filing of the plat nor, in the Nevada case, the giving of a deed to a person not occupying the same, could deprive the occupant of his right of possession in the property and that such right could be shown to defeat the claim of another or the public, although the court could not quiet title to the property in the occupant or his successors. In the case of City of Globe v. Slack, supra, the occupant at the time of the entry filed claim to only the part platted as a lot, but did not make any claim to the adjacent land which he occupied which was platted as a street. Later the original occupant sold his land to the defendant but by his deed only conveyed the lot and not the part platted as a street; however, his successor occupied the part platted as a street the same as the original occupant. The court held that the defendant had sufficient title to defeat the claim of the city.

Nor did the city or town of North Ogden obtain the right to take for public streets the land in question without compensation under the provisions of section 1175, C. L. U. 1876. This section provides in substance that the probate judge shall cause the unclaimed lands to be surveyed and laid out into suitable lots and blocks and shall cause all necessary streets to be laid out. Obviously the intent of that statute was only to authorize the laying out of streets adjacent to the unclaimed lots and blocks which were necessary for the use thereof, and not to authorize the laying out of streets necessary for the use of the lots and blocks which had already been claimed. In any event, if the intent of that section was to take from an occupant land which he occupied at the time the townsite was entered in the land office, and which was still occupied by him or his successors in interest, it would be null and void as an attempt to deprive him of his vested interests under the act of Congress of the equitable title to the lands which he occupied at the time of the entry. See cases above cited and especially City of Pueblo v. Budd, supra.

I therefore conclude that the plaintiffs in this action obtained the equitable ownership of all the land which they occupied at the time of the entry in the Land Office of the townsite of North Ogden by the County Judge, and that under the act of Congress neither the City of North Ogden nor the public ever obtained any right, title or interest therein through the plat or survey which was filed, and therefore unless the plaintiffs have dedicated that land to the public the plaintiffs must succeed in this action.

Neither the plaintiffs nor their predecessors in interest have dedicated these lands which are platted as streets to the public as such. Such a dedication can only be done by some act of the owner with the intention that it will have that effect or by estoppel. The filing of this plat in the recorder's office by the public officials, not being an act of the owner, does not have that effect. If it were otherwise, then the public officials could take anyone's property merely by filing a plat showing such property to be platted as property for a public use. All of the cases above cited hold that such a plat does not constitute a dedication of the property by the owner to a public use. The fact that the predecessors of plaintiffs merely made claim to the lands which were shown on the plat as lots and blocks does not constitute such a dedication of the lands platted as streets, City of Globe v. Slack, supra; nor does the acceptance of a deed from the County Judge of only the lands so platted have that effect, Bingham v. Walla Walla, supra; nor does the conveyance of the land to his successor without describing the land covered by the street as platted constitute a dedication, City of Globe v. Slack, supra. The failure of the occupant to file any claim to the land covered by the street as platted does not constitute a dedication thereof to that use. City of Helena v. Albertose, supra, and City of Pueblo v. Budd, supra. This, I think, is the proper rule. All of these cases hold that the occupant asserts ownership in the land platted as streets by occupying such lands at the time of the entry in the land office, and the continuance of such occupancy by him and his successors in interest show that there is no dedication to a public use.

The defendants and interveners rely on the case of Tooele City v. Elkington, 100 Utah 485, 116 P. 2d 406. In that case no [**230] mention is made of who was in possession of the lands in question at the time of the entry in the land office of the townsite, and no discussion of who was entitled to the land at that time is made. But it is assumed that the fact that the predecessors in interest of the defendants conveyed only with reference to the lots and blocks as platted and did not expressly convey the parts in question which was platted as an alley; that such conveyance constituted a dedication of the alley as platted to the public. No discussion of what is necessary to constitute a dedication is made except that the case of Wallace v. Cable, 87 Kan. 835, 127 P. 5, 6, 42 L.R.A. N.S. 578, is referred to. But that case is certainly no authority for the assumed proposition, as the dedication therein relied upon was a part of the deed which expressly provided: " ... meaning hereby to convey to the party of the second part block 73 in the city of Wyandotte, according to the plan of said city published by John H. Miller, made March 18, 1857, with half of the adjoining street, and it is hereby stipulated that these streets shall not be closed up except by mutual consent."

The great weight of authority is contrary to the rule assumed in the Tooele case. See note to Wallace v. Cable, supra, 87 Kan. 835, 127 P. 5, 42 L. R. A., N. S., 587.

There is no contention that there was any dedication by estoppel and no facts on which such estoppel could be based. I therefore think the decision of the trial court should be reversed.

APPENDIX 1b

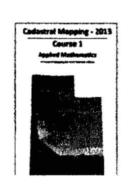
Salt Lake County,
Plaintiff and Appellant,

v.

Metro West Ready Mix, Inc., a Utah corporation, and
Monterra Rock Products, Inc., a Utah corporation,
Defendants and Appellees.

No. 20020701
F I L E D March 23, 2004

2004 UT 23



APPENDIX 2:

This opinion is subject to revision before finalpublication in the Pacific Reporter.

IN THE SUPREME COURT OF THE STATE OF UTAH

----oo0oo----

Salt Lake County,

Plaintiff and Appellant,

v.

Metro West Ready Mix, Inc.,

a Utah corporation, and

Monterra Rock Products, Inc.,

a Utah corporation,

Defendants and Appellees.

No. 20020701

F I L E DMarch 23, 2004

2004 UT 23

Third District, Salt Lake City

The Honorable Sandra Peuler

Attorneys: David E. Yocom, Don H. Hansen, Salt Lake City, for plaintiff

Mark R. Clements, Mark H. Richards, Salt Lake City, for defendants

---

DURRANT, Associate Chief Justice:

¶1 In this appeal we address whether a purchaser who obtains title to property through a wild deed can be a bona fide purchaser under Utah's Recording Statute.

¶2 The dispute at issue is between Salt Lake County (the "County") and Metro West Ready Mix, Inc. ("Metro West") over the ownership of Parcel G, a fifteen-acre piece of property located in Utah County near the border between Utah and Salt Lake Counties (the "Property"). The County received legal title to the Property on December 4, 1878; however, it failed to record its deed in the Utah County Recorder's Office until June 17, 1998.

¶3 Nine years before the County recorded its deed in Utah County, Metro West purchased the Property from the Property's purported owners. Metro West did so even though a record title search revealed that the owners had no record title to the Property. Metro West subsequently recorded its quitclaim deed in the Utah County Recorder's Office on April 14, 1989.

¶4 After learning of Metro West's claimed ownership, the County filed suit in 1999 to quiet title to the Property. The trial court found that Metro West was a bona fide purchaser under Utah's Recording Statute and granted summary judgment in favor of Metro West. The County appealed, and the court of appeals affirmed based on its newly-enunciated "apparent title rule." We reverse.

FACTUAL BACKGROUND

¶5 The United States Government conveyed the Property by land patent to William Turner in 1878. The Property is located in Utah County, with its northern border abutting the Utah County/Salt Lake County line. In 1878, Turner conveyed the Property to the County by warranty deed, which the County immediately recorded in Salt Lake County. In 1998, approximately 120 years later, the County recorded its warranty deed in Utah County.

¶6 Nine years before the County recorded its warranty deed in Utah County, Metro West's predecessor-in-interest, Lamona Farms, approached the purported owners of the Property, Darhl and Roena Tingey ("the Tingeys"), and inquired as to the purchase price. The Tingeys represented to Lamona Farms's owners that the Tingey family had been in possession of and had used the Property since the turn of the century. The Tingeys would only agree to convey the Property, however, by quitclaim deed. The Tingeys explained that they were unable to convey the Property by warranty deed due to a 1,000-foot discrepancy in the Property description. This discrepancy, the Tingeys claimed, was created when the state of Utah began staking its section markers at both the north and south ends of the state. The Utah County Recorder's Office confirmed this discrepancy.

¶7 Before purchasing the Property, one of Lamona Farms's owners and a title company both reviewed the records at the Utah County Recorder's Office and determined that nothing in the records conflicted with the Tingeys' ownership representations. It is undisputed, however, that the Utah County Recorder's Office never listed the Tingeys as owners of the Property, and that the Tingeys were strangers to the record title.

¶8 The Tingeys conveyed the Property to Lamona Farms by quitclaim deed on April 14, 1989, for $25,000. Lamona Farms promptly recorded its quitclaim deed in Utah County. In 1991, Lamona Farms transferred the Property to Monterra Rock Products, Inc., which merged two years later into Metro West.

PROCEDURAL HISTORY

¶9 In February 1999, the County filed an action to quiet title to the Property. Metro West filed a motion for summary judgment, arguing that it possessed legal ownership of the Property because (1) it was a bona fide purchaser under Utah's Recording Statute, see Utah Code Ann. § 57-3-103 (2000); (2) it had adversely possessed the Property under Utah's Adverse Possession Statute, see Utah Code Ann. §§ 78-12-7 to -13 (1996); and (3) principles of equity and public policy supported its ownership entitlement. The trial court granted Metro West's motion based upon Utah Code section 57-3-103 and the "undisputed facts that [Metro West] purchased [the Property] for valuable consideration and in good faith, and recorded its deed in Utah County prior to any recording there by [the County]." The County appealed.

¶10 The court of appeals affirmed the trial court's decision. Salt Lake County v. Metro West Ready Mix, Inc., 2002 UT App 257, 53 P.2d 499. In so doing, the court of appeals announced an "apparent title rule," under which a purchaser is entitled to bona fide purchaser protection where the records are silent with respect to ownership "if the grantor had apparent legal title, even if he or she did not have perfect legal title." Id. at ¶ 11. Apparent title, according to the court of appeals, is determined by considering "(1) whether the purported owner claimed to own the property; (2) whether the purported owner possessed the property; and (3) whether there was any activity or indication on the property that would raise questions as to who owned the property." Id. at ¶ 13. Under this apparent title doctrine, the court of appeals reasoned that the trial court was correct in extending bona fide purchaser protection to Metro West because the records were silent as to the Tingeys' ownership; the Tingeys had asserted that they had owned the Property since the turn of the century; the Tingeys were in possession of the Property; and the County did not have any signs or carry out any activity on the Property that would lead Metro West to believe that the Tingeys were not the legal owners of the Property. Id. at ¶¶ 15-16. The County petitioned for certiorari review of the court of appeals' decision, which we granted pursuant to Utah Code section 78-2-2(5) (2002).

STANDARD OF REVIEW

¶11 "When exercising our certiorari jurisdiction, we review the decision of the court of appeals, not of the trial court." Mitchell v. Christensen, 2001 UT 80, ¶ 8, 31 P.3d 572 (internal quotations omitted). Summary judgment is appropriate when there is no genuine issue of material fact and the moving party is entitled to judgment as a matter of law. Utah R. Civ. P. 56(c). "[B]ecause a summary judgment presents questions of law, we accord no particular deference to the court of appeals' ruling" and review it for correctness. Mitchell, 2001 UT 80 at ¶ 8 (citing Ron Case Roofing & Asphalt Paving, Inc. v. Blomquist, 773 P.2d 1382, 1385 (Utah 1989)).

ANALYSIS

I. UTAH'S RECORDING STATUTE

¶12 The County argues that the court of appeals erred in holding that a purchaser is protected under Utah's Recording Statute when he is on notice that his grantor has no record title to the property conveyed. We agree.

¶13 Utah's Recording Statute provides as follows:

Each document not recorded as provided in this title is void as against any subsequent purchaser of the same real property, or any portion of it, if:

(1) the subsequent purchaser purchased the property in good faith and for a valuable consideration; and

(2) the subsequent purchaser's document is first duly recorded.

Utah Code Ann. § 54-3-103 (2000). To be in good faith, a subsequent purchaser must take the property without notice of a prior, unrecorded interest in the property. See Ault v. Holden, 2002 UT 33, ¶ 31, 44 P.3d 781. In addition, to be in good faith a subsequent purchaser must also take the property "without notice of any infirmity in his grantor's title." Pender v. Bird, 119 Utah 91, 96, 224 P.2d 1057, 1059 (1950); see also Paldevco Ltd. P'ship v. City of Auburn Hills, No. 202134, 1998 Mich. App. LEXIS 626, at *5 (Mich. Ct. App. Dec. 18, 1998) (unpublished per curiam decision) (noting "[a] good-faith purchaser is one who purchases without notice of a defect in the vendor's title" and that "[n]otice need only be of the possibility of the rights of another, not positive knowledge of those rights"). This notice is not confined to situations in which a subsequent purchaser has actual notice of an unrecorded interest or infirmity in the grantor's title. Rather, it includes circumstances where a purchaser has constructive notice of such information, including both (1) record notice "'which results from a record or which is imputed by the recording statutes,'" and (2) inquiry notice "'which is presumed because of the fact that a person has knowledge of certain facts which should impart to him, or lead him to, knowledge of the ultimate fact.'" First Am. Title Ins. Co. v. J.B. Ranch, Inc., 966 P.2d 834, 837 (Utah 1998) (quoting 66 C.J.S. Notice § 6 (1950)).

¶14 The County argues that because a search of the Utah County records revealed that the Tingeys had no record title to the Property, Metro West was necessarily on inquiry notice of a defect in the Tingeys' title. According to the County, under this inquiry notice analysis the lack of record title would have led a reasonable person to have discovered the County's recorded title to the Property in the Salt Lake County Recorder's Office. Consequently, the County asserts that Metro West cannot be a good faith purchaser without notice under the recording statute.

¶15 We agree that the Tingeys' lack of record title put Metro West on notice of a defect in the Tingeys' title. However, we do so not because Metro West was on inquiry notice of the defect, but because Metro West had both actual and constructive record notice of the defect, which precluded it from taking the Property in good faith.

¶16 Because the Tingeys had no record title to the Property when they transferred it to Metro West, the conveyance was carried out through what is commonly referred to as a "wild deed." See Poladian v. Johnson, 85 So. 2d 140, 140 (Fla. 1955) (en banc) ("A 'wild deed' is one executed by a stranger to the record title . . . ."); 11 Thompson on Real Property, Thomas Edition, § 92.11(c) (David A. Thomas ed., 1994) ("[A] 'wild deed' [is one] executed by a grantor with no record ownership of the interest . . . ."). Few courts have addressed whether a purchaser who acquires title through a wild deed takes free of an unrecorded interest in the same property under the recording statutes. However, at least one commentator has expressly noted that a purchaser who takes title through a wild deed is not the type of purchaser that recording statutes protect. See Joyce Palomar, Patton and Palomar on Land Titles, § 10 (3d ed. 2003) ("[T]he term 'bona fide purchaser' as used in recording acts does not include one who buys from a stranger to the record."). And the majority of courts that have addressed the issue have held that a purchaser who receives property through a wild deed is not entitled to take free of an unrecorded interest simply by virtue of having recorded a purported conveyance of title executed by a stranger to the record title. See Holland v. Hattaway, 438 So. 2d 456, 470 (Fla. Dist. Ct. App. 1983) (noting that because "recording statutes do not support or validate a wild deed or any conveyance of an interest that the grantor does not have," an appellant whose chain of title traced back to the sovereign had superior interest over an appellee who could only trace title to a wild deed executed by a stranger to the record); Zimmer v. Sundell, 296 N.W. 589, 591 (Wis. 1941) ("If one who has no title under the laws governing conveyances is to have a superior one under the recording acts, it should be because he has relied upon the record, and when he purchases from one who is a stranger to the record title he has no grounds for such reliance."). But see Hyland v. Kirkman, 498 A.2d 1278, 1284, 1289, 1295-96 (N.J. Super. Ct. Ch. Div. 1985).

¶17 We conclude that a purchaser whose chain of title is founded on a wild deed cannot be a bona fide purchaser under Utah's Recording Statute. As previously noted, to be in good faith a purchaser must purchase the property without notice of any defect or infirmity in the grantor's title. Pender, 119 Utah at 96, 224 P.2d at 1059. Because "[o]ne who deals with real property is charged with notice of what is shown by the records of the county recorder of the county in which the property is situated," Crompton v. Jenson, 78 Utah 55, 70, 1 P.2d 242, 247 (1931), and by implication charged with notice of what the records should show but do not, i.e., a lack of record title in a grantor, we hold that by definition a purchaser whose title is founded on a wild deed is on notice that his grantor had no record title to the property purportedly being conveyed. This is true in instances where, as in this case, the subsequent purchaser has obtained actual notice of this absence by searching the records. Moreover, it is also true even when the purchaser has no actual notice of the title defect, since all grantees of wild deeds are necessarily charged with constructive record notice by virtue of the recording statutes. Accordingly, a purchaser who acquires property through a wild deed will be held to have been on notice of a defect in his grantor's title and will not qualify as a subsequent purchaser in good faith for purposes of Utah's Recording Statute.

¶18 This is not to say that a purchaser who acquires property through a wild deed can never acquire good title. We merely hold that the recording statutes do not protect such a purchaser as against an unrecorded interest in the same. See Huntington City v. Peterson, 30 Utah 2d 408, 410, 518 P.2d 1246, 1248 (1974) (explaining that recording a deed does not pass title); Horman v. Clark, 744 P.2d 1014, 1016 (Utah Ct. App. 1987) ("The recording statute's purpose is not to make the transfer of property effective as between the parties . . . ."). If a purchaser can establish that his grantor possessed and conveyed valid title to the property independent of the recording statutes, a conveyance through wild deed will be effective as against any competing claims.

¶19 Applied in this case, because the Tingeys had no record title to the Property, Metro West was on notice of a defect in the Tingeys' record title and therefore did not purchase the Property in good faith for purposes of Utah's Recording Statute. As such, it is not a bona fide purchaser and is not entitled to quiet title to the Property as against the County's unrecorded interest simply because it recorded its purported title first. If, however, the Tingeys acquired valid title to the Property through a means not reflected by the record, such as through adverse possession, the Tingeys conveyed valid title and Metro West would be entitled to quiet title to the Property as against the County. Because the record is insufficient for us to determine whether the Tingeys had valid title to the Property, we remand for further proceedings consistent with this opinion.

II. ADVERSE POSSESSION

¶20 Although not an issue reached by the trial court below, Metro West argues in the alternative that it is entitled to summary judgment because it established ownership of the Property through its own adverse possession. The County contends that not only do disputed material facts exist that preclude summary judgment in Metro West's favor, but that even if Metro West established all the necessary elements of adverse possession, Metro West is barred from acquiring title through adverse possession under Utah Code section 78-12-13 because the County designated the Property for a public use. See Utah Code Ann. § 78-12-13 (2000).

¶21 "[A]n appellate court may affirm a trial court's ruling on any proper grounds, even though the trial court relied on some other ground." DeBry v. Noble, 889 P.2d 428, 444 (Utah 1995). To do so, however, the facts established in the record must be sufficient to support the alternative ground. See Renn v. Utah Bd. of Pardons, 904 P.2d 677, 685 (Utah 1995).

¶22 In Utah, a person without legal title is deemed "to have been under and in subordination to" the owner with legal title unless that person has adversely possessed the property. Utah Code Ann. § 78-12-7 (2000). When an occupant has entered into possession of property under a claim of title, the occupant may establish adverse possession by demonstrating that (1) the property was "occupied and claimed for the period of seven years continuously," id. § 78-12-12; (2) "the party, his predecessors and grantors have paid all taxes which have been levied and assessed [on the property]," id.; and (3) the property was, in pertinent part, "usually cultivated or improved," "protected by a substantial inclosure[,]" or "used . . . for the ordinary use of the occupant," id. § 78-12-9.

¶23 To be entitled to summary judgment on the basis of adverse possession, Metro West must affirmatively show all the statutory elements of adverse possession. See English v. Openshaw, 28 Utah 241, 247, 78 P. 476, 477 (1904) ("To overthrow [the] presumption [that the adverse party is not under subordination to the legal owner's title], the party claiming adversely ha[s] the burden to establish the fact[] by competent evidence . . . ."); see also Martin v. Kearl, 917 P.2d 91, 93 n.5 (Utah Ct. App. 1996) ("[B]ecause of the gravity of adverse possession claims--wresting title from otherwise rightful owners--claimants must strictly comply with all requirements."); 3 Am. Jur. 2d Adverse

Possession § 311 (1986). In this case, we find at least two key areas of dispute sufficient to render summary judgment inappropriate.

¶24 First, there are disputed facts as to whether Metro West's use of the Property was continuous during the required seven-year period. Metro West argued before both this court and the trial court that "[b]eginning in 1990 and continuing thereafter, Metro West improved [the Property] by bulldozing numerous access roads across the property[,] excavating and drilling numerous holes on [the Property,] and conducting regular sample testing of the underground materials." However, as the County argued to the trial court, whether the nature of this use was sufficiently continuous is in dispute.

¶25 For example, prior to transferring the Property to Metro West, Dr. Richards, Lamona Farms's owner, could not specify during what period he or his employees occupied the Property. Dr. Richards also could not recall how many days his employees spent on the Property conducting testing. And, though he claimed that roads were being made on all of Lamona Farms's property continuously, Dr. Richards could not identify any times when such roads were made on the Property specifically because he did not know where the Property's boundaries were. Metro West also had no record of how many days its employees spent on the Property. Roy McNeil, one of Metro West's principals, testified that he knew Metro West had begun exploring dirt on the Property in 1998; however, he could not say when exploration had occurred between 1990 and 1995.

¶26 Second, a dispute exists regarding whether Metro West paid property taxes on the Property for the seven-year period. As the County correctly asserts, the tax receipts submitted to the trial court do not indicate that the taxes being paid were specifically for the Property. In fact, not only do descriptions provided in the "property description" portion of the receipts differ from year to year, but the total acreage of land identified in the tax receipts changes as well. Based on the tax receipts provided in the record, it is impossible for us to determine whether the varied descriptions on the tax receipts were, in fact, attributable to the Property.

¶27 Because there is a genuine issue of material fact as to whether Metro West continously used and paid taxes on the Property, Metro West is not entitled to summary judgment and we need not address whether disputed facts exist as to the remaining elements of adverse possession under Utah Code section 78-12-9. Moreover, having determined that Metro West is not entitled to summary judgment on the basis of adverse possession, we need not reach the County's alternative argument that summary judgment would be barred by the "designated for public use" exception under Utah Code section 78-12-13.

CONCLUSION

¶28 To take free of an unrecorded interest in property under Utah's Recording Statute, a subsequent purchaser must purchase the property in good faith without notice of any defect in his grantor's title. Because a purchaser whose chain of title is founded on a wild deed necessarily has constructive record notice that his grantor's record title is defective by virtue of taking title from a stranger to the record, a purchaser who acquires property through a wild deed cannot be a good faith purchaser. Accordingly, Metro West is not a bona fide purchaser under Utah's Recording Statute and cannot take free of the County's unrecorded interest in the Property unless it establishes that the Tingeys obtained valid title to the Property through a means other than a recorded conveyance. Moreover, since genuine issues of material fact exist as to whether Metro West adversely possessed the Property, summary judgment is inappropriate. We therefore reverse and remand for further proceedings consistent with this opinion.

¶29 Chief Justice Durham, Justice Wilkins, Justice Parrish, and Justice Nehring concur in Associate Chief Justice Durrant's opinion.

1. Many of the facts recited herein are taken from the court of appeals' opinion. See Salt Lake County v. Metro West Ready Mix, Inc., 2002 UT App 257, ¶¶ 2-4, 53 P.3d 499.

2. To avoid confusion, we hereinafter refer to Metro West's predecessors-in-interest simply as Metro West.

3. Because this current section is identical to the code section applicable at the time the County filed its quiet title action, we cite the current code provision. Compare Utah Code Ann. § 57-3-3 (1994), with Utah Code Ann. § 57-3-103 (2000).

4. Metro West asserts that the County's failure to set forth in its opposing memorandum "a statement of facts it claims are in dispute as [required by] [r]ule 4-501(2)(B) of the Utah Code of Judicial Administration" should result in our finding that Metro West's facts "be deemed admitted for purposes of summary judgment and this appeal." It is true that the County's opposing memorandum did not set forth disputed facts listed in numbered sentences in a separate section as required by the Utah Rules of Judicial Administration. See Utah R. Jud. Admin. 4-501(2)(B). However, given that the disputed facts were clearly provided in the body of the memorandum with applicable record references, we find the failure to comply with the technical requirements of rule 4-501(2)(B) to be harmless in this case. See Hall v. NACM Intermountain, Inc., 1999 UT 97, ¶¶ 19-21, 988 P.2d 942

(noting the failure to specifically set forth a legal basis for the award of attorney fees in compliance with rule 4-505 of the Utah Code of Judicial Administration was a harmless error because the court and both counsel always knew the purpose behind and the basis for the proposed award of fees). Accordingly, we examine whether the County's disputed facts are sufficient to preclude summary judgment in favor of Metro West.

This opinion is subject to revision before publication in the Pacific Reporter.

IN THE UTAH COURT OF APPEALS

----ooOoo----

Salt Lake County, a body corporateand politic of the State of Utah,Plaintiff and Appellant,

v.

Metro West Ready Mix, Inc.,a Utah corporation;and Monterra Rock Products, Inc.,a Utah corporation,Defendants and Appellees.

OPINION(For Official Publication)
Case No. 20010276-CA
F I L E DAugust 1, 200
2002 UT App 257

Third District, Salt Lake DepartmentThe Honorable Sandra Peuler

Attorneys:Donald H. Hansen, Salt Lake City, for AppellantMark Clements, Mark F. James, and Mark H. Richards, Salt Lake City, for Appellees

-----

Before Judges Bench, Greenwood, and Orme.

GREENWOOD, Judge:

¶1 Salt Lake County (the County) appeals the trial court's grant of Metro West Ready Mix, Inc.'s (Metro) motion for summary judgment. The judgment granted Metro quiet title to Parcel G (the Property) on the basis that Metro was a bona fide purchaser (BFP) under Utah Code Ann. § 57-3-102, -103 (2000) (the Recording Statute). We affirm.

BACKGROUND

¶2 In 1878, the United States Government conveyed the Property by land patent to William Turner. Turner recorded his patent in Salt Lake County, even though the Property is located entirely within Utah County. The Property's northern border is the Utah County/Salt Lake County line. In 1878, Turner conveyed the Property to the County by warranty deed. The County immediately recorded the warranty deed in Salt Lake County, but more than 120 years passed before the County recorded the warranty deed in Utah County in 1998.

¶3 Nine years before the County recorded in Utah County, Metro became interested in acquiring the Property to further its business interests. Metro principals approached the Tingeys, who were reputed to be the owners of the Property, and inquired as to the purchase price. The Tingeys represented that the Property had been in their family since the turn of the century, and a title search Metro conducted indicated that the Property had no record owner in Utah County. Furthermore, the Tingeys were in possession of the Property and neither the County nor anyone else had posted any signs or carried on any activities that would indicate that anyone other than the Tingeys owned the Property. The Tingeys conveyed the Property to Metro by quitclaim deed for $25,000 on April 14, 1989. Metro promptly recorded its quitclaim deed in Utah County, and incorporated the Property into its gravel pit operations on adjacent land owned by Metro.

¶4 Shortly after discovering the competing interests in the Property, the County filed an action to quiet title. Metro filed a motion for summary judgment, arguing that Metro was entitled to BFP protection under the Recording Statute. The trial court granted Metro's motion and this appeal followed.

ISSUE AND STANDARD OF REVIEW

¶5 The dispositive issue on appeal is whether the trial court correctly granted Metro's motion for summary judgment, holding that Metro is a BFP meriting protection under the Recording Statute even though the Tingeys never had legal title (1) to the Property. We review a trial court's summary judgment ruling for correctness, affording no deference to the trial court's conclusions of law. See Peterson v. Coca-Cola U.S.A., 2002 UT 42, ¶7, 446 Utah Adv. Rep. 27. Summary judgment is appropriate only if "there is no genuine issue as to any material fact." Utah R. Civ. P. 56(c).

ANALYSIS

¶6 The Recording Statute reads in part as follows:

Each document not recorded as provided in this title is void as against any subsequent purchaser of the same real property, or any portion of it, if:

(1) the subsequent purchaser purchased the property in good faith and for a valuable consideration; and

(2) the subsequent purchaser's document is first duly recorded.

Utah Code Ann. § 57-3-103 (2000). Although the plain language of section 57-3-103 does not specifically require that the subsequent purchaser take title without notice of prior conveyances, Utah courts have included this element as inherent in the good faith requirement. See Ault v. Holden, 2002 UT 33, ¶31, 44 P.3d 781; Johnson v. Higley, 1999 UT App 278, ¶24, 989 P.2d 61. Furthermore, this court has stated that the purpose of the Recording

Statute "is not to make the transfer of property effective as between the parties, but to protect the purchaser's interest against the asserted interest of any third parties, and to inform third parties of the existence of pre-existing encumbrances on the property." Horman v. Clark, 744 P.2d 1014, 1016 (Utah Ct. App. 1987) (citations omitted). However, the issue of whether a subsequent purchaser may obtain BFP protection when his or her grantor never had legal title to the property is an issue of first impression in Utah. We therefore begin our analysis by looking to other jurisdictions to find the rule most in harmony with existing Utah law.

¶7 The County urges us to adopt a historic approach followed by many courts. Under the historic approach,

The doctrine of bona fide purchaser without notice does not apply where the purchaser buys no title at all. If he buys from one who has title, and it should afterwards appear that another had a better title which he had not recorded, or that there was a fraud in the title of which he had no notice, the purchaser would be protected. But this doctrine cannot apply where no title whatever existed in the vendor. The good faith of a purchaser cannot create a title.

Dodge v. Briggs, 27 F. 160, 166-67 (C.C.S.D. Ga. 1886) (emphasis omitted); see alsoIowa Land & Trust Co. v. United States, 217 F. 11, 13 (8th Cir. 1914) ("The equitable doctrine of a bona fide purchaser without notice does not apply where there is a total absence of title in the vendor. The good faith of a purchaser cannot create a title where none exists."); Lindblom v. Rocks, 146 F. 660, 663 (9th Cir. 1906) (same); Kaiser Energy, Inc. v. Department of Envtl. Res., 535 A.2d 1255, 1259 (Pa. Commw. Ct. 1988) ("It is axiomatic that a person cannot effectively convey property in which he has no ownership rights. The bona fide purchaser concept applies only to purchasers of legal title."); Cook v. Eller, 380 S.E.2d 853, 854 (S.C. Ct. App. 1989) ("An individual cannot claim bona fide purchaser status if his grantor never had title to the property."). Therefore, under the historic approach, BFP protection does not apply when a grantor lacks legal title because "[the purchaser's] good faith cannot create title" where none exists. Lindblom, 146 F. at 663.

¶8 However, as Metro points out, some modern courts have rejected the historic approach because of its logical inconsistencies when juxtaposed against the plain language of race-notice recording statutes that are similar to Utah's. For example, in Roberts v. Estate of Pursley, 718 A.2d 837 (Pa. Super. Ct. 1998), the appellants had an unbroken chain of title back to

1854, whereas the appellees' chain of title was broken in 1901. See id. at 840. Despite the appellees' broken chain of title, the appellees' predecessor in interest recorded first. See id.

¶9 Although appellants could trace their legal title to 1854, the court held in favor of appellees under the BFP doctrine. Based on the Pennsylvania recording statute, the court reasoned:

> If "legal title," within the Appellant[s'] definition, were required for a subsequent purchaser to qualify as a bona fide purchaser, the recording statute would not further its intended goals. For instance, in the typical recording statute situation, a grantor sells land to a grantee who does not record the deed; then, a subsequent buyer purchases the same land from the same grantor as the original grantee and this subsequent grantee records his deed before the first grantee. The subsequent grantee does not have "legal title" within the Appellant[s'] definition because at the time the land was sold to him, the grantor did not have legal title to give such right. Yet, notwithstanding the fact that he does not have "legal title," he is a bona fide purchaser if at the time of the sale he was without notice of an adverse interest and value was given for the purchase of the land. As evidenced here, imposing a requirement of "legal title" to the definition of a bona fide purchaser would nearly render the recording statute useless. We, therefore, find no merit in Appellants' argument.

Id. at 841 (citations and emphasis omitted). Therefore, under this view, the historical approach is illogical because the traditional recording act hypothetical does not require the seller to have legal title in order for the subsequent purchaser to receive BFP protection.

¶10 Despite the historical approach's logical inconsistencies, the County argues that reading the Recording Statute in the manner adopted in Roberts creates "absurd consequences" by contradicting the purposes of the Recording Statute and sanctioning fraud in land conveyances. See State v. Redd, 1999 UT 108,¶12, 992 P.2d 986 (stating that courts avoid interpreting statutes in such a manner as to render absurd consequences). The County's argument is compelling because the Roberts rationale fails to acknowledge that the Recording Statute was not enacted for the purpose of making otherwise worthless transactions effective as between the parties. See Horman, 744 P.2d at 1016. However, the reasoning in Roberts exposes the flaw of the historical approach. Thus, both of the approaches heretofore discussed have logical and practical failings that make them inadequate to solve the question at issue in this case. For this reason, we examine the rationale of other courts in adopting a third alternative approach.

¶11 A number of courts have adopted the rule that a purchaser may obtain BFP protection if the grantor had apparent legal title, even if he or she did not have perfect legal title. See, e.g., Muller v. Hallenbeck, 19 Cal. Rptr. 251, 255 (Cal. Ct. App. 1962) ("A bona fide purchaser is one who can prove possession of his grantor, the purchase of the property, and the payment of the purchase money in good faith and without notice, actual or constructive."); Montana Elec. Co. v. Northern Valley Mining Co., 153 P. 1017, 1018 (Mont. 1915) (same); First Nat'l Bank of Plattsmouth v. Gibson, 84 N.W. 259, 260 (Neb. 1900) (stating rule that purchaser acquires only title that his grantor had at time of sale, unless he bought and paid for property on faith of apparent owner, upon which he was legally justified in relying); Smith v. Huff, 164 S.W. 429, 431 (Tex. Civ. App. 1914) (holding that purchaser could not have BFP status because grantor never had actual or apparent title to land).

¶12 Courts that have adopted the apparent title rule implement the Recording Statute without sacrificing a plain language reading. For example, in Montana Electric, the court defined a bona fide purchaser as:

"One who at the time of his purchase advances a new consideration, surrenders some security, or does some other act which leaves him in a worse position if his purchase should be set aside, and purchases in the honest belief that his vendor had a right to sell, without notice, actual or constructive, of any adverse rights, claims, interest, or equities of others in and to the property sold."

153 P. at 1018 (emphasis added) (quoting Foster v. Winstanley, 102 P. 574, 579 (Mont. 1909)). Thus, the purchaser must take title without notice that the grantor is in fact not the owner.

¶13 Courts applying the apparent title rule have analyzed several factors to determine whether the purchaser was without notice that the grantor did not have at least apparent title to the property. If the records are silent on ownership, courts have considered the following: (1) whether the purported owner claimed to own the property, see Pender v. Bird, 119 Utah 91, 224 P.2d 1057, 1059 (1950); (2) whether the purported owner possessed the property, see Ault v. Holden, 2002 UT 33, ¶42, 44 P.3d 781; Muller, 19 Cal. Rptr. at 255; and (3) whether there was any activity or indication on the property that would raise questions as to who owned the property. See First Am. Title Ins. Co. v. J.B. Ranch, Inc., 966 P.2d 834, 838 (Utah 1998) (stating party on inquiry notice where railroad poles, guy wires, and trolley wires on property subject to adverse claim).

¶14 We adopt the apparent title rule because it harmonizes the purposes of the Recording Statute. First, by recognizing the need for a grantor to have at least apparent title to make a valid conveyance, the Recording Statute does not validate sham conveyances. Second, it protects purchasers whose honest belief that the purported owner owns the property is supported by legally significant facts.

¶15 Applying the apparent title rule to this case, we conclude that the trial court was correct in extending BFP protection to Metro. (2) To determine whether Metro was legally justified in its belief that the Tingeys owned the Property, the threshold question is whether the records were silent as to ownership. Metro contends the records were silent regarding ownership because the Utah County Recorders Office had no record owner listed for the Property. The County argues, however, that Metro had a duty to inquire beyond the Utah County records, and had it done so, would have discovered that the Tingeys were not the owners. The Recording Statute states that if a document is recorded in the appropriate county, it provides notice to the world. See Utah Code Ann. § 57-3-102(1) (2000). No statute or Utah case requires a party to conduct investigation beyond the county records unless the party has knowledge of certain facts and circumstances requiring further inquiry. See Johnson v. Higley, 1999 UT App 278, ¶24, 984 P.2d 61. However, the facts and circumstances requiring further inquiry do not arise from records unless a statute so provides. See First Am. Title Ins., 966 P.2d at 839 (holding road maps on file in county clerk's office did not provide constructive notice).

¶16 Since the Utah County records relevant to the Recording Statute were silent as to the owner, the apparent title approach requires that we address the secondary factors. First, the Tingeys asserted that the Tingey family had owned the Property since the turn of the century. Second, the Tingeys were in possession of the Property. Third, the County did not have any signs on the Property or carry out any activity that would lead Metro to believe that the Tingeys were not the owners. Therefore, applying this rule, Metro is entitled to BFP protection because it was legally justified in relying on the Tingeys' claims of ownership. (3)

CONCLUSION

¶17 In sum, the trial court was correct in concluding that Metro was a BFP because Metro was legally justified in concluding that the Tingeys were the owners based on the lack of record evidence, the Tingeys' possession of the Property, and the County's failure to post signs or carry on any activity that would raise questions about the Tingeys' title to the Property. Accordingly, we affirm.

______________________________ Pamela T. Greenwood, Judge

¶18 WE CONCUR:

______________________________ Russell W. Bench, Judge

______________________________ Gregory K. Orme, Judge

1. As used in this opinion, the term "legal title" means title superior to all others, based on real property legal principles other than the Recording Statute. It is also the term used in several of the opinions cited herein.

2. It is undisputed that Metro purchased the Property for value and recorded first. The only issue we address in regard to Metro's BFP status is whether it had notice that the Tingeys did not have legal title to the Property.

3. Because we affirm the trial court based on the Recording Statute, we do not reach Metro's adverse possession argument.

APPENDIX 1c

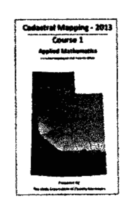
Lynn S. **SCOTT** and
Ann B. Scott, his wife and Frank H. Bjorndal and Audrey K. Bjorndal, his wife,
Plaintiffs and Appellants,

v.

Willford **HANSEN** and
Viola Hansen, his wife, Cecil Hansen and LaDonna Hansen, his wife, Marjorie Baker,
Darrell A. Tate, Barbara Buckley and Michael E. Tate,
Defendants and Respondents.

No. 10580.
Supreme Court of Utah.

422 P.2d 525, 18 Utah 2d 303
Dec. 29, 1966.



APPENDIX 3:

Lynn S. **SCOTT** and Ann B. Scott, his wife and Frank H. Bjorndal and Audrey K. Bjorndal, his wife, Plaintiffs and Appellants,

v.

Willford **HANSEN** and Viola Hansen, his wife, Cecil Hansen and LaDonna Hansen, his wife, Marjorie Baker, Darrell A. Tate, Barbara Buckley and Michael E. Tate, Defendants and Respondents.

No. 10580.
Supreme Court of Utah.

422 P.2d 525, 18 Utah 2d 303
Dec. 29, 1966.

Kirton & Bettilyon, F. Burton Howard, Salt Lake City, for appellants.
Dwight L. King, Salt Lake City, for respondents.

CROCKETT, Justice.
Plaintiffs sued to quiet title to about five acres of land in the Butlerville area in southeastern Salt Lake County. The district court gave judgment for the defendants, adjoining landowners, and plaintiffs appeal.

The issues are these: (1) Whether the boundary between the plaintiffs' and the defendants' lands is along a county road, referred to in their respective deeds, as the road actually exists, or along a line south of it as the county road is shown on the county recorder's plats; and (2) whether the defendants have nevertheless by adverse possession under color of title established ownership of the land under Secs. 78-12-8 and 9, U.C.A.1953.

The grantor of both properties, one Magie Thompson, in the year, 1906, conveyed the plaintiffs' tract to their predecessors, describing the north boundary thereof as "along the south side of the county road." Seven years later, in 1913, she conveyed to the defendants' predecessors their tract to the north, describing the south boundary of the tract similarly as being, "along the south side of the county road." The circumstance which gives rise to this controversy is that the county road as it actually exists has always been a meandering road as shown in the drawing below whereas, it is shown on the county recorder's plat as a straight east-west line.

As will be seen from the drawing, if the boundary between the properties is regarded as the road as it actually exists, the disputed property belongs to the plaintiffs; whereas, if the boundary is as defendants contend, the straight line marked as a county road on the recorder's plat, the defendants own the land.

[1,2] In a situation such as this where a dispute arises as to the boundary between tracts conveyed to the parties by a common grantor, it becomes important to determine if possible the intent of the parties at the time of the conveyance.[1] There are rules of construction which have been adopted for the purpose of assisting in ascertaining and giving effect to such intent.[2] One of these is that fixed monuments or markers of a permanent nature which can be definitely identified and located take precedence over calls of courses or distances,[3] or plats,[4] or amounts of acreage.[5] This is so because it is reasonable to assume that the parties are more apt to be familiar with such monuments or markers than with precise measurements, or with recorder's plats; consequently, giving precedence to the call to such a monument or marker results in less possibility of error and a greater likelihood of giving effect to the intent of the parties.[6]

[3] In applying the principle just stated to this case, the conclusion seems clear that in specifying the county road as the north boundary of the plaintiffs' property; and again in designating it as the south boundary of the defendants' property, the deference was to the county road as it actually existed and was observable by the parties involved, rather than to the theoretical county road shown by the straight line on the county plat. Nor is the point made by the defendants persuasive that the distance along the existing road to the west quarter-section line is farther than the deed's call of 80 rods. Both deeds give the distance as "westerly along the south side of said road 80 rods more or less" to the said quarter-section line. This could just as well be taken to argue against defendants' position: that it relieves the call of exactness and shows the intent that the distance may be "more or less" and that in either event the call runs to said quarter, section line.[7] The defendants' claim of title by adverse possession under "color of title" is based upon a document subsequent to their deed. In 1935 the decree of distribution in the estate of their father, Andrew Hansen, Jr., who owned before them, distributed the property to them by metes and bounds without mentioning the county road. This is the basis of the defendants' claim of "color of title" to the disputed area. As a result of it, both the defendants and the plaintiffs have paid taxes on the property in their respective descriptions. The plaintiffs have not been in actual physical

possession of their property, although they did put up one or two "no dumping" signs and a few feet of fence along the road to prevent the public from dumping refuse upon it. The defendants have a residence to the north on their tract and have farmed portions of it for many years, but none of their farming operations has extended south of the road onto the disputed area.

[4] Section 78-12-8 and 9, U.C.A.1953, which the defendants rely on in claiming title by adverse possession provide in substance that if one is in possession of a "known farm" or other tract of land under a written instrument or conveyance giving color of title he may establish ownership by adverse possession without having actually possessed, cultivated or fenced the whole tract. This appears to be but a codification of the common law doctrine which arose out of teh practicalities of possessing land. It is obvious that a man cannot be in actual possession of every square rod of extended tracts of land such as are used, for example, in farming or grazing. The purpose of these statutes is to recognize and deal with this situation by providing that if one takes possession of the portion of a piece of land known as an integrated or unified tract, under circumstances such that others would know that he was claiming the whole tract, that constitutes possession of the whole. The pivotal consideration here is that there must be some actual occupation[8] of the property of such character or under such circumstances that the owner knows, or as a man of ordinary prudence should know, that the land was being held as his own by the adverse claimant.[9]

[5, 6] The difficulty with the defendants' position is their assumption that the requirement of possession of the claimed property is fulfilled if a part of the tract described in the document which gives "color of title" is so possessed whether it would give notice to the true owner or not. Inasmuch as the area in dispute here is physically separated from the defendants' property by the existing county road, and the defendants have never farmed or been in actual possession of any part of it, there is no basis upon which it could be regarded as part of a "known farm" or as otherwise unified with the defendants' tract of land. Our attention has not been directed to anything which the defendants have done with respect to possession of the disputed area which was hostile to and inconsistent with the rights of the plaintiffs so they would have notice that the defendants were claiming the property adversely.[10] Accordingly, the defendants' claim of ownership fails because neither mere "color of title" to property, nor their subjective intent to possess land adversely to the owner, is sufficient to establish title by adverse possession unless the possession is such as to give notice to the owner. This requisite is not met under the circumstances shown here by the defendants' possession of their own adjacent land. If the rule were otherwise, landowners would be placed under the unduly burdensome necessity of periodically checking the property descriptions of their neighbors to see that some document had not been placed of record which encroached upon their land.[11]

[7] Further support of the conclusions we have reached, and correlating with the principle just stated concerning the necessity of notice to the true owner that his property right is being claimed adversely, is found in considering the nature of the adverse possession statutes. They are contained in the chapter entitled, "Limitation of Actions." Like other statutes of limitation, they provide for a time limit in which a claim or defense must be asserted, or it is barred. But all statutes of limitation are predicated upon the proposition that the prescribed period does not begin to run against a party until a cause of action has arisen. In actions for possession of land,

this does not occur until the true owner's right of possession has been so invaded as to give rise to a cause of action. In this case the plaintiffs' right to possession of their land had not been so disturbed or encroached upon; consequently the statute had not run against them.[12]

The decree is vacated and the case is remanded to enter judgment in favor of the plaintiffs. Costs to plaintiffs (appellants).

CALLISTER and TUCKETT, JJ., concur.

McDONOUGH, J., heard the arguments but died before the opinion was filed.

HENRIOD, Chief Justice (concurring).

I concur for one reason: The metes and bounds description specifically mentioned exactly 80 rods for three of the courses, but described the fourth as 80 rods "more or less" along a county road. This raised a red flag which a title examiner would question, for inconsistency and lack of specificity in relation to the other three courses. This would indicate that the grantor meant the actual meandering road to be the boundary, not any physically unused road not reflected accurately in the plat.

. Losee V. Jones, 120 Utah 385, 235 P2d 132; Campbell V. Weisbrod, 73 Idaho 82, 245 P.2d 1052; Delphey V. Savage, 227 Md. 373, 177 A.2d 249.

. Delphey V. Savage, Supra; Wheeler V. Stanolind Oil and Gas Co., 151 Tex. 418, 252 S.W.2d 149; Re West Tenth Street, Borough of Brooklyn, City of New york, 267 N.Y. 212, 196 N.E. 30, 98 A.L.R. 634.

. Johnson Real Estate Co. V. Nielson, 10 Utah 2d 380, 353 P.2d 918; Home Owners' Loan Corp. V. Dudley, 105 Utah 208, 209, 141 P.2d 160; Lainhart v. Shepherd, (Ky.) 246 S.W.2d 460; 6 Thompson on Real Property 571 (Perm.Ed.) Sec. 3044.

. Rowell v. Weinemann, 119 Iowa 256, 93 N.W. 279.

. Calder v. Hillsboro Land Co., (Fla.App.), 122 So.2d 445; Thompson on Real Property, supra. 576.

. Bridges v. Thomas, (Fla.App.), 118 So.2d 549; Thompson on Real Property, supra, 576; Curran v. Maple Island Resort Assn., 308 Mich. 672, 14 N.W.2d 655, citing Moran v. Lezotte, 54 Mich. 83, 88, 19 N.W. 757, 759.

. Losee v. Jones, supra; Dwelle V. Greenshields, (Okl.) 305 P.2d 1038; Brewer v. Schammerhorn, 183 Kan. 739, 332 P.2d 526.

. Dougherty v. Looney, 108 Okl. 279, 236 P. 583; Wachovia Bank & Trust Co. v. Miller, 243 N.C. 1, 89 S.E.2d 765; McCoy v. Anthony Land Co., Inc., 230 Ark. 244, 322 S.W.2d 439; Chilton v. White, 72 W.Va. 545, 78 S.E. 1048; United Fuel Gas Co. v. Dyer, 185 F.2d 99 (4th Cir. 1950); Pender v. Jackson et al., 123 Utah 501, 260 P.2d 542.

. 3 Am.Jur.2d Adverse Possession § 47 (1962) and cases therein cited.

. See Salt Lake Investment Co. v. Fox, 32 Utah 301, 90 P. 564, 13 L.R.A.,N.S., 627.

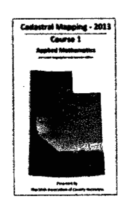
. McCoy v. Anthony Land Co., Inc, footnote 8, supra.

. Wheately v. San Pedro, Los Angles and Salt Lake Ry. Co., 169 Cal. 505, 147 P. 135.

APPENDIX 2a

Mapping And Parcel Identification Standards of Practice

Utah State Tax Commission
Property Tax Division
Rev. July 2010



Mapping and Parcel Identification Standards of Practice

8

Utah State Tax Commission
Property Tax Division
Rev. July 2010

Table of Contents

Section VIII.I

General Information

Purpose

The principal responsibility of the assessor is to locate, inventory, and appraise all property within the jurisdiction. A complete set of ownership plats is necessary to perform this function. The creation and updating of ownership plats are the responsibility of the county recorder (Section 17-21-21), but become important to the county assessor and the State Tax Commission during the valuation (assessment) of real estate. Plat maps help determine the location of property, indicate the size and shape of each parcel, and its relationship to pertinent features affecting value.

Assessors are no longer required by statute to keep a book of ownership plats of the parcels within the county. The county recorder is required to "prepare copies of ownership maps and plats" and to transmit these to the county assessor by January 30. (Section 17-21-22)

Scope

These standards of practice clarify the procedures of property identification and mapping for ad valorem taxation purposes.

Definitions

Ownership Plats

An ownership plat includes: graphic descriptions of parcels of land indicating their relative size and position to other properties, rivers, creeks, roads, and major geographic features. They are drawn to scale and delineate property dimensions and parcel serial number.

Parcel

A contiguous area of land described in a single description in a deed or as one of a number of lots on a plat, preferably in one general use, separately owned, and capable of being separately conveyed.

Geographic Information System (GIS)

Ownership plats and related tabular information stored and transferred in electronic format.

Automated Geographic Reference Center (AGRC)

The state agency charged by the legislature with the responsibility of coordinating government GIS data storage and transmission standards.

Section VIII.II

Standard 8.1 Required Maps

8.1.0 Mapping System

A basic mapping system for ad valorem tax purposes should include the following components:

Ownership Plats (Cadastral Maps)

Ownership plats contain detailed information prepared at appropriate scales. Plats should clearly indicate property boundary lines, dimensions, and should include any other pertinent identifying criteria such as parcel numbers.

City or County Index Systems

This index uses entire city or county maps to show the location (plat book and page) of each individual ownership plat.

Subdivision Index

A subdivision index lists all recorded subdivisions. The index also contains an ownership plat reference page for each subdivision with subdivisions listed in both alphabetical and numerical order.

Electronic Mapping and Storage

Ownership plats may be maintained in a geographic information system (GIS). Related index systems and associated data may be maintained either in GIS or within an electronic database, so long as all information required by *Standard 8.1.1* is included.

8.1.1 Plat Content – Basic Information

Ownership plats shall contain the following:

- Boundaries of individual parcels–ownership plat parcel boundaries shall be located by utilizing the most recent conveyance of property for each parcel,
- Parcel dimensions and bearings when applicable,
- Names and boundaries of subdivisions, plats and surveys, including lot, block and survey numbers,
- County, township, range, section and government lot boundaries and numbers,
- Boundaries of political subdivisions, e.g., county lines, city limits, and service districts comprising the various tax areas and entities,
- Location and names of streets, highways, alleys, railroads, rivers, lakes, etc., that are a matter of public record,
- Parcel identification numbers,
- Date and source of the most recent plat change,
- The scale to which the plat is drawn,
- Coordinate-geometry traverse definitions derived from legal descriptions when plats are maintained in a GIS,

- Other basic plat information, including a plat number, title block, plat key, north arrow, (unless the top of the map is assumed to be north), keys to adjoining plats, and,
- Abstract information, such as vesting document numbers and dates.

8.1.2 Plat Content – Supplemental Information

Ideally, supplemental information should be recorded on overlays or in a computer database. This allows access to as much or as little data as required without changing the original plats. It also facilitates use of the map data by others. Ownership plats may contain:

- Names of recorded owners
- Parcel address
- Parcel acreage
- Location of streets, trails, streams, etc., that are not a matter of public record.

8.1.3 Plat Design

Ownership plats shall incorporate the following essential design:

Uniform Plat Sizes

A uniform size for ownership plats facilitates handling and storage. The accepted size for plats is 18" x 18". Plats should be drawn in a size adequate to show scales of 1" = 100' for quarter/quarter sections, 1" = 200' for quarter sections, and 1" = 400' for sections. Headings and title blocks must be included on the ownership plats.

Standard Plat Scales

The proper scale is one, which covers the largest possible area and at the same time shows the necessary detail. The size of the majority of the parcels in the area to be covered by a single plat and the lettering standards employed are determining factors in choosing plat scales. In general, large-scale plats should be used for urban areas and small-scale plats should be used for rural areas. Plat scales also should be easily convertible from feet to meters. The following are commonly used platting scales:

Urban areas: 1"=50', 1"=100',1"=200'

Rural areas: 1"=400', 1"=800'

Exceptions to these scales are situations of multiple-section ownerships, e.g., federal and state land.

The following standards apply to ownership plats:

Standardized Plat Symbols, Lines, and Letters

The use of standardized symbols, lines, and letters is essential to maintaining useable plats.

Standardized Plat Layouts

Plat layout should be standardized containing a title heading, a plat key, and keys to adjoining plats.

Plat Boundaries

Streets and survey lines should be utilized as plat boundaries whenever possible. Plat boundaries should be drawn so parcel extensions between plats are minimized.

Recorded Plats of Subdivisions

Subdivisions shall be drawn on the ownership plats in a legible scale.

Plat Material

All ownership plats shall be produced on stable base, reproducible, polyester film. If film cannot be used, a drafting media that closely approximates the advantages of film shall be used. Additionally, ownership plats may be developed and maintained in a GIS capable of producing accurate hard-copy prints on a stable base. Backup copies of disks or tapes shall be stored off site.

Coordination of GIS Plat-Design Standards

To facilitate uniform plat design, counties shall conform to the current state plat standards. Counties utilizing GIS should also coordinate with AGRC for additional GIS uniformity.

8.1.4 Plat Maintenance

Ownership plats shall be continuously updated. A set of ownership plats represents a substantial capital investment, so regular and proper maintenance is critical. Plat maintenance involves recording description changes and making plat corrections. There should also be a plan for re-platting areas at a larger scale in order to satisfactorily depict new subdivisions.

Standard 8.2 Identification of Properties Being Assessed

8.2.0 Parcel Identification Systems

Proper identification of properties is essential to accurate valuation. A parcel identification system provides a method for referencing land parcels or data associated with parcels, using a number or code instead of a complete legal description. The correlation of plats and individual property records requires all property files to be indexed using uniform parcel identifiers. Each parcel shall be assigned a unique identification number or code, as required for the permanent appraisal record. (R884-24P-37) The assessment roll and all other assessment files should be organized and filed according to the parcel identification number. These parcel identification numbers shall be used on all tax maps, property record cards, assessment and tax rolls, the notice of valuation and tax change, the property tax notice, and other property tax and land records.

8.2.1 Kinds of Parcel Numbers

It is important that the parcel numbering system allow the user to locate the parcel by means of the unique number. There are three kinds of numbering systems, which are suitable for this purpose.

Map-base

In a map-base system, the first number sequence is assigned to a map, the second number sequence to a block or other organized map division, and the last to the individual parcel. An alternative map-base system is called a book-page system. The first number sequence is assigned to a book of maps, the second number sequence to a map page within the book, and the third to a parcel on the map. See Appendix 8C for an example.

Government Survey

The government survey takes for a base the existing land survey system of townships and ranges. The numbering sequences correlate with sections, quarter sections and other such subdivisions as necessary until the last number sequence, which refers to the individual parcel. See Appendix 8D for an example.

Geographic Coordinate Code (Geocode)

The geocode system is typically used on a computerized (GIS) system where the approximate center (paracentroid) of each parcel is identified. The parcel is numbered based on its x (east-west) location and y (north-south) location. Although the geocode system is accurate and quick, the parcel numbers do not tie directly to any given map or block. See Appendix 8E for an example. When a GIS mapping system is utilized, coordination with the AGRC is required to facilitate uniformity, data transfer, numbering schemes and formats used in parcel identification.

8.2.2 Desirable Characteristics

Parcel identifiers should incorporate the following attributes: (1) uniqueness, (2) permanence, (3) simplicity, (4) ease of maintenance, (5) flexibility and (6) reference to geographic location. Of these, uniqueness is most important.

Uniqueness

Uniqueness refers to a one-to-one relationship between a parcel and its identification number. An identification number shall be assigned to only one parcel.

Permanence

Parcel identifiers should be permanent and change only if the boundaries of the parcel change and a new parcel is created. However, in areas where there is extensive subdivision requiring re-platting, it may become necessary to assign new parcel identification numbers even though some parcel boundaries have not changed. Whenever a new parcel is created, it should be assigned a new parcel identification number.

Simplicity

Parcel identification numbers should be easy to understand and have as few digits as possible. A parcel identification number that is uncomplicated and easily understood will help reduce errors in its use.

Ease of Maintenance

The parcel identification system should be easy to maintain and should efficiently accommodate changes, such as subdivision or consolidation of parcels.

Flexibility

The parcel identification system should be reasonably flexible. It should be capable of serving a variety of uses and be convenient for both field and office operations.

Reference to Geographic Location

The parcel identification system based on geographic location makes it possible to locate a parcel using only the identifier. The identifier becomes an abbreviated legal description.

Standard 8.3 Assigning Parcel Identification Numbers

8.3.0 Assigning Numbers

The responsibility for assigning parcel identification numbers belongs with the county recorder. The recorder shall maintain a complete and accurate record of all information necessary to assign appropriate numbers. This information includes abstract, ownership, tax entities, tax areas and individual parcel legal descriptions.

8.3.1 Divisions and Combinations

When parcels are divided (split) or combined, new parcel numbers shall be assigned and old parcel numbers permanently retired.

8.3.2 Number Assignment

Responsibility for assigning new parcel numbers and eliminating old parcel numbers should be clearly identified and closely controlled within the recorder's office. Only a few individuals should have authority to assign or change parcel numbers. If changes are automated, access to the computer program should be limited.

8.3.3 Requiring Parcel Identification Numbers on Documents

To facilitate the abstracting of recorded documents, the county recorder may require that the parcel identification number(s) (tax serial number) appear on each document before it is accepted for recording, where appropriate. This requirement must be posted in a conspicuous place in the recorder's office. The parcel identification number may not be considered to be part of the legal description of the parcel, and may be located on the margin of the document. If the identification number is in error, it does not affect the validity of the document or the effectiveness of its recording. (Section 17-21-20)

Standard 8.4 Special Property Identification Considerations

8.4.0 Special Identification Considerations

In general, identification is a simple linking of a property identification number to a plat or to a legal description. However, the variations listed in this standard represent special problems in identifying real property.

8.4.1 Multiple Taxing Districts

If a property straddles two or more taxing areas within a county, each portion must be identified separately. Separate identification is necessary since each area has its own tax rate and will collect taxes only on the portion of the property within its boundaries. The system of identification must be designed to accommodate this circumstance.

8.4.2 Divided Interest

When land and improvements have separate owners, the separate interests also must be identified. The parcel identifier must indicate if a particular parcel consists solely of improvements, or if it is only the land beneath those improvements.

8.4.3 Undivided Interest

An undivided interest in a property consists of ownership of only a percentage of the property, yet without a physical division of that property. The identification system must clearly indicate partial ownership to limit possible confusion.

8.4.4 Common area parcels on a Plat

A parcel designated as common area on a plat may not be separately owned or conveyed independent of the other parcels created by the plat. For purposes of assessment, the ownership interest is to be divided equally among all parcels created by the plat, unless a different division of interest is indicated on the plat or an accompanying document. The ownership interest is considered to be included in the description of each instrument describing a parcel on the plat by its identifying plat number, even if the common area interest is not explicitly stated in the instrument. (Section 10-9-806.5)

8.4.5 Abandonment of Roads

When a highway, street or road is abandoned, the county recorder's office, upon an order executed by the proper authority, is to vest title to the vacated or abandoned highway, street or road to the adjoining record owners; one half of the width of the highway, street or road is to be assigned to each of the adjoining owners. [Section 72-5-105(2)]

- If a property description of an owner of record extends into the vacated or abandoned road, that portion is t be vested to the record owner as well as the additional property up to ½ of the width of the road.
- If a property description of an owner of record extends beyond ½ of the width of the road, that portion is to be vested to the recorded owner, with the remainder of the road vested to the other adjoining owner.

8.4.6 Other Special Considerations

Other special considerations to be identified include air rights, mining claims, and privilege tax properties. Assessors and recorders must develop the identification system so as to clearly distinguish such variations.

Standard 8.5 Property Identification Aids

8.5.0 Property Identification Aids

A number of aids are available to assist the assessor and recorder in identifying and describing property. Some are listed in this standard. Although these tools and methods will assist property tax administrators in their work, they are not intended to replace the established procedure for describing property. This established method is to extract the description from the recorded vesting document and mathematically determine the description of the remaining parcel. Any photographic, digitized or Global Positioning System (GPS) location information is merely extra information to aid in the identification and location of properties.

8.5.1 Aerial Photography

Aerial photography is an important tool in the discovery of real property improvements and verification of real property use. Aerial photos are particularly useful to help discover escaped improvement assessments in remote areas, change of agricultural land use, and as an additional mapping layer in GIS. The photography should be taken with a sufficient number of ground control monument, paneled or targeted so they can be identified.

Information and descriptions from aerial photos can be digitized into a GIS system. However, such information is for reference only and must not be used to replace descriptions created by the recorder's office from vesting documents.

8.5.2 Global Positioning System (GPS)

Hand-held instruments are now available to identify position, elevation and distance anywhere on the earth. This Global Positioning System (GPS) is based on the relative location from any of twenty-four NAVSTAR satellites in geo-synchronous orbit 11,000 miles above the earth. The primary advantage of GPS is that the property tax administer can determine the location of any point without maps in the field. Comparing information from field sightings with ownership plats in the office can determine property ownership. This will help identify improvements, which are not on the assessment roll, location of gravel pits and oil drilling rigs, and other properties far from urban areas. GPS is an important tool for the property tax administrator, but is not to be used to create parcel descriptions.

Appendix 8A

Sources of Maps

Aerial Photographs

- United States Department of Agriculture
- Farm Service Agency (FSA)
  Phone (801) 975-3503, Fax (801) 975-353
  2222 West 2300 South
  Salt Lake City, UT 84119
- Regional and County Planning and Zoning Office
- Private aerial photography providers and contractors

Base or Section Maps

- County Surveyor's Office

Contour Maps or Topographic Maps

- Utah Geological Survey Department of Natural Resources
  Phone: (801) 537-3300, Fax: (801) 537-3395
  1594 W. North Temple
  Salt Lake City, UT 84116
- Private vendors

Plat Books and Subdivisions

- Commercial Mapping Companies
- County Recorder's Office

Soil Survey Maps

- Natural Resources Conservation Service (NRCS)
  Phone: (801) 524-4572, Fax: (801) 524-4403
  125 South State, Room 4010
  Salt Lake City, UT 84147
- Utah State University Extension Services
  Offices located in each county

Computerized Parcel Layers

- Automated Geographic Reference Center (AGRC)
  Phone: (801) 538-3165
  3162 Capital Office Building
  Salt Lake City, UT 84114

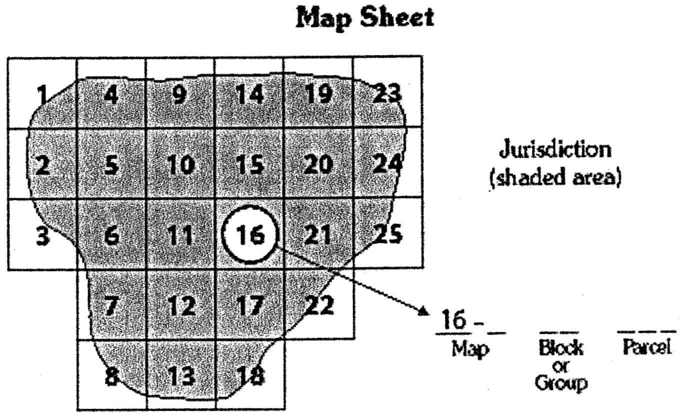
Appendix 8B

Standard Abbreviations Permitted in Property Tax Proceedings

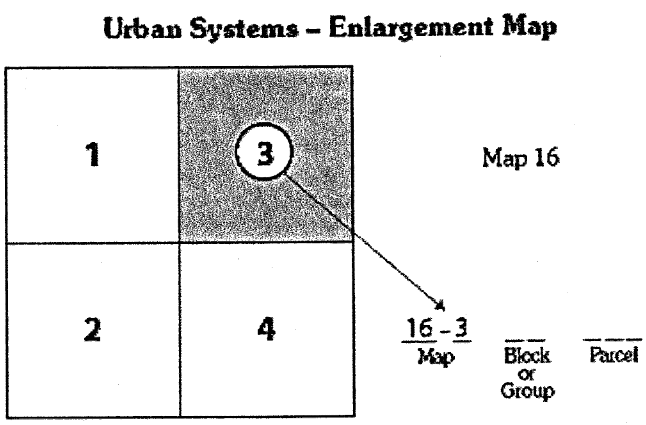
a., ac	for acre, acres
add.	for addition
ave.	for avenue
beg.	for beginning
blk.	for block
bet.	for between
bdy., bdrs.	for boundary, boundaries
ch., chs.	for chain, chains
com.	for commencing
cont.	for containing
deg.or °	for degree, degrees
dist.	for distance
E	for East
E'ly	for easterly
ft.	for foot, feet
frac.	for fractional
in., ins.	for inch, inches
lk., lks.	for link, links
lt., lts.	for lot, lots
m., min., or '	for minute(s)
m. or l.	for more or less
N	for north
NE	for northeast
NE'ly	for northeasterly
N'ly	for northerly
NW	for northwest
NW'ly	for northwesterly
pt.	for point
¼ sec.	for quarterly section
r., rs.	for range, ranges

rd., rds.	for rod, rods
ROW	for right of way
s. or "	for second, seconds
SE	for southeast
SE'ly	for southerly
st.	for street
sub.	for subdivision
S.L.M.	for Salt Lake Meridian
SW	for southwest
T., tp., tps.	for township, townships
th.	for thence
U.S. sur.	for Unites States Survey
U.S.M.	for Uintah Special Meridian
W	for West
W'ly	for westerly

Appendix 8C
Map-based Parcel Identification System

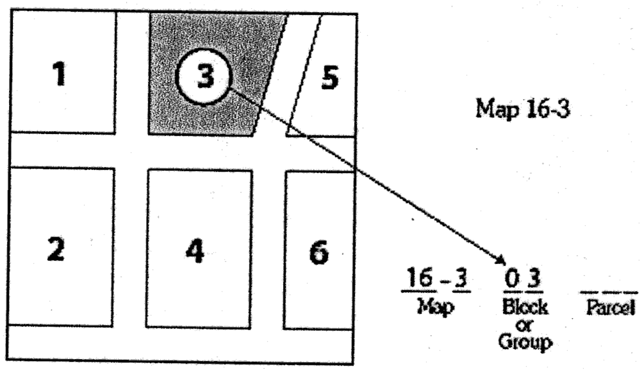


The map sheet is the basic component. In this example, the sheets are numbered consecutively north to south beginning in the northwest corner of the jurisdiction.



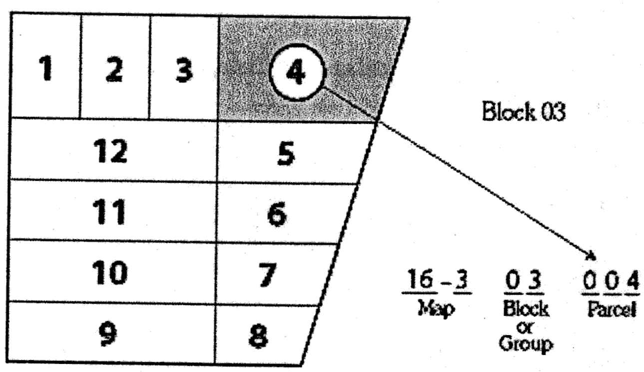
In urban or congested areas, the second component is an enlargement map. In this example, the enlargement maps are designated as suffixes of the basic (rural) map sheet.

Block Number



In urban areas, the third component is the block number. A block or group is one or more contiguous parcels completely surrounded by roads, streams, railroads, or the margin of the map.

Parcel Number

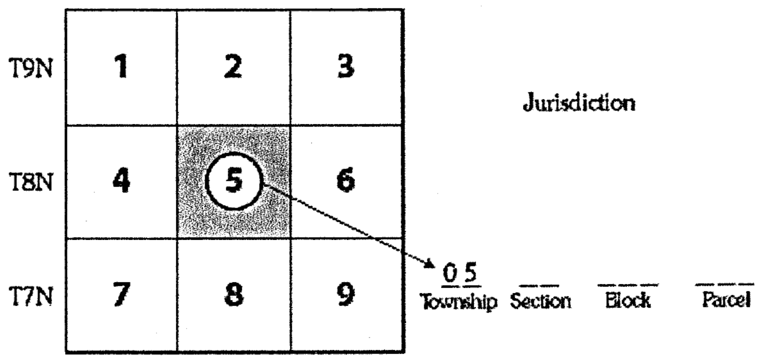


In urban areas, the fourth component is the parcel identifier, assigned consecutively within each block.

Appendix 8D

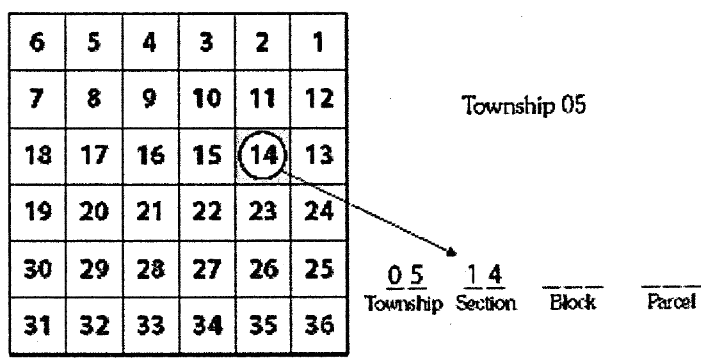
Government Survey Parcel Identification System

Townships



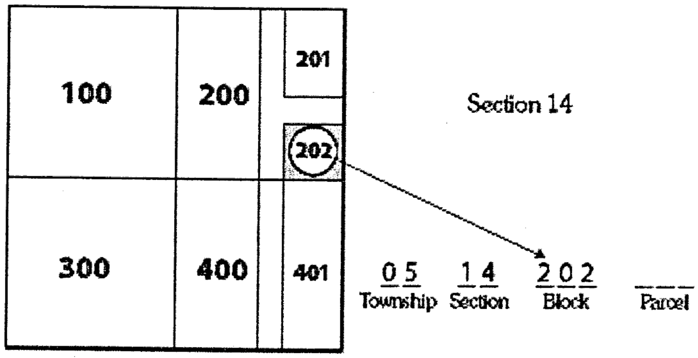
The township is the basic component of this system. In this example, all townships within the jurisdiction are assigned consecutive numbers from west to east. An alternative approach would be to use tier and range directly, such as: Tier – 08; Range – 14

Section Number



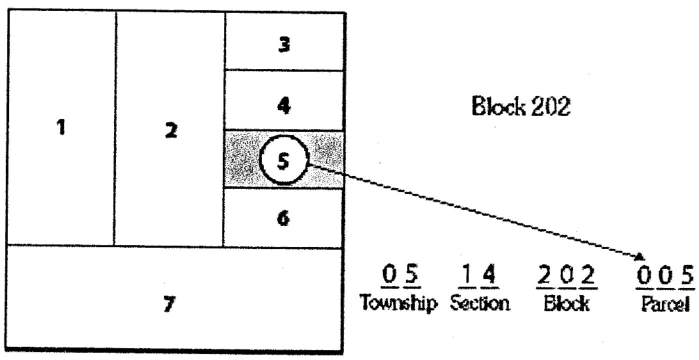
The second component is the section number.

Block Number



The third component is the block designation. In this example, the northwest quarter is assigned 100–199, the northeast quarter is 200–299, the southwest quarter is 300–399, and the southeast quarter is 400–499.

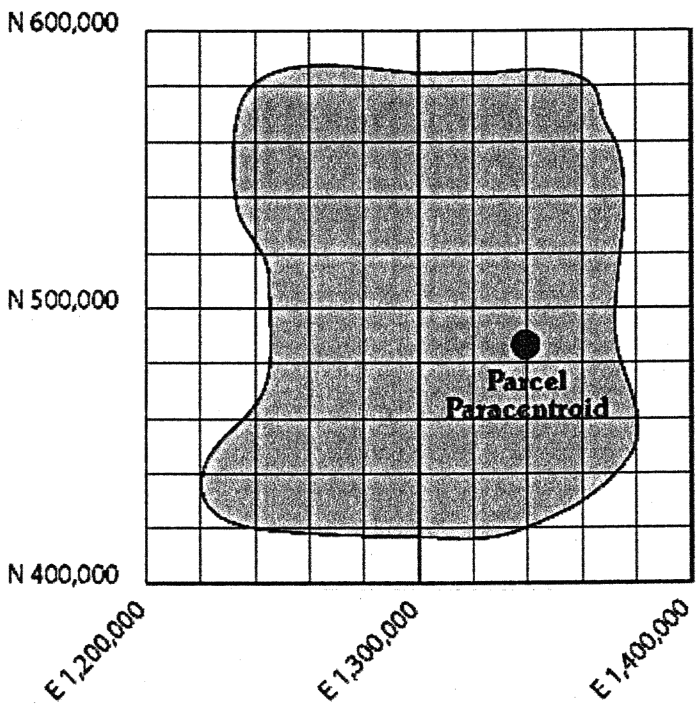
Parcel Number



The fourth component is the parcel designation. In rural areas, it is common for an entire quarter section to be under one ownership. If so, the parcel number is always 001. For example, if the southwest quarter of section 14 above is one parcel, the designation would be 05 14 202 001.

Appendix 8E

Geographic Coordinate Code Parcel Identification System



1340000	0482416
Easting	Northing

or

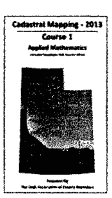
10	34	48	02	04	01	06
EN	EN	EN	EN	EN	EN	EN

The entire jurisdiction is covered by a contiguous grid system. The geocode of the paracentroid (visual center) of the example above is 1,340,000 feet east and 482,416 feet north. This may be written using either of the methods shown. Note that the number of digits could be shortened by identifying the paracentroid to the nearest ten feet.

APPENDIX 2b

Standard Abbreviations

Authorized by Utah Code
(UCA 59-2-1335)



Appendix 8B

Standard Abbreviations Permitted in Property Tax Proceedings

a., ac	for acre, acres
add.	for addition
ave.	for avenue
beg.	for beginning
blk.	for block
bet.	for between
bdy., bdrs.	for boundary, boundaries
ch., chs.	for chain, chains
com.	for commencing
cont.	for containing
deg.or °	for degree, degrees
dist.	for distance
E	for East
E'ly	for easterly
ft.	for foot, feet
frac.	for fractional
in., ins.	for inch, inches
lk., lks.	for link, links
lt., lts.	for lot, lots
m., min., or '	for minute(s)
m. or l.	for more or less
N	for north
NE	for northeast
NE'ly	for northeasterly
N'ly	for northerly
NW	for northwest
NW'ly	for northwesterly
pt.	for point
¼ sec.	for quarterly section
r., rs.	for range, ranges

rd., rds.	for rod, rods
ROW	for right of way
s. or "	for second, seconds
SE	for southeast
SE'ly	for southerly
st.	for street
sub.	for subdivision
S.L.M.	for Salt Lake Meridian
SW	for southwest
T., tp., tps.	for township, townships
th.	for thence
U.S. sur.	for Unites States Survey
U.S.M.	for Uintah Special Meridian
W	for West
W'ly	for westerly

APPENDIX 2c

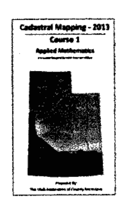
Standard on Digital Cadastral Maps

and

Parcel Identifiers

Approved January 2012

by the IAAO



Standard on Digital Cadastral Maps and Parcel Identifiers

Approved January 2012

International Association of Assessing Officers

This standard revises and replaces the July 2003 *Standard on Digital Cadastral Maps and Parcel Identifiers*.

The assessment standards set forth herein represent a consensus in the assessing profession and have been adopted by the Executive Board of the International Association of Assessing Officers. The objective of these standards is to provide a systematic means by which concerned assessing officers can improve and standardize the operation of their offices. The standards presented here are advisory in nature and the use of, or compliance with, such standards is purely voluntary. If any portion of these standards is found to be in conflict with the *Uniform Standards of Professional Appraisal Practice* (*USPAP*) or state laws, *USPAP* and state laws shall govern.

Acknowledgments

At the time of the 2011 revision (approved January 2012) the Technical Standards Committee was composed of Alan Dornfest, AAS, chair; Doug Warr, AAS; Bill Marchand; Robert Gloudemans; Mary Reavey; Dennis Deegear, associate member; and Chris Bennett, staff liaison.

Published by
International Association of Assessing Officers
314 W 10th St
Kansas City, Missouri 64105-1616

816/701-8100
fax: 816/701-8149
http://www.iaao.org

ISBN 978-0-88329-203-7 edition

Printed in the United States of America.

Contents

Standard on Digital Cadastral Maps and Parcel Identifiers

1. Scope

This standard provides recommendations on the development and maintenance of digital cadastral assessment maps, parcel data layers in a geographic information system, and parcel identifiers. It describes digital mapping system components, content, design, creation, maintenance, and contracts. It also discusses deed processing and parcel identification systems. This standard addresses computerized mapping systems; see the *Standard on Manual Cadastral Maps and Parcel Identifiers* (IAAO 2004) for information on manual parcel mapping. Technical aspects of the standard and technical recommendations may apply universally; however, many of the specific recommendations are more pertinent for local jurisdictions maintaining in house parcel-mapping systems. State mapping functions are not addressed in this standard, although many aspects of the standard may be applicable to state and provincial systems provided for local assessor use.

2. Introduction

The principal responsibility of the assessor is to locate, inventory, and appraise all property within the jurisdiction. A complete set of maps is necessary to perform this function. Maps help determine the location of property, indicate the size and shape of each parcel, and reveal geographic relationships that affect property value. Maps and map data are important not only for assessors, but also for other governmental agencies, the public, and the land information community (such as realtors, title companies, and surveyors). In addition, the assessor must track current ownership of all parcels, so that the proper party can receive assessment notices and tax bills. Computerization of map and parcel data can enhance the capability to manage, analyze, summarize, display, and disseminate geographically referenced information.

2.1 Computerized Mapping

Computerized mapping systems may be referred to by several names. They include:

- Geographic information system (GIS)
- Land information system (LIS)
- Digital multipurpose cadastre
- Multipurpose land information system (MPLIS)
- Land parcel database

For purposes of this standard and for consistency with other IAAO standards the term "digital cadastral mapping system" will be used.

2.2 The Value of Digital Cadastral Maps

Working with digital cadastral maps and tabular parcel-related data in a GIS, users can selectively retrieve and manipulate layers of parcel and spatial information to produce composite maps with only the data they need. Sharing GIS files over an internal or external data network makes parcel maps and related attribute information widely available, and reduces the duplication of effort inherent in separate map systems. Such sharing is becoming increasingly sophisticated, ranging from allowing users to download data or prepared maps, to allowing users to make sophisticated queries that may draw on the power of the host GIS's software and hardware.

2.3 Components of a Digital Cadastral Mapping System

A digital cadastral mapping system should have the following components:

- Reference to a geodetic control network
- Current base map layer (ideally, photogrammetrically derived)
- A cadastral layer delineating all real property parcels
- Vertical aerial photographs and/or images (ideally, orthorectified)
- A unique parcel identifier assigned to each parcel
- A means to tie spatial data to attribute data (ownership and parcel characteristic files)
- Additional layers of interest to the assessor, such as municipal boundaries, zoning, soil types, and flood plains

2.4 The Role of the Assessor

The assessor may assume many different roles in the management of a jurisdiction's digital mapping system or GIS. The assessor may have the lead role in mapping parcels and also street centerlines, crime scenes, zoning, and other layers for other offices. In contrast, the role may be limited to maintaining a parcel layer, leaving responsibility for other layers to the relevant offices (for example, crime scenes to the sheriff) or to a central office (a jurisdiction-wide GIS agency, or an information technology department). In any case, the assessor must retain the ultimate authority to inventory, create, and define parcels and parcel identifiers for property tax purposes.

3. Elements of a Digital Cadastral Mapping System

A mapping system for assessment purposes includes the maps, accompanying records, and resources to support mapping. In the digital environment it should be designed to work seamlessly as a key component within a GIS. The data format and map projection should be compatible with other GIS data layers in the jurisdiction, and it should be well documented, with metadata that explains how it was created and how it will be maintained. At its core, it should contain the following elements:

3.1 Geodetic Network

A geodetic control network consists of monumented points whose locations on the surface of the earth are defined with certainty. These points may be described in terms of latitude and longitude, but are more commonly used when projected to state plane coordinates. Additional points in the field are often collected in preparation for georeferencing vertical aerial photography and base maps. Density and placement of control points should be related to map scale, population density, property value, accuracy specifications, and anticipated product lifespan. Today, professional land surveyors are using global positioning systems (GPS) to locate such points with a very high level of accuracy.

In the future, improvements in GPS satellite signals, receiver equipment, satellite based augmentation systems (SBAS), and positioning techniques may reduce the need for dense on-the-ground control networks. A few precise control points tied to continuously operating reference stations (CORS)—especially those that broadcast their signals, may fulfill this function. Assessors should support efforts to create such stations and to mandate their use when preparing plats, surveys, and property boundary descriptions.

3.2 Base Map Layers

Base maps locate the major physical features of the landscape such as roads, water features, elevation contours, fence and hedge lines, and building footprints. In some jurisdictions, they contain the fundamental information from which the cadastral maps are prepared. Base maps should be tied to the geodetic network. They are typically prepared using photogrammetric methods and include attributed: points (power poles, fire hydrants, etc.); lines (curbs, ditches, and fences, etc.); and polygons (ponds, building footprints, etc.). Base map development in urban areas requires very sophisticated techniques and equipment. This work is typically performed by professional photogrammetry firms.

In more rural and remote areas, base map needs may be met by a national mapping program's digital topographic maps or orthophotoquads, or by other orthoimages. Examples are the U.S. Geological Survey's Digital Raster Graphics (DRG's) or Digital Line Graphs (DLGs), and the National Aerial Photography Program (NAPP) orthophotos.

Another key map layer comprises elevation data. An advanced GIS should contain a digital terrain model (DTM), or digital elevation model (DEM) that will enable a three-dimensional representation of the ground. Thus, the assessor can visualize geographic features such as flood plains or view lots. The DEM/DTM will also provide the foundation for development of orthophotos (orthorectification). The traditional DEM/DTM product was developed through stereo image models obtained in conjunction with aerial orthophotography. However, many jurisdictions now acquire high-resolution digital elevation data through a separate remote sensing process that employs airborne LiDAR (light detection and ranging) technology.

3.3 Cadastral Map Layers

The cadastral map layers should be tied to the base map layer and should show all parcels in the assessing jurisdiction. Each parcel polygon should be attributed with a unique parcel identifier. Parcel boundary lines should be attributed or annotated with bearing, distance, and curve data. Details on map creation and maintenance are provided in sections 4 through 7.

3.4 Additional Map Layers

A multipurpose cadastre should have a variety of layers. Polygon map layers that can be of great value to the assessor include municipal and taxing district and school district boundaries, appraisal neighborhoods, soil types, zoning, subdivision boundaries, and flood insurance rate areas. Linear map layers may include features such as street centerlines, utility lines, and transit lines. Point map layers may include locations of fire and police stations, public buildings and schools. In smaller jurisdictions, the assessor may be responsible for creating and maintaining some or all of these GIS layers; in a large jurisdiction, they may be developed and maintained by other offices or agencies.

3.5 Parcel Identifiers

Each parcel should be keyed to a unique identification number or code that links the cadastral layer with files containing data such as ownership, value, use, and zoning. The parcel identifier provides a common index for all property records and may help track changes in legal descriptions in a rigorous and more manageable way. Parcel identification systems are detailed in section 8.

3.6 Ownership Information

The current owner, owners, and/or parties of interest should be identified for each parcel. In addition, the basis of ownership (recorded deed, contract, court decree, and so on) should be documented. It is desirable to maintain records of past ownership history. Deeds and other ownership documents should be processed within two weeks of recording (National Research Council 1983, 56), although with adequate staffing and technology, deeds can be processed within twenty-four hours or less. Details on ownership database maintenance are in section 6.

3.7 Imagery

Vertical aerial photographs based on film have long been an essential product for base map development. They are being increasingly replaced by digital images from aircraft. Imagery has greater value when it has had all distortions removed so that it closely matches the geodetic control, can serve as a base map, and meets the measurement tolerances required for the cadastral layer. Such images are called orthophotos, orthorectified images, or "orthos." Jurisdictions should acquire new imagery of urban areas at least every five years and of rural areas at least every ten years. Jurisdictions experiencing rapid or slow growth may need to adjust this timetable. Aerial imagery (and photogrammetric work done to create base maps) should meet industry recognized standards for scale, positional accuracy, resolution, and other requirements (URISA/IAAO 1999; U.S. Geological Survey 1986; Federal Geographic Data Committee 1996, 1998a, 1998b; and American Society of Photogrammetry and Remote Sensing 1989).

3.8 Map Products

The assessor should make cadastral data available in a variety of formats. Providing Internet maps can allow easy public access; an Intranet can give similar access to all offices in the jurisdiction; a virtual private network (VPN) can facilitate data dissemination and sharing with stakeholders at remote sites. No matter how advanced the capabilities of the office, a set of current printed maps should be available to the public and staff.

Digital cadastral maps, whether viewed on-screen or in printed form, should include the following elements:

- Boundaries of all parcels
- Parcel identifiers
- Parcel dimensions and areas
- Easements that influence value
- Subdivision or plat boundaries, as well as block and lot numbers
- Boundaries and names of political subdivisions, such as counties, towns, townships, and municipalities
- Boundaries and names of geographic subdivisions, such as sections, townships, government lots, land districts, and land lots
- Locations and names of streets, highways, alleys, railroads, rivers, lakes, and other geographic features

Printed maps should include other basic information: a map sheet number, title block, map scale, map legend, north arrow, key or link to adjoining maps, quality standard achieved, update log, and a disclaimer.

3.9 Facilities and Equipment

Cadastral mappers and deed processors should have a minimum of 100 square feet (10 square meters) of workspace per person. Additional adequate space should be provided for specialized equipment and storage. The process of converting from manual to digital maps may create a temporary need for additional space if the project is performed in-house; see *Standard on Facilities, Computers, Equipment, and Supplies* (IAAO 2003).

3.10 Program Management

Responsibility for mapping program management should be clearly assigned. Managers must ensure that map products meet assessment needs. A manager's duties may include

- Producing new cadastral and associated map layers (see sections 4 and 5)
- Maintaining existing layers and ownership records (see section 6)
- Controlling quality of map production and maintenance (see section 7)
- Contracting for mapping services such as aerial imagery
- Coordinating map layer responsibilities with other stakeholders
- Sharing and selling map products
- Creating and maintaining metadata that describes map products
- Public relations with the land information community and the public
- Purchasing hardware and software
- Creating and maintaining procedure manuals
- Staying current with national and industry standards
- Training personnel
- Creating and maintaining budgets

3.11 Staff and Training

An effective digital cadastral mapping and deed-processing program requires approximately one staff person per 10,000 to 20,000 parcels. This number may be modified depending on the following factors:

- Degree of automation and efficiency in deed processing and mapping work flows

- Economies of scale in larger jurisdictions
- Need to create or recreate digital map layers
- Volume of deed processing work
- Ratio of ownership name changes to transfers that create new parcels.
- Volume of new subdivision and condominium plats filed
- Need to respond to public requests for map and ownership information
- Reliance on contracted mapping services
- Need to create and maintain layers for non-assessment purposes, such as zoning, transportation planning, and emergency response
- Ease of deed processing, especially if transfers contain an accurate parcel identifier

All mapping personnel should receive training in procedures that are appropriate to the jurisdiction. At a minimum, mapping and deed-processing staff should understand the engineering basis of highway and railroad rights-of-way, the surveying basis of boundary creation and description throughout the history of the jurisdiction, and appropriate legal principles of boundary and title law. Once these basic competencies have been achieved, staff should be trained in techniques of mapping with coordinate geometry (COGO), computer aided drafting (CAD), and/or GIS software.

3.12 Procedures, Standards, and Records

A procedure manual for deed processing, as well as for the production and maintenance of cadastral and other map layers, should be developed to ensure that this work is accomplished in a uniform manner. The manual should include detailed standards for map layers and associated databases. It must be continually maintained to reflect procedural changes. In accordance with applicable statutes and ordinances, records used in preparing and maintaining the maps should be retained. These map creation and maintenance processes should be included in metadata that accompany map layers. The Urban and Regional Information Systems Association (URISA) has published a complete digital cadastral mapping procedure manual (URISA 1999).

4. Preparation for a Digital Cadastral Mapping Program

Extensive preparation, planning, and testing are necessary before fully embarking on a new or revised digital mapping program. An introductory level guide to many of these issues has been published (URISA/IAAO 1999); a much more detailed guide is also available (FGDC 1994).

4.1 Needs Analysis

Adequate preparation is essential before a new digital mapping program is undertaken or an existing one is revised. The jurisdiction must first evaluate the assessor's mapping needs, with outside assistance if needed. This analysis should include the following steps:

- Review applicable statutes, rules, regulations, and standards
- Inventory office functions, practices, and resources
- Determine the type of finished product and the accuracies desired and required
- Evaluate existing and needed personnel, facilities, software, hardware, and operating systems
- Determine the funding available

This process should then be extended to the entire jurisdiction and to potential stakeholders and end users. At that point, decisions should be made on the following factors:

- Type of digital maps to create (see section 4.2)
- Need for cooperation with other entities that have
  - already created useful map layers
  - want access to cadastral map layers
- Mix of in-house versus contracted work required to accomplish goals
- Optimal work flow
- Technical specifications for the digital map layers
- Preliminary schedule for the work

4.2 Selection of Type of Digital Cadastral Map

Based on the needs analysis, a type of digital cadastral map must be selected. In general, there are five types of digital maps, ranging from scanned images of the manually prepared legacy maps to maps created through a parcel data model. The desired quality of the digital cadastral map layer is a critical decision depending on budget and goals. A jurisdiction might decide to proceed through all five types over a period of years. However, the fifth type, based on a parcel data model, is the best and should be the goal of all jurisdictions.

4.2.1. Scanned Maps

Existing legacy paper maps or drafting film sheets can simply be scanned into a raster image. These can be difficult to maintain in this raster form, and do not have linked data, but can be easily accessed and shared by anyone with a computer. They are relatively inexpensive to produce, with minimal training and software needs. They also can perpetuate problems that result from legacy maps that were not based on a geodetic network, or were subject to compilation errors. Such maps can be useful during the migration to the next three types of maps (4.2.2, 4.2.3 and 4.2.4), and they can become valuable historical records documenting the end of manual map maintenance, and the start of digital mapping.

4.2.2. Scanned and Georeferenced Maps, with Data Points

Such scanned maps may be referenced to a geodetic control network, so they can be displayed with other georeferenced data layers. These are difficult to maintain, but easily shared. The scanned maps can have a data point placed in each parcel's approximate center (centroid). The attributes of these data points—such as owner name, situs address, assessed value and property characteristics, can be displayed and queried.

4.2.3. Trace-Digitized Polygons

Digital polygons can be created by tracing images (orthophotography or scanned images of hard copy maps) on a large high-resolution monitor (heads-up digitizing) or on a digitizing table. The resulting cadastral layer can be adequate for many appraisal and planning analysis functions; however, parcel boundary lines do not have bearing and distance attributes and are usually less accurate than lines created with COGO methods. If manual maps with parcel identifiers and bearing and distance annotation have been scanned and georeferenced (as in section 4.2.2), they can be displayed as a background layer to the traced polygons. Beginning at this level of digital cadastral maps, the vector data (such as parcel lines) can be displayed on top of raster images (such as aerial photographs); this is a powerful tool for assessment purposes.

4.2.4. COGO'd Polygons

Metes and bounds on source documents, such as deeds and plats, can be used to create parcels through coordinate geometry methods that process bearing, distance, and curve attributes to describe lines. Such maps can be the most accurate and useful if the survey data is accurate and reliable, and they are designed to retain the input bearings and distances as attributes to the parcel lines. Parcel boundaries defined without bearing and distance information can be created by digitizing or transferring base map lines as described in 4.2.3.

4.2.5 Parcels Created within a Data Model

The most sophisticated digital cadastral map system does not use individual map layers; instead, it uses a data model that defines spatial relationships (topological rules) between different components and layers, creating an integrated suite of layers. An example could be made of a subdivision. First the subdivision exterior is mapped; then blocks are mapped, which must fit within that exterior; and finally lots are mapped, which must fit within the blocks. Rules may also be set for individual parcels, which must close, and cannot have gaps or overlaps unless desired. A publication is available which exhaustively describes the FGDC's parcel data model (von Meyer 2007).

4.3 Technical Specifications

Specifications should be prepared that clearly define technical aspects of the aerial imagery or digital mapping project. They should include such items as:

- Quality and quantity of the mapping products
- Layers to be delivered, and associated data
  - This should include, at a minimum, the map portions of the Cadastral Core Data Set standard of the FGDC (FGDC 2006a): Metadata, Parcel Outline, Parcel Centroid, Parcel Number, and Parcel Area
  - Layer and data field names should follow relevant standards, including the FGDC's Cadastral Data Content Standard (FGDC 2008)
- Positional accuracy requirements
- Geographic areas to be flown or mapped
- Tiling scheme for data (typically township, range, and section, UTM grid cells, or state plane coordinate grid cells)
- Preliminary activities to be performed (e.g., the establishment of horizontal and vertical ground control)
- Map layers to be produced
- Data to be captured as attributes or annotation
- Sources of data to be used
- Topology rules for use in data models
- Procedures for quality control and product acceptance
- Designs for printed products and format for digital map files
- Documentation of processes
- Metadata to be provided

4.4 Pilot Project

Any major mapping or re-mapping program—whether conducted in-house or under contract—should begin with a pilot project. This should focus primarily on representative areas of the jurisdiction, not on the easiest or most difficult to map. Completing a pilot project provides guidance on technical specifications, training needs, fitness of hardware and software, need for outside assistance, program costs, effectiveness of quality control, and work schedule.

4.5 Contracting for Mapping Services

Consideration should be given to whether the new map layers will be prepared in-house or obtained from an outside source. Many assessing offices do not have the expertise or resources necessary to plan for and create digital cadastral maps. In that case, local, regional, state, or provincial agencies may be available to provide assistance. If this is not possible, the jurisdiction must either acquire needed personnel and equipment or contract with a professional mapping firm for the production of map layers. Adequate staff time must be allocated to quality control (see 7.2.). Adequate quality assurance mechanisms must be in place to check the final results. The jurisdiction or assessor should become familiar with accepted contracting procedures, such as those contained in the IAAO *Standard on Contracting for Assessment Services* (2008).

5. Digital Cadastral Map Creation

5.1 Assembling Source Data

The first step in creating a new or revising an existing digital cadastral map layer should be to assemble all relevant information. This includes the following:

- A list of the parcels in the area to be mapped
- Maps of taxing district and municipal boundaries
- Geodetic control network information
- All General Land Office (GLO), Dominion Land Survey, and Bureau of Land Management cadastral survey plats and field notes in areas covered by a township, range, and section system
- Railroad, highway and, utility route surveys
- Subdivision, town site, township, and town plats and surveys
- Private land surveys and associated corner records
- Current orthoimagery of the area
- Deed descriptions for unplatted parcels and for parcels that vary from lot and block boundaries
- Court decisions that affect parcels in the area to be mapped
- Relevant base map data, such as edge of pavement, street and railroad centerlines, water features, and fence and field lines
- Information on rights-of-way, whether dedicated by plat, purchased in fee, vacated, abandoned, or unopened and, if required, held as an easement
- Information on utility easements, if required
- Previous maps on vellum, paper, drafting film and in digital form
- Other imagery of the area, from non-ortho or older photographic sources, especially if it is in paper form and has parcel boundaries
- Highway maps, street name databases, and other sources of information for official names of roads
- Sources of geographic names
  - National map series topographic maps, such as U.S. Geological Survey topographic quadrangles
  - Geographic name databases (for example, the U.S. Geographic Names Information System)

5.2 Mapping Parcels in the COGO and Data Model Environments

COGO-based mapping should employ the sources listed in section 5.1. Creation of the digital cadastral map layer should proceed in the following general sequence:

- The geodetic control layer
- The township/range/section, UTM grid, municipal boundary, or other tiling framework
- Well-surveyed linear features affecting large areas, such as highways, railroads, or canals
- The largest and best-surveyed areas of parcels, such as subdivisions and townsites
- Parcels with good quality metes and bounds descriptions
- Those parcels whose boundaries must be traced or transferred from base map elements, such as creeks and fences

If base map data are insufficient for tracing, there should be field checks and discussions with owners to establish agreed-upon boundaries.

In the data model environment, a similar procedure is followed, moving from larger areas to smaller, following established topology rules.

5.3 Problem Resolution

Digital cadastral map layers commonly contain areas in which individual parcels or groups of parcels have gaps, overlaps, or closure errors. Decisions on resolving such problems should be made with great care, based on the following:

- Knowledge of mapping and boundary law, such as principles of junior and senior rights, water boundaries, and adverse possession
- Knowledge of surveying techniques and technology, such as the need to rotate descriptions to a common basis of bearing

- Knowledge of land division systems affecting the jurisdiction, such as the evolving PLSS and/or Spanish, French, Dutch, or English colonial practices (Price 1995) in North America.
- Understanding of the capabilities and limitations of the software being employed, such as the ability to snap, extend, trim, generalize, adjust closure by compass rule, and use fuzzy tolerances
- Good judgment and common sense

The goal should be to produce a final cadastral map layer with seamless, clean polygons without gaps or overlaps. It may be necessary to work with property owners, attorneys, private surveyors, and county surveyors to resolve problems and achieve this goal.

Decisions on problem areas should be well documented. This may be done on worksheets (which should be filed and preserved) or in text files. The best practice for documentation would be annotations or memo fields attached to points, lines, or areas on the map.

6. Mapping System Maintenance

Digital cadastral map layers and ownership databases should be maintained on a continuous basis by qualified personnel. Map and ownership data represent a substantial capital investment, which can be lost unless all changes and corrections are made on a regular basis. The Bureau of Land Management (BLM) has developed a complete system for maintaining digital cadastral map layers and associated ownership databases. Its National Integrated Land System (NILS) (BLM 2001) is based on a work flow that can be adapted for use in local government cadastral programs.

6.1 Ownership Maintenance

Maintenance of ownership databases involves the following:

- Collecting all relevant deeds, contracts, plats, court cases, owner requests, and other muniments of title
- Identifying the parcels these documents affect
- Determining whether the documents have any effect, are simple ownership changes, or require changing parcel boundaries through splits, combinations, property line adjustments, new subdivisions, or map edits
- Interacting with property owners, surveyors, attorneys, title insurance staff, and other land information professionals to resolve problems when necessary
- Entering the changes in appropriate databases
- Controlling quality of the data (see section 7.2.)

6.2 Cadastral Layer Maintenance

Maintenance of the digital cadastral map layer involves the following

- Obtaining information about needed changes (through the processes in section 6.1)
- Making required changes of parcel lines, parcel identifiers, and associated data
- Performing quality control (see section 7.2.)
- Distributing map information to appropriate parties
- Constantly correcting and improving the cadastral layer when new and more accurate data become available
- If needed, remapping areas or map tiles at a higher scale or with greater accuracy

6.3 Backing Up Data

For computerized map and ownership data, a back-up copy should be made at the end of each workday and periodically moved to be stored at a remote site.

7. Quality Control

In both creation and maintenance of digital cadastral maps and ownership databases, accuracy must be ensured through adequate quality control.

7.1 Map Accuracy

Digital cadastral map layers should be tested for spatial accuracy, and the results should be documented in metadata. Map accuracy is typically expressed in one of three ways.

- The National Map Accuracy Standard (NMAS) (U.S. Bureau of the Budget 1947) for large-scale maps typically requires that 90% of all well defined points on a printed map should vary no more than 1/30 of an inch from their true location. Thus, if a map is drawn or compiled at a scale of one inch equals 100 feet, then an easily identified point on the ground should be within 3.33 feet of its true location. The NMAS is most appropriate for paper maps which are only viewed at the printed scale. This standard would only be applicable to the digital mapping environment if accuracy was described for a particular map scale (for example, "This map layer meets NMAS at a scale of one inch equals 100 feet").
- The American Society of Photogrammetry and Remote Sensing (ASPRS 1990) developed standards that define three classes of positional accuracy, based on limiting root mean square error. The quality standard is based on full (ground) scale and is well suited to large scale base maps prepared through digital orthoimagery.

- The National Standard for Spatial Data Accuracy (NSSDA) (FGDC 1998b) presents a rigorous statistical methodology to evaluate the positional error observed when a sample of well defined map points varies from their true geospatial location. However, the standard does not provide positional accuracy thresholds; it merely provides a way of describing the accuracy of a digital map.

A major problem with any cadastral map, manual or digital, is that positional accuracy tends to vary within a single map layer. In the township, range, and section environment, parcels close to a section corner tend to be mapped more accurately than parcels in the center of a section. In the metes and bounds environment, a new subdivision's parcels will be very accurate, but nearby parcels described by fields, fences, creeks, and roads may be very inaccurate. Thus, while accuracy should be field tested and documented in metadata, accuracy measures must be used judiciously; their greatest value may be in pointing to areas where additional survey work or map effort should be employed.

One last consideration is that no one accuracy standard meets all needs. In an urban environment, accuracies of one foot or less (0.25 meters) may be necessary. In rural areas, it may be sufficient to specify an accuracy of +/– 8 feet (2 meters).

7.2 Quality Control

Quality control is a vital process in both map creation and maintenance. Software should be designed and configured with built-in testing for data integrity and validity. For mapping purposes, this is easier in the parcel data model environment.

In map creation, tests should be conducted and queries performed to ensure that all relevant documents (Section 5.1.) are reflected in parcel boundary layers. Queries should be run to ensure that all parcels in tabular databases are found in the digital cadastral map layer, and vice versa. Polygon parcel layers should be viewed with orthoimagery and older scanned maps in the background, to visually inspect for misregistration. Parcel area attributes on the digital cadastral map layer should be compared to areas in tabular databases, for staff review of significant differences.

In map and ownership maintenance, tests should be conducted and queries performed to ensure that all relevant documents (Section 6.1.) have been gathered and properly processed, with correct ownership and map changes made. Near-perfect correlation must be maintained between parcels in tabular databases and parcels in digital map layers.

8. Parcel Identifiers

The greatest assessment use of a digital cadastral mapping system is not just maintaining parcel and ownership information, but using it in the appraisal process. To do so, parcels in a digital cadastral map layer must be linked to assessment data such as year built, square feet, sale price, sale date, etc. The key link between parcels and tabular data is the parcel identifier (also referred to as the parcel identification number, PIN, or parcel ID). A PIN uses a number or code instead of a complete legal description to uniquely identify one parcel.

The jurisdiction's tax parcel number should be legally defined and recognized as the official reference to all documents or data for each parcel. All jurisdictions in a state or province should use the same primary system of parcel identification. Because agencies have different needs, various secondary identifiers also may be used to index parcel data; however, all of the secondary identifiers must be cross-indexed to the legally recognized, unique tax parcel identifier (National Research Council 1983, 63).

8.1 Desirable Characteristics

Many types of parcel identifier are in use. A PIN, in use or proposed, should be judged based on six attributes.

8.1.1 Compliance with Standards

If a state, regional, or local parcel identifier format has been adopted, a jurisdiction should follow it. In addition, various national PIN formats have been proposed (PRIA 2003), but not yet mandated.

It is likely that, at least initially, a "national parcel number" would simply add an appropriate Federal information processing standards (FIPS) code, developed by the National Institute of Standards and Technology, to the front of each jurisdiction's existing PINs (National Academy of Science 2007). In 1995, The Federal Geographic Data Committee (FGDC) Cadastral Subcommittee developed a *Cadastral Data Content Standard* (2003) that identifies core parcel data useful to many stakeholders and suggests that this information be captured and maintained by assessors. The core data elements are described in Appendix A.

8.1.2 Uniqueness

Uniqueness is the most important attribute of a PIN. It refers to a one-to-one relationship between a parcel and its identifier. An identifier should be assigned to one and only one parcel.

8.1.3 Permanence

Parcel identifiers should be permanent and change only when absolutely necessary. This is especially important when stakeholders such as planning departments or tax payment tracking services link their own databases to the assessor's.

8.1.4 Simplicity and Ease of Use

Parcel identifiers should be easy to understand and use and have as few digits as possible. A parcel identifier that is uncomplicated and easily understood helps to reduce errors in its use.

8.1.5 Ease of Maintenance

The parcel identification system should be easy to maintain and should efficiently accommodate changes, such as subdividing or consolidating parcels.

8.1.6 Flexibility

The parcel identification system should be reasonably flexible. It should be capable of serving a variety of uses: not just land parcels, but multi-story condominiums, subsurface rights, leases, easements, and so on.

8.2 Kinds of Parcel Identifiers

There are five basic types of parcel identifiers, described as follows. The first two types, which incorporate clues to a parcel's geographic location, are recommended for assessment purposes.

8.2.1 Geographic Coordinate Systems

The geographic coordinate system is a method of locating a point on the Earth's surface based on its distance from each of two intersecting grid lines known as x and y axes. These grid lines can be based on latitude and longitude, the Universal Transverse Mercator (UTM) system, or state plane coordinates. Parcel identifiers using this system are composed of the coordinates for a single point, usually the parcel centroid.

Parcel identifier systems based on geographic coordinates are easy to maintain, because new numbers are quickly assigned by picking parcel centroids. They are easy to use in the field, because the PIN can help locate the parcel when using a GPS. Such PINs also meet the criteria of uniqueness and permanence.

However, geographic coordinate-based PIN's may not meet the criteria of simplicity, because a complete parcel identifier could be very long. This is partially due to the need to include not just x and y coordinates, but a z (elevation) coordinate as well. This is required for multi-story condominiums and apartments, where parcels at different levels could have the same x-y centroid. The elevation problem could also extend to sub-surface parcels, such as underground parking or mineral rights.

8.2.2 Rectangular Survey System

This system of parcel numbering is based on township/range/section systems such as the United States PLSS. Parcel identifiers based on a rectangular survey system are developed by using the township, range, section, quarter-section, and quarter-quarter-section numbers, along with individual parcel identifiers assigned to each tract. This kind of PIN provides an approximate geographic location of each parcel, is relatively easy to understand and maintain, and meets the criteria of uniqueness and permanence. However, it could never be the basis of a national parcel numbering system, because many areas do not use a township/range/section system.

8.2.3 Assessors' Map-based Systems

A map-based system is relatively simple and easy to use. Under this system, the assessment map itself is incorporated into the parcel identifier. The parcel identifier consists of a map (or page), block (or group), and parcel number such as 32–02–16, where 32 represents the map on which the parcel is found, 02 indicates the block on the map, and 16 identifies the parcel in that block. Map-based identifiers may, to some extent, reference a geographic area and are convenient for use with printed maps in the field. However, they have limited usefulness in the digital cadastral mapping environment, where the goal is one seamless map of the entire jurisdiction, rather than individual sheets.

8.2.4 Name-related Identifiers

Name-related identifiers use the names of individuals claiming an interest to a parcel as the parcel identifier. A common example of this is the use of name codes in the grantor–grantee index. Use of such identifiers is discouraged because they do not meet the criteria of permanence and reference to geographic location.

8.1.3 Alphanumeric Identifiers

An alphanumeric code is often an arbitrary number associated with the parcel. An example is the sequential numbering system in a tract index. This may have advantages of permanence and ease of maintenance (new parcels are simply assigned the next available number). However, ease of use is limited, as adjoining parcels could have wildly different PINs.

8.3 Assignment of Parcel Identifiers

Parcel identifiers should be assigned to all parcels, whether taxable or exempt, during the initial phase of a digital cadastral mapping program. These PINs should be considered provisional until the mapping program has been completed and all maps have been formally approved.

Maintenance of parcel identifiers should be done on an ongoing basis by a single agency as new parcels are created. This raises of the question of assigning PINs to an existing ("parent") parcel which has been divided ("split") into two or more ("child") parcels. Some jurisdictions retain the original PIN of the parent, and only assign a new number to each new child. However, since the parent parcel's boundaries are now changed, the parent should also be assigned a new PIN, and its original PIN should be retired. In this case, the parent's new parcel identifier puts stakeholders on notice that it has changed in some way.

Glossary

This glossary defines mapping terms used in this standard and its appendices and other commonly used mapping expressions. Some of these definitions were compiled from the textbook, *Definitions of Surveying and Associated Terms* (ACSM 2005), and are used with permission of the publisher.

Assessment map. (See *cadastral map.)*

Base map. A map containing the background upon which geographic data is overlaid. Contains basic survey control and reference framework for integrating all of the other map features of a particular area. Orthophotos are commonly used as a cadastral base map.

Bearing. Direction of a line measured from north or south to east or west, not exceeding 90 degrees.

CAD. Computer aided design. A digital software technology used for the design, drafting and presentation of graphics. It is commonly employed in drafting work for engineering and manufacturing, and may also be used to design maps.

Cadastral map. A map showing the boundaries of subdivisions of land, usually with the bearings and lengths thereof and the areas of the individual tracts, for the purposes of describing and recording ownership. A cadastral map may also show culture, drainage, and other features relating to the value and use of the land.

Compilation. (1) Cartography: The production of a new or revised map from an existing map, aerial photograph, survey, or other source material (see *delineation*). (2) Photogrammetry: The production of a map or chart, or portion thereof, from aerial photographs and geodetic control data, by means of photogrammetric instruments, also called stereocompilation.

Contour line. A line drawn on a topographic map connecting points with equal terrain surface elevation.

Control (ground and geodetic). A system of points which are used as fixed references of position (horizontal) or elevation (vertical) or both. Ground control are points obtained from ground surveys. These points can be used to rectify the accuracy of cartographic products to the actual area on the ground that is represented. Geodetic control takes the size and shape of the earth into consideration.

Coordinates. Linear or angular quantities that designate the position of a point in a given reference frame or system. The x and y values, or three-dimensional x, y & z values that define a location in a planar or three-dimensional coordinate system.

Data model. A generalized, user-defined view of data representing the real world. A description of the structure of a database. It describes how data is represented and accessed.

Delineation. The visual selection and distinguishing of map worthy features by outlining, or on a map manuscript (as when operating a steroplotting instrument); also, a preliminary step in compilation.

DEM Digital Elevation Model. A digital representation of bare-earth elevations (z values) that is referenced to a common datum. Digital elevation models are typically used to represent terrain relief without vegetation, buildings or improvements.

DTM Digital Terrain Model. A digital representation of the Earth's surface. Its construction includes a basic elevation model (i.e. a DEM) that is typically enhanced with break line data to accentuate abrupt changes terrain features features, such as pavement edges, road crowns, riverbanks, ridgelines, creek beds, etc.

Feature. Points, symbols, lines, and areas on a map representing natural and man-made geographic features. An object in a geographic or spatial database with a distinct set of characteristics.

Geocode. A code (usually numerical) used to locate or identify a point on a map, such as the center of a parcel.

Geodetic coordinates. The quantities of latitude and longitude that defines the position of a point on the surface of the earth with respect to the reference spheroid, frame or system. (See also *coordinates*.)

Government lot. A partial section of land established, measured and computed by the U.S. Government's survey of the public lands. Often used synonymously with "fractional lot" or 'fractional section".

Grid. A uniform system of rectilinear lines superimposed on an aerial photograph, map, chart, or other representation of the earth's surface; used in defining the coordinate positions of points.

Index map. (1) A map of smaller scale on which are depicted the locations (with accompanying designations) of specific data, such as larger-scale topographic quadrangles or geodetic control. (2) Photogrammetry: A map showing the location and numbers of flight strips and frame images.

Land information system. A system for capturing, retaining, checking, integrating, manipulating, analyzing and displaying data about land and its use, ownership and development.

Layer. Set of related geographic features, such as streets, parcels, or rivers, and the attributes (associated characteristics of those features) logically organized into groups that can be displayed independently.

LiDAR. Light detection and ranging. A remote sensing tool for generating very accurate digital surface models. It uses an aircraft mounted sensor that emits rapid pulses of infrared laser light to determine ranges to points on the terrain below. The point data may be used to construct a digital surface model (DSM), digital elevation model (DEM), or digital terrain model (DTM.).

Lot. A plot of land, generally a subdivision of a city, town, or village block, or some other distinct tract, represented and identified by a recorded plat.

Map. A representation (usually presented on a two dimensional medium) of all or a portion of the earth or other celestial body, showing relative size and position of features to some given scale or projection. A map is a model that may emphasize, generalize or omit the representation of certain features to satisfy specific user requirements.

Map projection. An orderly system (mathematical model) to portray all or part of the earth, which is an irregular sphere, on a planar or flat surface. Some distortions of conformality, distance, direction, scale, and area will always result from this fitting process. Examples include the Mercator and the Lambert Conic Conformal Map Projection to name a few.

Monument A permanent physical structure marking the location of a survey point or boundary line. Common types of monuments are inscribed metal tablets set in concrete post, solid rock or parts of buildings; distinctive tone posts; and metal rods driven in the ground.

Orthophotograph. A photograph having the properties of an orthographic projection. It is derived from a conventional perspective vertical photograph (for mapping purposes) by simple or differential rectification so that image distortions caused by camera tilt and relief of terrain are removed.

Overlay. A map recorded on a transparent medium that may be superimposed on another record; for example, maps showing original land grants (or patents) prepared as tracing cloth overlays so that they can be correlated with maps showing the present ownership. Also, any of the several overlays that may be prepared in compiling a manuscript map; usually described by name, for example, lettering overlay.

Parcel. A single, discrete piece of land having defined physical boundaries and capable of being separately conveyed.

Photo delineation. The selection and identification of mapworthy features on a photograph or digital image.

Photogrammetry. The art, science and technology of obtaining reliable information about physical objects and the environment through processes of recording, measuring and interpreting images and patterns of electromagnetic radiant energy. (See also orthophotography)

Plane rectangular coordinates. A system of coordinates in a horizontal plane used to describe the positions of points with respect to an arbitrary origin by means of two distances perpendicular to each other. (See also *coordinates.*)

Planimetric map. A map that presents only the horizontal positions for the features represented; distinguished from a topographic map by the omission of relief in measurable form.

Plat. A diagram drawn to scale showing all essential data pertaining to the boundaries and subdivisions of a tract of land, as determined by survey or protraction.

PLSS. Public Land Survey System. A rectangular survey system used in much of the United States dividing land areas into townships of 36 one-square mile sections. Sections can be further subdivided into quarter sections, quarter-quarter sections, or irregular government lots.

Point. Single x, y (optionally z) location points in space. Dimensionless geometric feature having no other spatial properties except location. Many different natural and man-made features are modeled as points in a spatial database including trees, hydrants, poles, buildings, parcel centroids etc.

Positional accuracy. The degree to which the coordinates define a point's true position on the earth's surface.

Rectification. The process of projecting the image of a tilted aerial photograph onto a horizontal reference plane to eliminate the image displacement caused by tile of the aerial camera at the time of exposure.

Remote sensing. The process of obtaining information about an object while physically separated from it. Practically, this is a term used to describe the process of using sensors mounted on satellites to capture images and to observe the Earth's geology, surface and atmosphere.

Resolution. (Spatial Resolution). 1) The minimum distance between two adjacent ground features that can be detected by remote sensing. 2) The smallest possible map feature that can be accurately displayed at a specified map scale.

Scanning. Capturing an image using an optical or video input device that uses light sensing technology. A process by which photographs, printed data, or drawn maps are converted to a digital format.

Spatial. Relating to space or a space. Refers to the shapes, location, proximity, and orientation of objects with respect to one another in space.

State plane coordinate systems. A series of grid coordinate systems prepared y the U.S. Coast and Geodetic Survey for the entire United States, with a separate system for each state. Each state system consists of one or more zones. The grid coordinates for each zone are based on, and mathematically adjusted to, a map projection. (See also *coordinates*.)

Topology. A set of defined relationships between links, nodes, and centroids. Topology describes how lines and polygons connect and relate to each other. Among the topological properties of concern in GIS are connectivity, order and neighborhood.

Vector. Vector data is the storage of X, Y, Z coordinates connected to form points, lines, areas, and volumes. A vector can be a straight line joining two data points.

References

American Congress on Surveying and Mapping. 2005. *Definitions of Surveying and Associated Terms.* Gaithersburg, MD: American Congress on Surveying and Mapping.

Bureau of Land Management. 2001. National Integrated Land System (NILS).

Federal Geographic Data Committee. May, 2003. *Cadastral data content standard version 1.3.* Washington, D.C.: Federal Geographic Data Committee [http://www.nationalcad.org/data/documents/CADSTAND.v.1.3.pdf]

Federal Geographic Data Committee. 1998a. *Content standard for digital geospatial metadata.* Washington, D.C.: Federal Geographic Data Committee.

Federal Geographic Data Committee. 1998b. *National standard for geospatial positioning accuracy, Part 3: National standard for spatial data accuracy (NSSDA).* Washington, DC: Federal Geographic Data Committee.

Federal Geographic Data Committee. 2008. *Cadastral data content standard for national spatial data infrastructure.* Washington, DC: Federal Geographic Data Committee.

IAAO. 2008. *Standard on contracting for assessment services.* Kansas City: IAAO.

IAAO. 2003. *Standard on facilities, computers, equipment, and supplies.* Kansas City: IAAO.

IAAO. 2004. *Standard on manual cadastral maps and parcel identifiers.* Kansas City: IAAO.

National Academy of Science. 2007. *National land parcel data: A vision for the future.* Washington, DC: The National Academies Press.

National Research Council. 1983. *Procedures and standards for a multipurpose cadastre.* Washington, DC: National Academy Press.

Price, E.T. 1995. *Dividing the land: Early American beginnings of our private property mosaic.* Chicago: University of Chicago Press.

URISA/IAAO. 1999. *GIS guidelines for assessors*, Second Edition, Chicago, URISA/IAAO.

U.S. Geological Survey. 1986. *Large-scale mapping guidelines. Open file report 86-005.* Reston, VA: U.S. Geological Survey.

Selected Bibliography

Arnoff, S., 2005. *Remote sensing for GIS managers.* Redlands, California: ESRI Press.

Bluestein, S.. 2005. A self-evaluation guide to digital parcel maps and parcel identifiers. *Fair & Equitable.* 3(1):3-6.

Brown, P.M., and D.D. Moyer, eds. 1989. *Multipurpose land information systems: The guidebook.* Rockville, MD: FGDC.

Deck, J. and M. Tsui. 1999. *Aerial imagery guidelines.* Park Ridge, Illinois: The Urban and Regional Information Systems Association.

Donatello, G. 2009. Everyone wants assessors, parcel data—federal, state, local and private: An overview of parcel data issues. *Fair & Equitable.* 7(2):3-7.

Ghilani, C.D. and P.R. Wolf. 2008. *Elementary Surveying: an introduction to geomatics.* 12th ed. Upper Saddle River, New Jersey: Prentice-Hall

Federal Geographic Data Committee. 2008. *Cadastral data content standard for the national spatial data infrastructure.* Reston, Virginia.

Federal Geographic Data Committee. 2007. *Cadastral NSDI reference document.* Reston, Virginia.

Federal Geographic Data Committee. 2002. *Content standards for digital geospatial metadata: extensions for remote sensing metadata.* Reston, Virginia.

Federal Geographic Data Committee. 1999. *Content standards for digital orthoimagery.* Reston, Virginia.

Federal Geographic Data Committee. 2008. *Geographic information framework data content standard.* Reston, Virginia.

Federal Geodetic Control Committee. 1989. *Multipurpose land information systems: the guidebook.* Washington, DC: National Ocean Survey.

International Association of Assessing Officers. 1990. *Property appraisal and assessment administration.* Chicago.

International Association of Assessing Officers and Urban and Regional Information Systems Association. 1999. *GIS guidelines for assessors second edition.* Park Ridge, Illinois: The Urban and Regional Information Systems Association

Kennedy, M. and S. Koop. 2000. *Understanding map projections.* Redlands, California: ESRI Press.

Moyer, D.D. 1980. *A multipurpose land data system: Structure, content and capabilities.* Proceedings of the Eighteenth Annual Conference of the Urban and Regional

Information Systems Association (URISA). Toronto.

National Research Council. 1980. *Need for a multipurpose cadastre*. Washington, DC: National Academy Press.

National Research Council. 2007. *Land parcel database: a vision of the future*. Washington, DC: National Academy Press.

Robinson, A., R. Sale, and J. Morrison. 1978. *Elements of cartography.* 4th ed. New York: John Wiley & Sons.

McGlone, J.C.. 2004. *Manual of photogrammetry.* 5th ed. Bethesda, Maryland: American Society of Photogrammetry.

National Digital Elevation Program. 2004. *Guidelines for digital elevation data v 1.0.*

Parrish, K. 1999. *Digital parcel mapping handbook.* Park Ridge, Illinois: The Urban and Regional Information Systems Association

Parr, D. 2000. *GIS glossary of terms*. Park Ridge, Illinois: The Urban and Regional Information Systems Association

Raber, B.R. and J. Cannistra. 2005. *LIDAR guidebook: concepts, project design, and practical applications.* Park Ridge, Illinois: The Urban and Regional Information Systems Association

Somers, R. 2001. *Quick guide to GIS development.* Park Ridge, Illinois: The Urban and Regional Information Systems Association. Quick guide to GIS Implementation and Management

U.S. Geological Survey. 1986. *Large-scale mapping guidelines.* Open File Report 86-005. Reston, Virginia: U.S. Geological Survey.

von Meyer, N. 2004. *GIS and land records.* Redlands, California: ESRI Press.

von Meyer, N. 2002. *Surveying and land records glossary of terms.* Park Ridge, Illinois: The Urban and Regional Information Systems Association.

von Meyer, N. 2009. Sharing the data you have—Getting the data you need. *Fair & Equitable.* 7(2): 9–12.

Wade, T. and S. Sommer. 2006. *A to Z GIS: An illustrated dictionary of geographic information systems.* Redlands, California: ESRI Press.

Wolf, P.R. 2000. *Elements of photogrammetry with applications in GIS.* New York: McGraw-Hill.

Appendix. Core Parcel Data Elements

Land record/GIS integration makes strong economic and business sense. With integration, an assessing jurisdiction's land records, GIS, tax/assessment/valuation and addressing data are more valuable together! The collective records are much more useful to more people. With key record cross-referencing beginning before instrument recordation, Land Record/GIS integration begins near the start of many key workflow processes. To allow for such integration, assessors should consider capturing Parcel Core Data to the extent practical.

Parcel Core Data describes the minimum set of attributes about land parcels and associated reference data that can provide essential information to meet business needs without publishing the complete set of parcel characteristics.

Background

The Federal Geographic Data Committee (FGDC) Cadastral Subcommittee completed the Cadastral Data Content Standard1 in 1995. [ADD language referring to core date listed below] This information was published in the May, 2003 FGDC Cadastral Data Content Standard version 1.3. (http://www.nationalcad.org/data/documents/CADSTAND.v.1.3.pdf)

In 2000 the Subcommittee began a series of studies on the uses for and applications of cadastral data. Many business needs were studied , includinghurricane and wildland fire response, energy management needs, uses of parcel data by federal agencies and most recently mortgage and real estate analysis. From this body of work the Subcommittee defined a limited set of attributes (core parcel data) that provide a platform supporting multiple business needs.

It is important to recognize that publication data are not the same as operation and maintenance data or production data. Production data are structured to optimize maintenance processes are integrated with internal agency operations, and contain much more detail than publication data. Publication data are a subset of the more complete production data and are intended to be integrated across jurisdictional boundaries and be presented in a consistent and standard form nationally. To the extent that assessors consistently capture and make available Parcel Core Data, this goal will be attainable.

Parcel Core Data provides a platform thatrecognizes a basis upon which many other themes and data sets are referenced. For example land parcel data could be used to spatially enable business license information, voter registration or health statistic information.

The parcel core data speaks to the standardization of the small, but most crucial set of attributes. Jurisdictions may expand upon the minimum set and some applications may need additional attributes, but having a short list of standardized attributes should make linkage or other data sets possible and allow for the expansion and individualization of published data.

There are two other important notes about the cadastral data platform. Parcel data changes frequently and needs to be updated regularly. Many of the initial needs of the business applications studied can be met with annual parcel updates, but in the end all business applications need current data. Therefore, even though assessors' records may be subject to updating on an annual cycle to accommodate property tax needs, unlike many other spatial framework data sets, cadastral information to be used to satisfy multiple business needs should be continually updated. The second note is that all spatial data should have accompanying metadata describing the source agency, contact information, spatial referencing and accuracy and currency.

The following is the list of attributes defined in the core data set. [This list was developed by FGDC and assessors should try to capture this information and make this core data set available.] In the physical file structure the address elements are defined as individual components and as a single concatenated field.

Metadata—The metadata will contain information about the entire data set such as the data steward, the parcel contact, a description of the basis for the assessment system (sale price, use, market value etc), the date of the file, information on interpretation of the assessment classifications and any other metadata that would support the use and application of the information.

Parcel Outline (Polygon)—This is geographic extent of the parcel, the parcel boundaries forming a closed polygon. The parcel geometry may be a polygon or a point. The Parcel centroid and the polygon are not both required.

Parcel Centroid (Point) - This is a point within the parcel that can be used to attach related information. This may be a visual centroid or a point within the parcel. It may not be the mathematical centroid as this point needs to be contained within the parcel polygon.

Parcel ID—A unique identifier for the parcel as defined by the data steward or data producer. The parcel identifier should provide a link to additional information about the parcel and should be unique across the data steward's geographic extent.

National Parcel ID—This is a nationally unique identified constructed from either the GNIS code for the jurisdiction or the Census codes plus the local identifier.

Source Reference—This field is often called the Volume/Page or Liber/Page in local records. This is a pointer to, or an attribute describing, the source reference for the parcel.

This could be a deed, plat, or other document reference.

Source Reference Date—The date of the Source Reference, which is essentially the last update date for this parcel. The entire data set may have a last updated date or an "unloaded for publication" date that is different than the specific currency or update date for each individual parcel.

Owner Type—The type of ownership is the classification of owner. In some local governments tax parcels are tagged as either taxable or exempt and the owner classification is not known.

Improved—This is an attribute to indicate whether or not there is an improvement on the parcel.

Owner Name—An indication of the primary owner name, recognizing that there may be multiple owner names or that some owner names may be blocked for security reasons or that some jurisdictions may not allow the distribution of owner names. For publicly held lands the owner name is the surface managing agency, such a Bureau of Land Management, Department of Transportation, etc.

Assessment/Value for Land Information—This is the total value of the land only. The basis of the value, such as market value, resale value, sale price or use value should be described in the metadata.

Assessment/Value for Improvements Information—This is the total value of improvements on the parcel. The basis of the value, such as market value, resale value, sale price or use value should be described in the metadata.

Assessment/Value Total—This information is the total value of the land and improvements. The basis of the value, such as market value, resale value, sale price or use value should be described in the metadata.

Basis of the Values—An indication of the type of values that are provided (taxable, market, assessed or other). This may be included in the metadata if it is the same for all of the records in a data set.

Assessment Parcel Use Code—This is the parcel use classification for the tax parcel based on the classification of the parcel for the purposes of valuation.

Tax Bill Mailing Address—This is the US Postal Service address for the tax bill mailing.

Site Address—This is the street address (site address) for the parcel. If there is more than one, select the first or primary site address.

Parcel Area—The area of the parcel expressed in acres.

Reference

Cadastral Subcommittee. 2003. *FGDC Cadastral Data Content Standard version 1.3*, May 2003, http://www.nationalcad.org/data/documents/CADSTAND.v.1.3.pdf (accessed January 27, 2012).

Assessment Standards of the International Association of Assessing Officers

Guide to Assessment Administration Standards

Standard on Assessment Appeal

Standard on Automated Valuation Models

Standard on Contracting for Assessment Services

Standard on Digital Cadastral Maps and Parcel Identifiers

Standard on Facilities, Computers, Equipment, and Supplies

Standard on Manual Cadastral Maps and Parcel Identifiers

Standard on Mass Appraisal of Real Property

Standard on Oversight Agency Responsibilities

Standard on Professional Development

Standard on Property Tax Policy

Standard on Public Relations

Standard on Ratio Studies

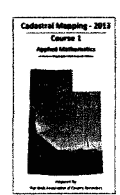
Standard on Valuation of Personal Property

Standard on Valuation of Properties Affected by Environmental Contamination

Standard on Verification and Adjustment of Sales

APPENDIX 3

Engineering Standard Lettering Height Guides



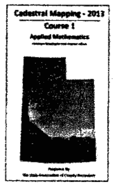
ENGINEERING SCALES

THIS CHART IS USED WHEN ONE INCREMENT EQUALS ONE FOOT

SCALE	PLOT SCALE	CONVERSION FACTOR	1/16" TEXT SIZE	3/32" TEXT SIZE	1/8" TEXT SIZE	5/32" TEXT SIZE	ARROWHEAD SIZE	7/16" REFERENC CIRCLE	9/16" REFERENCE CIRCLE
1"=1'-0"	1	0	0.0625	0.0938	0.125	0.1563	0.0938	0.4375	0.5625
2"=1'-0"	2	0.5	0.125	0.1875	0.25	0.3125	0.1875	0.875	1.125
3"=1'-0"	3	0.33	0.1875	0.2813	0.375	0.4688	0.2813	1.3125	1.6875
4"=1'-0"	4	0.25	0.25	0.375	0.5	0.625	0.375	1.75	2.25
5"=1'-0"	5	0.2	0.3125	0.4688	0.625	0.7813	0.4688	2.1875	2.8125
6"=1'-0"	6	0.1667	0.375	0.5625	0.75	0.9375	0.5625	2.625	3.375
8"=1'-0"	8	0.125	0.5	0.75	1	1.25	0.75	3.5	4.5
1"=10'-0"	10	0.1	0.625	0.9375	1.25	1.5625	0.9375	4.375	5.625
1"=12'-0"	12	0.083	0.75	1.125	1.5	1.875	1.125	5.25	6.75
1"=15'-0"	15	0.0667	0.9375	1.4063	1.875	2.3438	1.4063	6.5625	8.4375
1"=20'-0"	20	0.05	1.25	1.875	2.5	3.125	1.875	8.75	11.25
1"=30'-0"	30	0.033	1.875	2.1825	3.75	4.6875	2.1825	13.125	16.875
1"=40'-0"	40	0.025	2.5	3.75	5	6.25	3.75	17.5	22.5
1"=50'-0"	50	0.02	3.125	4.6875	6.25	7.8125	4.6875	21.875	28.125
1"=60'-0"	60	0.0167	3.75	5.625	7.5	9.375	5.625	26.25	33.75
1"=80'-0"	80	0.0125	5	7.5	10	12.5	7.5	35	45
1"=100'-0"	100	0.01	6.25	9.375	12.5	15.625	9.375	43.75	56.25
1"=120'-0"	120	0.0083	7.5	11.25	15	18.75	11.25	52.5	67.5
1"=150'-0"	150	0.0067	9.375	14.0625	18.75	23.4375	14.0625	65.625	84.375
1"=180'-0"	180	0.0056	11.25	16.875	22.5	28.125	16.875	78.75	101.25
1"=200'-0"	200	0.005	12.5	18.75	25	31.25	18.75	87.5	112.5
1"=300'-0"	300	0.0033	18.75	28.125	37.5	46.875	28.125	131.25	168.75
1"=400'-0"	400	0.0025	25	37.5	50	62.5	37.5	175	225
1"=500'-0"	500	0.002	31.25	46.875	62.5	78.125	46.875	218.75	281.25
1"=600'-0"	600	0.0017	37.5	56.25	75	93.75	56.25	262.5	337.5
1"=800'-0"	800	0.0013	50	75	100	125	75	350	450
1"=1000-0"	1000	0.001	62.5	93.75	125	156.25	93.75	437.5	562.5
1"=2000'-0"	2000	0.0005	125	187.5	250	312.5	187.5	875	1125
1"=2400'-0"	2400	0.0004	150	225	300	375	225	1050	1350

APPENDIX 4

Conversion Factors



APPENDIX 4 - CONVERSION FACTORS AND CONSTANTS:

Imperial Units:

1 US Survey Foot (US ft)	= 12 Inches
	1.64141 Links
	0.060606 Rod
	0.015152 Chain
	3.0480060 Decimeters
	0.3048006096 meter (m)
	1200/3937 meters exactly
1 International Foot (SI ft)	= 0.3048 m exactly
1 Link (Gunter or Surveyor)	= 7.92 Inches
	0.66 US ft
	0.04 Rod
	2.011684 Decimeters
	0.001 Furlong
1 Rod, 1 Pole, 1 Perch (lineal)	= 198 Inches
	16.5 US ft
	25 Links
	0.25 Chain
	5.0292099 Meters
	0.025 Furlong

Note: The term Perch is also used in square measure.

1 Rope	= 240 Inches
	20 US Ft
	6.0960120 meters
1 chain	= 100 Links
	4 Rods
	66 US ft
	0.0125 Statute mile
	2.011684 Decameters
1 Statute Mile (mi)	= 5,280 US Survey Feet
	8,000 Links
	320 Rods
	80 Chains
	1.6093472 Kilometers
	1.12500 Scottish mile
	0.868978 Meridian (French Nautical) mile
	0.868976 International nautical (or international air) mile
	0.868383 Geographic (Nautical, Knot, Air or Sea) mile

1 degree	= 1 / 360th of a circle
1 radian	= 180° / π
1 grad	= 0.9 degree or 1 / 400th of a circle
1Vara	= 33.3333 Inches (Approx.) 2.7778 Feet (approx.) 8.4666667 Decimeters (Approx.)
1 acre	= 43560 US ft^2 160 perches 10 square chains 4 roods 1.17869 Arpents 0.786990 Scottish acre 0.617347 Irish acre 0.472656 Cheshire acre 0.704225 Yoke (or Joch)
1 Square Chain	= 10,000 Square Links 4,356 US Ft Sq. 16 Perches 0.4 Rood 0.117869 Arpent 0.1 Acre
1 Rood	= 10,890 Square Feet 40 Perches 2.5 Square chains 0.25 Acre
1 Arpent	= 36,956.2176 Sq. US Ft 8.48398 Sq Chains 3.39359 Roods 135.74368 Perches 0.848398 Acre
1 Perch (square measure)	= 0.007366825 Arpent 0.025 Rood

Metric Conversions:

Linear units:

1 meter (m)	= 3.28083333333... U.S. Survey Feet or 39.37 inches exactly (US ft)
1 meter (m)	= 3.28083989501... International Feet (SI ft)

Square units:

1 meter square (m^2)	= 10.7638673611 US survey feet square (US ft^2) or 10.7639104167 International feet square (SI ft^2)
1 kilometer (km^2)	= 247.104393 Acres
1 hectare	= 2.47104393 Acres

Metric Units:

1 millimeter (mm)	= 1000 micrometers (μm) or microns (μ)
1 centimeter (cm)	= 10 millimeter (mm)
1 decimeter (dm)	= 10 centimeter (cm
1 meter (m)	= 10 decimeter (dm) or 100 centimeter (cm)
1 kilometer (km)	= 1000 meter (m)

Approximation of the value of PI (pie) ≈3.141592653589793238462643383279

APPENDIX 5

Definitions